Contents lists available at GrowingScience

International Journal of Industrial Engineering Computations

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Inventory routing problem with backhaul considering returnable transport items collection

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C H R O N I C L E

Article history:
Received March 20 2023
Received in Revised Format
May 24 2023
Accepted June 26 2023
Available online
June, 26 2023

Keywords:
Inventory Routing Problem
Returnable Transport Items
Mathematical Modeling
Closed-Loop Supply Chain
Backhaul

A B S T R A C T

The Inventory Routing Problem (IRP) has been highlighted as a valuable strategy for tackling routing and inventory problems. This paper addresses the IRP but considers the forward delivery and the use of Returnable Transport Items (RTIs) in the distribution strategy. We develop an optimization model by considering inventory routing decisions with RTIs collection (backhaul customers) of a Closed-Loop Supply Chain (CLSC) within a short-term planning horizon. RTIs consider reusable packing materials such as trays, pallets, recyclable boxes, or crates. The RTIs represent an essential asset for many industries worldwide. The solution of the model allows concluding that if RTIs are considered for the distribution process, the relationship between the inventory handling costs of both the final goods and RTIs highly determines the overall performance of the logistics system under study. The obtained results show the efficiency of the proposed optimization scheme for solving the combined IRP with RTIs, which could be applied to different real industrial cases.

1. Introduction

Returnable transport items (RTIs) are an initiative developed in the '90s to move products between destinations in the supply chain, replacing disposable packaging to reduce the environmental impact generated by the inadequate disposal of waste, which makes the distribution of products sustainable (Sarkar et al., 2017). RTIs are used for the internal transport of materials, components, semi-finished products, and finished products' distribution. RTIs include all means of assembling goods for transport, storage, handling, and product protection in the supply chain, which are returned for further usage. The RTIs include returnable pallets and all forms of reusable crates, totes, trays, boxes, roll pallets, roll cages, barrels, trolleys, pallet collars, racks, lids, and refillable liquid or gas containers (Hellström & Johansson, 2010). A logistical challenge appears related to the storage and recovery of these elements. This challenge consists of deciding the period for which the RTIs should stay at the customer's facilities and the return process to the supplier's facilities to consider as available to distribute a new product. RTIs generally represent an essential asset to companies today. A single RTI could cost 10 Euros to thousands of Euros (Hellström & Johansson, 2010). However, the RTI cost generally does not exceed the holding product cost. RTIs often represent a significant capital investment and a considerable cost of holding and transportation. Therefore, it is not usual to leave the RTIs to the customer from a practical perspective. RTI, as its name suggests, should ideally be collected or returned. The RTI is mainly collected for two reasons: a) due to the need to use them for future deliveries, and b) due to the difficulty of leaving them at the customer's location for a long time, mainly when empty. When a certain amount of RTIs remains in a customer's facility, a cost is incurred to manage these inventories, ranging from the cost of the physical space that is occupied to managing it for its custody and protection. Johansson & Hellström (2007) mention that there is no adequate management of RTIs and

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ISSN 1923-2934 (Online) - ISSN 1923-2926 (Print)
2023 Growing Science Ltd.
doi: 10.5267/j.ijiec.2023.6.001

that there is no accounting for these, and even Carrasco-Gallego et al. (2012) showcase loss of RTIs, which are up to 10%. As Glock & Kim (2014) state, the benefit obtained using RTIs can be lost if its return is not managed correctly.

The challenge of RTIs management implies, from the cost point of view, minimizing the cost of maintaining inventories and the transportation cost of returning them. However, the decision must consider other aspects since the return activity of RTIs for their reuse makes the supply chain become a Closed-Loop Supply Chain (CLSC). Under this concept, forward logistics decisions are being integrated with reverse logistics. In forward logistics, inventory management decisions have been integrated with transportation decisions in the Inventory Routing Problem (IRP) strategy. Some strategies are evaluated for reverse logistics decisions, such as first delivery and then pick-up (backhaul), mixed delivery and pick-up, and simultaneous delivery and pick-up (Mahjoob et al., 2021).

Londoño et al. (2021a) present a problem integrating the optional backhaul collection of RTIs with final goods distribution decisions for a single day. The authors discuss the conflict between paying the penalty for keeping RTIs at the customer's facility versus the cost of visiting the customer to pick up empty RTIs. However, broader planning horizons must be considered to establish when the RTIs should be collected. The following facts justify the backhaul collection: a) the need to prioritize deliveries, b) to avoid cross-contamination of some products, or c) the operation of reorganizing the load at customer facilities (Koç & Laporte, 2018).

It is necessary to establish a decision-making process integrating product distribution decisions with RTI collection decisions to have a CLSC with the return of RTIs. This integration seeks to minimize the logistics costs for a given planning horizon, including delivery costs, RTI collection, and total inventory handling costs (final products and RTIs). In the context of the Periodic Vehicle Routing Problem, where routing is planned over a finite time horizon, if the supply chain conditions are stable or similar, decisions on a finite horizon could be replicated for planning horizons of equal length in the future.
This paper seeks to model and optimize the integration of forwarding logistics decisions for the delivery of goods with those of reverse logistics for collecting RTIs in closed-loop supply chains for a given planning horizon minimizing the total logistics cost of transportation and inventory holding. We introduce the problem and propose a mathematical model based on the selection of predefined customer visit programs, which correspond to one, two, three, and six periodic visits to customers for delivery or collection. The rest of the paper is organized as follows. Section 2 is devoted to the literature review of the related problems of IRP. Section 3 describes the proposed mathematical model with its assumptions and details. The obtained results are analyzed in Section 4. Finally, Section 5 describes the concluding remarks.

2. Literature Review

The IRP is a challenging problem integrating two relevant components of supply chain management: inventory management and transportation (Campbell et al., 1998). The IRP simultaneously optimizes visit sequence decisions for a set of customers and inventory decisions for each node by considering a discrete-time horizon. The IRP aims to determine the routes to satisfy the customers' demand and avoid stockouts by considering the minimum total cost. The following decisions are considered: i) the period to serve a customer, ii) the amount to be served for each customer, and iii) the routes to be performed.

Several methods have been proposed to model and solve the IRP and its variants. Farias et al. (2019) propose mathematical formulations for the IRP. In this work, computational experiments on randomly generated instances have been used to test the proposed formulations. Other authors, such as Campbell and Savelsbergh (2004) and Archetti (2007), propose IRP models without considering elements of periodic routing.

Reviews of IRP-related aspects have been developed by Coelho et al. (2014a), Malladi and Sowlati (2018), and Thinkaran et al. (2019). Coelho et al. (2014a) present an extensive review of thirty years of IRP. The paper's primary goal is to provide a comprehensive review of the literature based on a new classification of the problem. Malladi and Sowlati (2018) propose a comprehensive analysis-based literature review of IRP considering Sustainability aspects. This review covers different topics, including waste management (reverse logistics), returnable transport item management (CLSC), waste prevention and reduction (perishable products), and emission reduction (emission caps and carbon price). A brief review of some metaheuristic methods for IRP has recently been proposed by Thinkaran et al. (2019), and a detailed study of various algorithms and classes of IRP is presented. Finally, Londoño et al. (2021b) categorize IRP by considering RTIs.

Different researchers have extensively studied IRP variants. Agra et al. (2018) and Kleywegt et al. (2002) have proposed some relevant works considering stochastic aspects. Bertazzi et al. (2019) studied multi-depot considerations for the IRP. The Inventory-Routing Problem with transshipment is the IRP considering transshipments between retailers or between the supplier and a retailer. Lefever et al. (2018) and Coelho et al. (2012) propose solution methods for this problem based on exact approaches and approximate algorithms. Two variants of the well-known IRP, such as the IRP with Logistic Ratio and the multiobjective green cyclic IRP, have been studied by Benoist et al. (2011), Singh et al. (2015), Archetti et al. (2017), Alvarez et al. (2018), Archetti et al. (2019) and Rau et al. (2018). Real applications of the IRP have been introduced by Coelho et al. (2014b), Ghiami et al. (2019), Azadeh et al. (2017), and Onggo et al. (2019), and Wei et al. (2019). Finally, Stochastic IRP problems have been studied recently by Nikzad et al. (2019), Markov et al. (2020), and Alvarez et al. (2021).

However, there are two specific variants that are closely related to the problem of the Inventory Routing Problem with the consideration of Returnable Transport Items (CLIRPB): Inventory Routing Problem (IRP) and the Inventory Routing Problem with Backhauls (IRPB). The IRP considers routing decisions and inventory policies for retailers and suppliers for a multiperiod strategy with a minimal total cost function. The IRP has been extensively studied in the literature, and several authors have proposed exact, heuristic, metaheuristic, and matheuristic approaches for solving different variants. Exact algorithms have been proposed by Yadollahi et al. (2018), Yadollahi et al. (2019), and Aksen et al. (2012). Yadollahi et al. (2018) consider the IRP with stochastic demand, minimizing the total cost. The authors formulate a mathematical model as a safety-stock-based stochastic IRP (SIRP) considering variables such as the retailers' inventory storage capacity. Yadollahi et al. (2019) consider the SIRP when the retailers' demand' variability is relevant. The authors propose two mathematical modeling approaches: the first uses a safety stock-based deterministic model, where extra stock is kept at the retailers to face the demands' variability. The second approach uses the sample average approximation (SAA) to solve the considered problem. Aksen et al. (2012) introduced two different formulations for the Selective Periodic Inventory Routing Problem.

Different modeling strategies for IRP have been proposed by Bard & Nananukul (2009), Qin et al. (2014), Liu et al. (2016), Juan et al. (2014), Cárdenas-Barrón et al. (2019), and Fukunaga et al. (2014). An interesting approach to solve a real problem for supermarket chains in the Netherland was proposed by Gaur & Fisher (2004). In this work, a saving of 4% of distribution cost for the first year of implementation is obtained.

In the IRPB, products are shipped from the depot to line-haul customers. Products or RTIs are collected from backhaul customers, and the vehicle must return to the depot. It is noteworthy that the IRPB can be observed in various industries (e.g., the automotive industry) (Arab et al., 2018). This problem is significant to this paper due to the possible consideration of RTIs to protect products in the distribution process, an aspect that has not been traditionally considered within IRP contexts. The management of the RTIs is considered relevant in scientific literature (Kroon & Vrijens (1995), Johansson & Hellström (2007), and Kim et al. (2014)). Glock & Kim (2014) and Hariga et al. (2016) studied the integration of forward and reverse logistics of RTIs with inventory decisions along the supply chain. These decisions are involved in a concept called CLSC.

To the best of our knowledge, the combined problem of inventory and routing decisions with RTI collection (backhaul customers) within CLSC has not been addressed yet in the scientific literature. This paper considers an IRP problem with RTIs considering a short-term horizon (1 week) without RTIs deterioration. Although the concept of combining delivery and pickup for vehicle routing problems has been studied intensely, the integration of the decision of delivery and pickup simultaneously for a given collection point and the collection of RTIs for backhaul customers within an integrated IRP framework has not been studied within the literature review.

3. Proposed Approach

This paper considers a distribution system comprising a supplier and a set of n customers that must be served to meet their known demand on a discrete planning horizon of T periods (days within a week). We consider that the customer's demands are repeated each week identically or similarly (periodic). Therefore, the system becomes periodic (assuming a scenario with stable demands: deterministic and periodic). In other words, the solution is focused on a set of T periods, and the resulting delivery and collection plan could be repeated for the following T period horizon. At the same time, the demand conditions do not change significantly.

The supplier (depot) has a single vehicle per period with enough capacity to distribute and use their RTIs to handle and protect the products. When the supplier arrives at a customer's location, it delivers the RTIs with the product. Once the products are delivered and the customers locate them in their storage locations, the RTIs are available for collection. Naturally, the RTIs cannot be withdrawn from the customer's location if the demand for the product has not occurred first.

A holding cost is incurred for each unit of product in inventory at the customer, and each RTI unit causes additional holding costs. At the same time, it is not withdrawn from the customer's facilities. It is considered a fixed and known transportation cost to travel from a location i to j. The available vehicle has a fixed and known capacity defined in RTI units for both the delivery and collection processes (the RTIs are not collapsible). Product demands from customers are established in RTI units.

The collection process must be performed once the delivery process has been completed. A vehicle's route for each period of the planning horizon must start at the supplier's location, first visiting the customers who must receive products and then visiting the customers with empty RTIs to collect. The vehicle returns to the supplier at the end of the travels. In a delivery and collection route, one of four possible situations could occur: i) a customer is visited only once to make a delivery, ii) a customer is visited only once to make a collection, iii) a customer is visited once to make not only delivery but also a collection and iv) a customer is visited twice, the first time to make a delivery and the second time to make a collection.

The combined IRPB considers several decisions. The following main decisions are considered for the proposed model: the customers should be visited in any of the four possible situations for each period of the planning horizon, in which sequence the visits should be carried out, and the number of products that must be delivered, and the amount of RTIs to be collected at each visit while satisfying the demand. The solution seeks to minimize the total transport costs for delivery and collection, the costs of handling inventories, and the penalty for non-collection RTIs.

This problem is called a Closed Loop Inventory Routing with a Backhaul-type collection of RTIs (CLIRPB). The CLIRPB could be represented as a complete graph problem. Let $G = (V, E)$ be a complete undirected graph, where $V = \{0,1,\dots,n\}$ is the vertices set representing customers with the supplier at vertex v_0. $E = \{(v_i, v_j): i \neq j\}$ is the set of edges over a given planning horizon of length T. With each edge $(v_i, v_j) \in E$ is associated a non-negative traveling cost c_{ij}. Each customer $j \in V \backslash \{v_0\}$ has a non-negative rate consumption d (volume per day) and could maintain a local inventory of products up to a maximum of C_j. The initial inventory at customer j is I_j at time 0. A homogeneous vehicle per period with capacity Q is available at the depot v_0. We propose a pattern strategy based on the idea proposed by Aksen et al. (2014) to solve this problem. The authors propose a set of patterns indicating whether or not there be a visit in period t. The chosen set P is shown in Table 1. Note that the mathematical model depends on the number of chosen patterns. However, the model is flexible and could be used with any patterns the managers consider.

Table 1
Schedule – Visit Patterns

Pattern	Period					
	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	1	0	0	0
4	0	0	0	1	0	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1
7	1	0	0	1	0	0
8	0	1	0	0	1	0
9	0	0	1	0	0	1
10	1	0	1	0	1	0
11	0	1	0	1	0	1
12	1	1	1	1	1	1

3.1 Model formulation

Assumptions
- Daily demand is fixed and known, and it is expected to remain unchanged shortly (e.g., some weeks or months).
- All cost parameters are deterministic along the planning horizon. It is also assumed to remain unchanged shortly.
- These two assumptions support the fact that the system solution can be periodically repeated for each T periods.
- Requests for withdrawing empty RTIs are assumed to occur at least one period after product delivery. This fact implies that all pick-up activities must be performed after all deliveries planned for the same period have already been executed. Suppose a delivery pattern occurs twice a week and the collection pattern occurs once a week (called example 1). Thus, the situation would be as follows: Assuming that the demand per period in the 6-day planning horizon of a customer is one unit, then the delivery is six units. Once the demand occurs, an RTI is available to be collected in the following period, another RTI in the following period, and it is kept until the customer is visited for collection. As the collection is twice per planning horizon, each time it is collected, the number of RTIs used to satisfy the demand divided by two is collected. This fact is independent of the moment in which the delivery occurs.
- All products to be delivered and empty RTIs to be withdrawn have to be 100% fulfilled.
- There is no additional loss and production of products and RTIs. These initial and final inventories of products and RTIs are equal.
- A full and empty RTI uses the same space inside the vehicle. Thus, the delivery and pick-up capacities are also the same.
- There is a vehicle capable of executing scheduled routes.
- There are no capacity constraints at the supplier. Therefore, we consider the entire available inventory to supply the scheduled shipments on the planning horizon.
- The vehicle has a limited capacity.

Notation for the CLIRPB

Sets:

- T: Periods on the planning horizon $T = \{1,..,H\}$, where H is the number of periods.
- V: Nodes of the graph, including the depot or supplier (node 0) and the N customers, i.e., $V = \{0,1,..,N\}$.
- V': Customers to be served $V' = \{1,..,N\}$.
- P: Visit pattern to be assigned to customers.

Parameters:

- d_i: Product demand for a given period for each customer $i \in V'$ (homogeneous across $t = 1, \dots, T$)
- Q: Vehicle capacity (same unit of customer demand).
- C_{ij}: Total transportation costs associated with a trip from node $i \in V$ to node $j \in V$.
- $h1_i$: Unit holding costs associated with end inventory of products at each customer or node $i \in V'$.
- $h2_i$: Unit holding costs associated with end inventory of RTIs at each customer or node $i \in V'$.
- B_{pt}: Binary parameter indicating if the visit pattern $p, p \in P$, involves a visit on period t (1 in a favorable case and 0 otherwise). An example of a visit pattern for a planning horizon of $T = 6$ periods is $P = [1\ 0\ 0\ 1\ 0\ 0]$, where a value of 1 indicates that a visit is scheduled in periods 1 and 4, and a value of 0 indicates periods 2, 3, 5 and 6 where no visits are scheduled.
- $L1_{pt}$: Integer parameters indicating the number of demand periods that the delivered products must cover in a visit on period t, $t = 1, \dots, T$, if a visit pattern $p, p \in P$ is employed. In other words, it counts the number of future periods, including the period t, in which a customer must not be visited, considering the visit pattern $p, p \in P$. As an example, given the pattern $P = [1\ 0\ 0\ 1\ 0\ 0]$, the amount of product delivered for period 1 must cover the demands of periods 1, 2, and 3. Besides, the amount of product delivered in period 4 must cover the demand of periods 4, 5, and 6. Therefore, $L1_{p1} = 3$, $L1_{p4} = 3$, $L1_{p2} = L1_{p3} = L1_{p5} = L1_{p\,6} = 0.$
- $L2_{pt}$: Integer parameters indicating the number of periods that the RTIs must cover in a visit on period t, $t = 1, \dots, T$, if a visit pattern $p, p \in P$ is considered. The values of $L2_{pt}$ are obtained as the values of $L1_{pt}$ by considering the periods from the end to the beginning. Indeed, for pattern $P = [1\ 0\ 0\ 1\ 0\ 0]$, the pickup of product is scheduled for backward period 3 (forward period 4) and backward period 6 (forward period 1).
- $TI1_p$: Total inventory level generated along the planning horizon on a given customer $i \in V'$ with a unitary demand, in case of serving it following the visit pattern $p, p \in P$. For example, considering pattern $P = [1\ 0\ 0\ 1\ 0\ 0]$, at the end of period 1, the final inventory must fulfill the demand of periods 2 and 3. At the end of period 2, the final inventory must fulfill the demand of period 3. Similarly, for period 4, the same number of inventory periods is established analogously. Therefore, the amount of holding inventory for the planning horizon is equal to $TI1_p = 6$ periods—the total inventory for any customer i can be computed as the simple product of this parameter by the daily demand d_i.
- $TI2_p$: Total inventory level generated by RTIs along the planning horizon on a given customer $i \in V'$, following the visit pattern p. For pattern $P = [1\ 0\ 0\ 1\ 0\ 0]$, the values of $TI2_p$ are calculated in an analogous way to those of $TI1_p$, considering the backward periods.

For example, 1, after the delivery occurs, there are five units left in the inventory; in the next period, four units remain, and so on until the last period of the horizon, so that in the planning horizon of 6 periods, the quantity of units $TI1_p$ in inventory is 15. For the inventory of empty RTIs, the situation is as follows: In the first period, there is one unit; in the next period, there are 2, and in the next, there are three until the vehicle arrives and picks it up. Thus, the number of units $TI2_p$ on the planning horizon is six.

Decision Variables:

- $Z1_i^t$: Binary decision variable that indicates if a customer $i \in V'$ is visited for product delivery during period t.
- $Z2_i^t$: Binary decision variable that indicates if a customer $i \in V'$ is visited for RTIs pick-up during period t.
- $Y1_{ij}^t$: Binary decision variable that indicates if the vehicle travel from customer i to customer j on period t for product delivery.
- $Y2_{ij}^t$: Binary decision variable that indicates if the vehicle travel from customer i to customer j on period t for RTIs pick-up at node j (not necessarily for node i).
- $Y3_i^t$: Binary decision variable that indicates if the vehicle consecutively performs a product delivery and an RTIs pick-up at a customer i on period t.
- $P1_i^p$: Binary decision variable that indicates if the visit pattern p is assigned to customer i for product delivery.
- $P2_i^p$: Binary decision variable that indicates if the visit pattern p is assigned to customer i for RTIs pick-up.
- $W1_i^t$: Product amount to be delivered from customer i on period t, which naturally depends on the visit pattern assigned to the customer for delivering.
- $W2_i^t$: RTI amount to be picked up from customer i on period t, which naturally depends on the visit pattern assigned to the customer for picking up.
- $I1_i$: Total product inventory produced by customer i along the overall planning horizon.
- $I2_i$: Total RTI inventory produced by customer i along the overall planning horizon.

Following (Gavish & Graves, 1978), the following fictitious variables are created to avoid subtours.

- $F1_{ij}^t$: Fictitious integer flow from node i to node j during period t, associated with the delivery process.
- $F2_{ij}^t$: Fictitious integer flow from node i to node j during period t, associated with the pick-up process.

Fig. 1 presents a small example to show the relationship between variables $F1_{ij}^t$, $F2_{ij}^t$, $Y1_{ij}^t$, $Y2_{ij}^t$, and $Y3_i^t$.

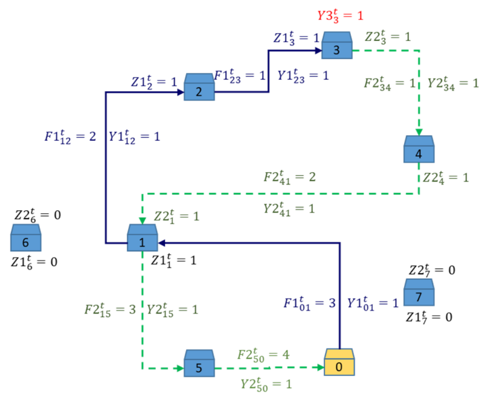


Fig. 1. Representation of a delivery and pickup (backhauls) route

3.1 Mathematical model

Objective function:
Minimize Transportation Costs + Inventory Holding Costs

$$\sum_{i\epsilon V}\sum_{j\epsilon V, i\neq j}\sum_{t\epsilon T} C_{ij} * Y1_{ij}^t + \sum_{i\epsilon V}\sum_{j\epsilon V, i\neq j}\sum_{t\epsilon T} C_{ij} * Y2_{ij}^t + \sum_{i\epsilon V'} h1_i * I1_i + \sum_{i\epsilon V'} h2_i * I2_i \qquad (1)$$

Eq. (1) represents the objective function considering the transportation and holding costs of products and RTIs, respectively.
Subject to

$$F1_{ij}^t \leq N \times Y1_{ij}^t \qquad \forall\ i\epsilon V, j\epsilon V', i \neq j\ \forall\ t\epsilon T \qquad (2)$$

$$\sum_{l\epsilon V', l\neq j} F1_{jl}^t - \sum_{i\epsilon V, i\neq j} F1_{ij}^t = -Z1_j^t \qquad \forall\ j\epsilon V', \forall\ t\epsilon T \qquad (3)$$

$$\sum_{j\epsilon V'} F1_{0j}^t = \sum_{j\epsilon V'} Z1_j^t \qquad \forall\ t\epsilon T \qquad (4)$$

$$F2_{ij}^t \leq N \times Y2_{ij}^t \qquad \forall\ i\epsilon V, j\epsilon V', \forall\ t\epsilon T\ i \neq j \qquad (5)$$

$$\sum_{l\epsilon V, l\neq j} F2_{jl}^t - \sum_{i\epsilon V', i\neq j} F2_{ij}^t = Z2_j^t \qquad \forall\ j\epsilon V', \forall\ t\epsilon T \qquad (6)$$

$$\sum_{j\epsilon V'} F2_{0j}^t = \sum_{j\epsilon V'} Z2_j^t \qquad \forall\ t\epsilon T \qquad (7)$$

Constraints (2) to (7) represent linehaul and backhaul unit flows (allow subtours to be eliminated). Constraints (2) ensure that a positive flow can occur if there is a vehicle traversing arc (i,j) for delivery when the variable $Y1_{ij}^t$ is used for each period $t \in T$. Similarly, constraints (5) ensure the same as (2) for RTIs. Equations (3) and (6) ensure the mass balance of the delivery process of products and the pick-up process for RTIs. The value of $-Z1_j^t$ in (3) indicates that a delivery has been performed, and the outflow must be negative. The value of $Z2_j^t$ in (6) represents a pick-up of RTIs, and the outflow must be positive for each period $t \in T$. Equations (4) and (7) ensure the number of maxima of accumulating flows for products and RTIs. In particular, the number of accumulating flows of products and RTIs must be equal to the sum of the total of backhaul nodes (4) or linehaul nodes (7).

$$Y3_i^t \leq \frac{(Z1_i^t + Z2_i^t)}{2} \qquad \forall\ i \in V', t \in T \qquad (8)$$

$$\sum_{j\epsilon V, i\neq j} Y1_{ji}^t = Z1_i^t \qquad \forall\ i\epsilon V', t\epsilon T \qquad (9)$$

$$\sum_{j\epsilon V, i\neq j} Y1_{ij}^t + \sum_{j\epsilon V, i\neq j} Y2_{ij}^t \leq 2 * Z1_i^t \quad \forall\ i\epsilon V', t\epsilon T \quad (10)$$

$$\sum_{j\epsilon V\ \ i\neq j} Y1_{ij}^t = Z1_i^t - Y3_i^t \quad \forall\ i \in V', t \in T \quad (11)$$

$$\sum_{j\epsilon V\ i\neq j} Y2_{ji}^t \leq 1 \quad \forall\ i \in V', t \in T \quad (12)$$

$$\sum_{j\epsilon V\ \ i\neq j} Y2_{ij}^t \leq 1 \quad \forall\ i \in V', t \in T \quad (13)$$

Constraints (8) establish the relationships between the decision variables of the visiting customer for delivery or collection, considering that the collection visit is performed immediately after the delivery is performed for the given node. Constraints (8) allow that the variable $Y3_i^t$ could be equal to 1 only if the node $i \in V'$ is visited to perform a delivery and collection consecutively at the period $t \in T$. Constraints (9) to (10) determine that delivery or collection decisions are conditioned if the node is visited or not. Constraints (9) and (10) ensure that a visit of a node $i \in V'$ (for collection or delivery) must be performed if $Z1_i^t = 1$ for a given period $t\ \epsilon\ T$. Equations (11) ensure that from a delivery node $i \in V'$ must not be performed another delivery or return to the supplier when a pick-up is performed for the same served node $i \in V'$ for each period $t\ \epsilon\ T$. The maximum number of input and output arcs for a given collection node is restricted by (12) and (13).

Note that variables $Y1_{ij}^t$ are activated according to variables $F1_{ij}^t$, while $F1_{ij}^t$ variables are activated according to variables $F2_{ij}^t$. Additionally, $F1_{ij}^t$ are modeled as flowing from supplier to the customers, while $F2_{ij}^t$are modeled to flow from the customers to the supplier.

$$Z1_i^t + Y3_i^t - 1 \leq \sum_{j\epsilon V, i\neq j} Y2_{ij}^t \quad \forall\ i \in V', t \in T \quad (14)$$

$$Z2_i^t = \sum_{j\epsilon V,\ \ i\neq j} Y2_{ij}^t \quad \forall\ i \in V', t \in T \quad (15)$$

$$Z2_i^t - Y3_i^t = \sum_{j\epsilon V\ \ i\neq j} Y2_{ji}^t \quad \forall\ i \in V', t \in T \quad (16)$$

$$\sum_{i\epsilon V'\ \ i\neq j} Y3_i^t \leq 1 \quad \forall\ \ t \in T \quad (17)$$

Constraints (14) to (17) establish the relationship between the visit decisions to deliver and collect (variables $Z1_i^t, Z2_i^t$) with the decisions to arrive at and leave a node (variables $Y1_{ij}^t, Y2_{ij}^t$) and the delivery and collection decisions immediately after (variables $Y3_i^t$). Constraints (14) and (15) ensure that the vehicle must visit another collection node or return to the supplier once a collection node has been served. Equations (16) ensure that a node for which a collection and pick up must be performed simultaneously, it must not be reached from any node $j\ \epsilon\ V$ avoiding this aspect by variable $Y2_{ji}^t$ (collection node). Finally, Constraints (17) ensure that it is possible to link one delivery node to one collection node for each route as maximum.

$$\sum_{j\epsilon V,\ \ i\neq j} \left(Y1_{ji}^t + Y2_{ji}^t\right) = \sum_{j\epsilon V,\ \ i\neq j} \left(Y1_{ij}^t + Y2_{ij}^t\right) \quad \forall\ i \in V', t \in T \quad (18)$$

$$\sum_{i\in V', i\neq 0} (Y1_{0i}^t + Y2_{0i}^t) = \sum_{i\in V', i\neq 0} (Y1_{i0}^t + Y2_{i0}^t) \quad \forall\ t\ \epsilon\ T \quad (19)$$

$$\sum_{i\in V', i\neq 0} (Y1_{0i}^t + Y2_{0i}^t) \leq 1 \quad \forall\ t\ \epsilon\ T \quad (20)$$

$$Z1_i^t \leq \sum_{j\in V'} Y1_{0j}^t \quad \forall\ i \in V', t \in T \quad (21)$$

$$Z2_i^t \leq \sum_{j\in V'} Y2_{j0}^t \quad \forall\ i \in V', t \in T \quad (22)$$

Eq. (18) guarantee the balance for each node. Equations (19) establish the balance between the number of input arcs and output arcs for each period $t\ \epsilon\ T$. Complementary constraints (20) determine that the sum of the number of input arcs from the supplier is equal to 1 for each period $t\ \epsilon\ T$. Besides, constraints (21) indicate that a delivery customer $j \in V'$must be visited on the sequence from the supplier or delivery arc $Y1_{0j}^t$ for a given period $t\ \epsilon\ T$. In addition, a collection customer must be connected to the supplier or collection arc $Y2_{j0}^t$ for a given period $t\ \epsilon\ T$ (Constraints 22).

$$I1_i = \sum_{p\in P} d_i \times TI1_p \times P1_i^p \qquad \forall\, i \in V' \qquad (23)$$

$$I2_i = \sum_{p\in P} d_i \times TI2_p \times P2_i^p \qquad \forall\, i \in V' \qquad (24)$$

$$Z1_i^t = \sum_{p\in P} B_{pt} \times P1_i^p \qquad \forall\, i \in V', t \in T \qquad (25)$$

$$Z2_i^t = \sum_{p\in P} B_{pt} \times P2_i^p \qquad \forall\, i \in V', t \in T \qquad (26)$$

$$\sum_{p\in P} P1_i^p = 1 \qquad \forall\, i \in V' \qquad (27)$$

$$\sum_{p\in P} P2_i^p = 1 \qquad \forall\, i \in V' \qquad (28)$$

$$W1_i^t = \sum_{p\in P} L1_{pt} \times d_i \times P1_i^p \qquad \forall\, i \in V', t \in T \qquad (29)$$

$$W2_i^t = \sum_{p\in P} L2_{pt} \times d_i \times P2_i^p \qquad \forall\, i \in V', t \in T \qquad (30)$$

$$\sum_{i\in V'} W1_i^t \leq Q \qquad \forall\, t \in T \qquad (31)$$

$$\sum_{i\in V'} W2_i^t \leq Q \qquad \forall\, t \in T \qquad (32)$$

Eq. (23) and Eq. (24) establish the inventory levels for each period, while Eq. (25) and Eq. (26) determine the decision to visit from the patterns. Note that the inventory level for each customer depends on the selected pattern and the demand d_i. Besides, a node $i \in V'$ must be assigned to a visit pattern for delivery and for collection for a given period $t \in T$ (Equations 27 and 28). Equations (29) and (30) determine that the number of products and RTIs for a customer $i \in V'$ are determined by the selected pattern $p \in P$. Finally, constraints (31) and (32) determine that the number of dispatched products and the collected RTIs from a customer must not exceed the vehicle capacity.

$Z1_i^t$ and $Z2_i^t$ are activated according to the binary pattern of visits. For the pattern $P = [1\ 0\ 0\ 1\ 0\ 0]$, constraints (26) and (27) make the decision variables take the value of 1 or 0 depending on the period. For this schedule, it is essential to visit a customer in periods 1 and 4 of the planning horizon. Thus, variables $Z1_i^t$ and $Z2_i^t$ would take the value of 1 in periods 1 and 3 and 0 in periods 2, 3, 5, and 6. Constraints (28) and (29) assign each customer a single pattern of visits. Constraints (30) and (31) define the number of products and RTIs to be delivered and collected according to the visit pattern and the number of periods L1 and L2. Constraints (32) and (33) are better explained with the above description. As observed before, the proposed CLIRPB model allows the scheduling of inventory management and transportation of both final goods and RTIs to be carried out. This schedule could be performed for a certain number of periods within an established planning horizon and are very useful in the predetermined planning of the use of vehicles and for the predefinition of visits to customers, which could even be a fixed distribution strategy over specific periods ahead. In fact, under the assumption of stability of market conditions, these decisions arising when solving the CLIRPB model could be replicated in the future in planning horizons of a similar number of periods compared to those studied in the base model. Nevertheless, these customer visit schedules help the supplier and the customer. For example, in a planning horizon of one week, a delivery program can establish whether a specific customer should be visited once a week (on any specific day), twice a week (indicating two specific days), or three times a week (on three specifically chosen days). These different visiting schedules allow the supplier and customers to establish schedules and resources to have available. A customer could know the amount of product and the time to be visited for product delivery or a collection of RTIs. On the other hand, the supplier could establish the inventory levels of products and RTIs and determine the available vehicle.

These results provided by the CLIRPB are potentially valuable for determining vehicle requirements, given the predetermined use that these would have according to the model in each period. The model must have accurate information on transportation costs and the costs of maintaining inventory to ensure the success of the solution and its implementation. Moreover, regarding transport costs, it should be made clear if there are differences between the costs of delivering and collecting, so these cost variations could be considered in the cost function.

4. Computational Experimentation

This section presents the results of computational experiments with the model proposed in Section 3.2. The model has been coded in AMPL and solved using CPLEX for AMPL Version 20.1 on a standard computer with an Intel Core I7 processor, 2.5 GHz. CPU 8 GB RAM. The effectiveness of the proposed model has been tested on a small example and a set of synthetic instances, consisting of a central supplier, a set of customers ranging from 10 to 30 customers with known demands, one single vehicle, a planning horizon, transportation costs for delivery and collection, and handling inventory costs for final goods and RTIs. Two experiments have been conducted for a single instance: the base problem described in Section 3.2 and the problem considering customers with equal demand. The experiments are explained below.

4.1 Small example

A case of a supply chain considering a supplier, five customers, and a planning horizon of four periods is presented to illustrate the CLIRPB. The proposed visit patterns (B_{pt}) are presented in Table 2.

Table 2
Visit patterns for small examples

B_{pt}	Period			
Pattern	1	2	3	4
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1
5	1	0	1	0
6	0	1	0	1
7	1	1	1	1

The value of the parameters $L1_{pt}$ and $L2_{pt}$, which indicates the demand periods that are covered with a delivery or collection node associated with a pattern, are presented in Table 3. For example, suppose pattern one is selected. In that case, the number of periods covered in the planning horizon with this pattern is four because when making a delivery, it must be large enough to satisfy the demand during $L1_{pt}$ until the next one occurs delivery. The same situation occurs for the collection, which would collect the $L2_{pt}$ periods back since the last collection was performed.

Table 3
Periods associated with parameters $L1_{pt}$ and $L2_{pt}$

$L1_{pt}$, $L2_{pt}$	Periods ($L1_{pt}$ and $L2_{pt}$)			
Pattern	1	2	3	4
1	4	0	0	0
2	0	4	0	0
3	0	0	4	0
4	0	0	0	4
5	2	0	2	0
6	0	2	0	2
7	1	1	1	1

The total number of inventory periods $TI1_p$ and $TI2_p$ per each pattern type are presented in Table 4. Table 4 shows that if it is pattern 1, after the first period, there are three days of coverage ahead in the inventory (and after the demand occurs 1 RTI). In the second period, there are two units in the inventory (2 units of RTIs), and finally, in the third period, one unit remains in the inventory (and three units of RTIs). Thus, six units were in the goods inventory and six in an inventory of RTIs to pick up.

Table 4
Inventory periods per each pattern for $TI1_p$ and $TI2_p$

	Pattern	1	2	3	4	5	6	7
$TI1_p$, $TI2_p$		6	6	6	6	2	2	0

Information of the nodes is presented in Table 5.

Table 5
Information of the nodes

Node	Coord X	Coord Y	Demand
0	0	0	0
1	353	109	52
2	72	342	95
3	123	57	25
4	17	6	64
5	362	407	45

The used values of $h_1 = 4$ and $h_2 = 1$.The visit assigned patterns when solving the model are shown in Table 6.

Table 6
Used patterns

Node i	Pattern $P1_i^p$	Pattern $P2_i^p$
1	6	6
2	7	5
3	7	6
4	7	7
5	6	2

Concerning $P1_i^p$, customers 1 and 5 are visited for delivery according to pattern 6. Therefore, the customers are visited twice (second and fourth period). Each period must cover a total of $L1_i = 2$ periods ahead of the planning horizon ($i = \{1, 5\}$). Tables 7 and 8 show the obtained results of patterns $P1_i^p$.

Table 7
Visit of pattern $P1_i^p = 6$

Customers	Pattern\Period	1	2	3	4
1,5	6	0	1	0	1

Table 8
Visit of pattern $P1_i^p = 7$

Customers	**Pattern\Period**	**1**	**2**	**3**	**4**
2,3,4	7	1	1	1	1

Customers 2, 3, and 4 are visited for delivery according to pattern 7. Therefore, these customers are visited each day of the planning horizon. Concerning $P2_i^p$, customers 1 and 3 are visited for collection according to pattern 6. Tables 9 and 10 show the obtained results of the collection patterns.

Table 9
Visit of pattern $P2_i^p = 6$

Customers	**Pattern\Period**	**1**	**2**	**3**	**4**
1,3	6	0	1	0	1

Twice in the planning horizon, in the second and fourth periods. Each collection must cover a total of $L1_{pt} = 2$ periods ahead of the planning horizon for $i = \{1,3\}$. Moreover, customer 2 is visited with collection pattern 5, customer 4 with collection pattern 7, and customer 5 with collection pattern 2. Table 10 shows the pattern summary.

Table 10
Visit of collection pattern for customers 2, 4, and 5

Customers	**Pattern\Period**	**1**	**2**	**3**	**4**
2	5	1	0	1	0
4	7	1	1	1	1
5	2	0	1	0	0

For customer 2, a pick-up visit is performed twice in the planning horizon (first and third period). Each collection must cover a total of $L2_i = 2$ periods ahead of the planning horizon for $i = \{2\}$. For customer 4, a collection visit is performed four times in the planning horizon (all periods). Each collection must cover a total of $L2_i = 0$ periods ahead of the planning horizon for $i = \{4\}$. For customer 5, a pick-up visit is performed only once in the planning horizon (period two). Each collection must cover a total of $L2_i = 4$ periods behind the planning horizon for $i = \{5\}$. The results of the variables $Z1_i^t$, $Z2_i^t$ and $Y3_i^t$ obtained when solving the IRPB for the proposed case are presented in Table 11.

Table 11
Variable solutions for the small problem

Period	Node	$Z1_i^t$	$Z2_i^t$	$Y3_i^t$
1	2	1	1	1
1	3	1		
1	4	1	1	
2	**1**	**1**	**1**	
2	**2**	**1**		
2	**3**	**1**	**1**	
2	**4**	**1**	**1**	
2	**5**	**1**	**1**	**1**
3	2	1	1	1
3	3	1		
3	4	1	1	
4	**1**	**1**	**1**	1
4	**2**	**1**		
4	**3**	**1**	**1**	
4	**4**	**1**	**1**	
4	**5**	**1**		

The visit schedule for each period of the planning horizon according to binary variables $Y1_{ij}^t$, $Y2_{ij}^t$, and flow variables $F1_{ij}^t$, $F2_{ij}^t$ is shown in Table 12. The quantities of products to be delivered to customers 1 and 5 are calculated by using (29) for customer 1 in period 2 is $W1_1^2 = L1_{12} * D_1 * P1_1^6 = 104$. The amount of product remaining in the inventory in period two is 52 units, which are demanded for the next period, leaving the inventory at zero. On the other hand, after visiting customers 2 and 5, the vehicle returns and picks up the RTIs in the inventory (from two periods ago), leaving zero units in inventory. Finally, for the next period, after the demand for that period has occurred, 52 RTIs remain in inventory.

Table 12
Visit Scheduling for planning horizon

Period	Node Start	Node End	Type	$Y1_{ij}^t$	$Y2_{ij}^t$	$Y31_i^t$	$F1_{ij}^t$	$F2_{ij}^t$	$W1_i^t$	$W1_i^t$	$I1_i^t$	$I2_i^t$
1	0	4	D	1			3		64		0	
1	4	3	D	1			2		25		0	25
1	3	2	D/P	1		1	1		95	95	0	0
1	2	4	p		1			1		25		0
1	4	0			1			3				
1	1	No visited	0	0		0	0	0	0	52	104	
1	5		No visited	0	0		0	0	0	0	45	135
2	0		4	D	1			5		64		0
2	4	3	D	1			4		25		0	
2	3	1	D	1			3		104		52	
2	1	2	D	1			2		95		0	95
2	2	5	D/P	1		1	1		90	180	45	0
2	5	1	P		1			1		95		0
2	1	3	P		1			2		104		0
2	3	4	P		1			4		64		0
2	4	0			1			5				
3	0	4	D	1			3		64		0	
3	4	3	D	1			2		25		0	25
3	3	2	D/P	1		1	1		95	95	0	0
3	2	4	P		1			2		64		0
3	4	0			1			3				0
3	1	No visited	0	0		0	0	0	0	52	104	
3	5	No visited	0	0		0	0	0	0	45	45	
4	0	4	D	1			5		64		0	
4	4		3	D	1			4		25		0
4	3		2	D	1			3		104		52
4	2	5	D	1			2		95		0	
4	5	1	D/P	1		1	1		90	90	45	0
4	1	3	P		1			1		95		0
4	3	4	P		1			2		104		0
4	4	0			1			3				
4	5	No									90	
1	0	4	D	1			3		64		0	
1	4	3	D	1			2		25		0	25
1	3	2	D/P	1		1	1		95	95	0	0
1	2	4	p		1			1		25		0

Fig. 2 to Fig. 5 show the Solution for the CLIRPB for all periods.

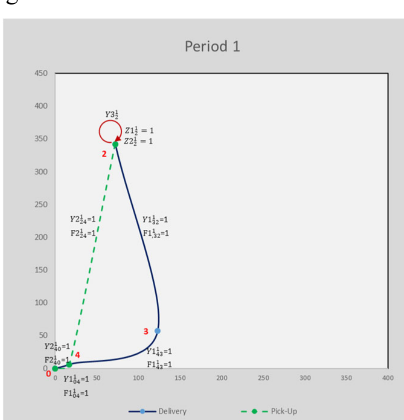


Fig. 2. Solution for the first period

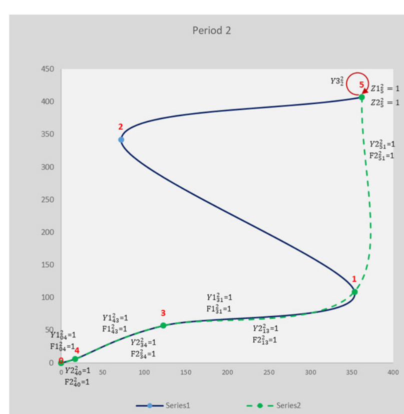


Fig. 3. Solution for the second period

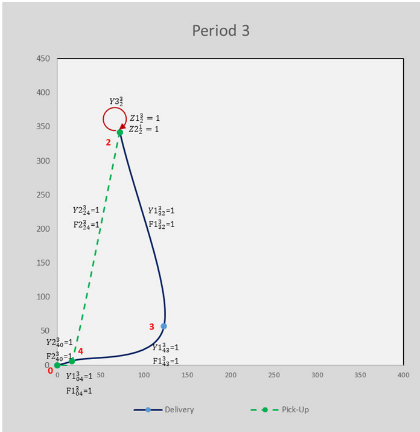


Fig. 4. Solution for the third period

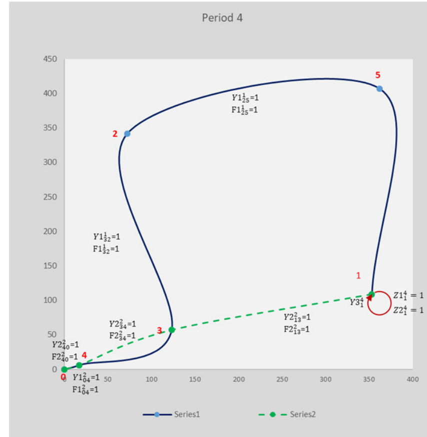


Fig. 5. Solution for the fourth period

4.1. Data Set Synthetic Instances

- It is assumed that the supplier has enough capacity to meet the known demands. The coordinates of the supplier location (X_0, Y_0) is generated in different coordinates. For example, in the lower corner of the grid (0,0), in the middle of the grid (250,250), and in random positions.
- Customers range from 10 to 30 nodes.
- Customer demand is known and is randomly generated with a uniform distribution in the interval $[10, 100]$, and the coordinates (X_i, Y_i) of each location are generated randomly with a uniform distribution in the interval [0,500].
- The transportation cost $C_{ij} = F\sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}$, where F is a factor used to convert the distance into the cost. The used value of $F = 1$.
- Vehicle capacity is 1.5 of total demand per period adapted from the works of (Archetti et al., 2007; Coelho et al. 2012).
- The planning horizon is six periods (e.g., days within a week).
- The cost of handling finished products in inventory is arbitrarily established for three different values: high ($h_1 = 4$), medium ($h_1 = 2$) and low ($h_1 = 1$).
- The cost of handling the RTI inventory is established with values proportional to h_1, thus, $h_2 = \gamma * h_1$ and γ represents a set of factors whose values are less than one, making the cost of keeping RTIs in inventory less $\gamma = [0.75, 0.50, 0.25, 0.125, 0.0625, 0.03125]$ is a used dynamic factor to test the proposed approach.

4.2. Results and discussion

The results of an instance with 15 customers with different depot location are shown in Tables 13, 14, and 15. For each table, the first and second columns show the different values that h_1 and h_2 could take, and the rest show the optimal solution under each scenario. The third and fourth columns show the C_1 delivery transportation cost, C_2 represents the collection cost, and column five shows the total transportation cost. Columns six and seven show the handling inventory cost of finished products (C_3) and RTIs (C_4), and column eight shows the total handling cost. The total logistics costs are in column 9. Column 10 shows whether the schedule is for a delivery (D) or a pick-up (P) node. The determined visit scheduling for delivery and collection of each customer is shown in columns 11 to 25. Indeed, the column "Visit Scheduling" values correspond to the selected pattern for each customer (customers numbered from 1 to 15). Note that the customers that must be visited during all periods of the planning horizon are highlighted in red, while in yellow are labeled customers who must have three visits on the planning horizon, and in green, customers who must have two visits. Finally, some customers have one visit on the planning horizon without color. The last column presents the processing time [seconds]. Tables 13 – 15 are color-coded to represent the frequency of visits to customers. The customers that are visited less frequently have no color; the others correspond to a traffic light where green is that these customers are visited twice on the horizon, yellow three times on the horizon, and red every period.

4.2.1. Base Case Results

Table 13 shows the results of the scheduling visits (both to make deliveries and to make collections) for different values of inventory handling costs h_1 and h_2, and transportation costs. In this table, the results are associated with the supplier's location in the lower-left corner of the grid with coordinates $X_0 = 0$ and $Y_0 = 0$. Later, solutions to other supplier locations are discussed.

Table 14
CLIRPB results for a single instance considering the supplier location at (250,250)

				Depot location (250, 250)						Visit Scheduling															Computing time
h_1	h_2	C_1 [\$]	C_2 [\$]	Transportation	C_3 [\$]	C_4 [\$]	Holding	Total	Deliver	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
4.00	3.000	9016	9016	18032	1056	792	1848	19880	D	12	12	12	12	12	10	12	12	12	12	12	12	12	10	9	13.67
									P	12	12	12	12	12	10	12	12	12	12	12	12	12	10	9	
4.00	2.000	9016	7198	16214	1056	2058	3114	19328	D	12	12	12	12	12	10	12	12	12	12	12	12	12	10	9	79.55
									P	10	12	10	10	12	10	12	10	10	12	12	10	12	10	9	
4.00	1.000	9792	4801	14593	444	2286	2730	17323	D	12	12	12	12	12	10	12	12	12	12	12	12	12	12	9	124.84
									P	10	10	10	10	11	10	11	10	10	11	11	10	10	11	3	
4.00	0.500	10024	3085	13109	288	2246	2533.5	15643	D	12	12	12	12	12	12	12	12	12	12	12	12	12	12	8	40.53
									P	9	8	8	8	7	8	7	9	9	2	7	8	8	7	2	
4.00	0.250	10034	2260	12294	288	1476	1764	14058	D	12	12	12	12	12	12	12	12	12	12	12	12	12	12	8	70.50
									P	9	8	5	8	7	5	7	3	9	2	7	8	8	6	2	
4.00	0.125	9879	1839	11718	444	1245	1689.38	13407	D	12	12	12	12	12	10	12	12	12	12	12	12	12	12	7	131.62
									P	5	2	3	3	1	3	7	5	5	4	1	3	2	6	4	
2.00	1.500	5139	5139	10278	4074	3055	7129	17407	D	11	10	10	10	11	10	12	11	11	10	11	10	10	11	3	15.03
									P	11	10	10	10	11	10	12	11	11	10	11	10	10	11	3	
2.00	1.000	5139	4775	9914	4074	2319	6393	16307	D	11	10	10	10	11	10	12	11	11	10	11	10	10	11	3	27.03
									P	11	10	10	10	11	10	11	11	11	9	11	10	10	11	3	
2.00	0.500	7251	3894	11145	2052	1848	3900	15045	D	11	12	11	11	12	11	12	11	11	11	12	11	12	11	10	367.17
									P	11	7	11	11	10	2	10	11	11	1	10	11	7	2	1	
2.00	0.250	7302	2641	9943	2052	1713	3765	13708	D	10	12	10	10	12	10	12	10	10	10	12	10	12	10	11	417.31
									P	3	11	1	1	8	1	8	3	3	5	8	1	11	5	2	
2.00	0.125	7426	1857	9283	2058	1245	3303.38	12586	D	10	12	10	10	12	10	12	10	10	12	12	10	12	10	9	314.8
									P	3	4	1	1	2	1	8	3	3	6	2	1	4	5	6	
2.00	0.062	7426	1806	9232	2058	669.4	2727.38	11959	D	10	12	10	10	12	10	12	10	10	12	12	10	12	10	9	451.53
									P	3	4	1	1	2	1	6	3	3	6	2	1	4	5	6	
1.00	0.750	4634	4510	9144	2439	1946	4385.25	13529	D	11	11	11	11	10	11	10	11	11	10	10	11	11	8	4	107.78
									P	11	11	11	11	10	6	10	11	11	10	10	11	11	8	4	
1.00	0.500	4634	4165	8799	2439	1593	4032	12831	D	11	11	11	11	10	11	10	11	11	10	10	11	11	8	4	60.09
									P	11	11	11	11	10	6	10	11	11	5	10	11	11	5	4	
1.00	0.250	4256	2005	6261	3162	2223	5385	11646	D	10	11	7	7	9	7	11	10	10	11	9	7	11	9	4	766.56
									P	5	2	1	1	9	1	9	5	5	4	9	1	2	6	4	
1.00	0.125	4616	1839	6455	2757	1245	4002.38	10457	D	7	10	10	10	11	10	11	7	7	8	11	10	10	11	3	580.48
									P	4	1	5	5	6	5	9	4	4	6	6	5	1	2	3	
1.00	0.062	4616	1839	6455	2757	622.7	3379.69	9835	D	7	10	10	10	11	10	11	7	7	8	11	10	10	11	3	861.83
									P	4	1	5	5	6	5	9	4	4	6	6	5	1	2	3	
1.00	0.031	4884	1828	6712	2472	334.7	2806.69	9519	D	10	10	10	10	11	10	11	10	10	9	11	10	10	9	4	410.3
									P	5	1	3	3	2	3	4	5	5	4	2	3	1	6	4	

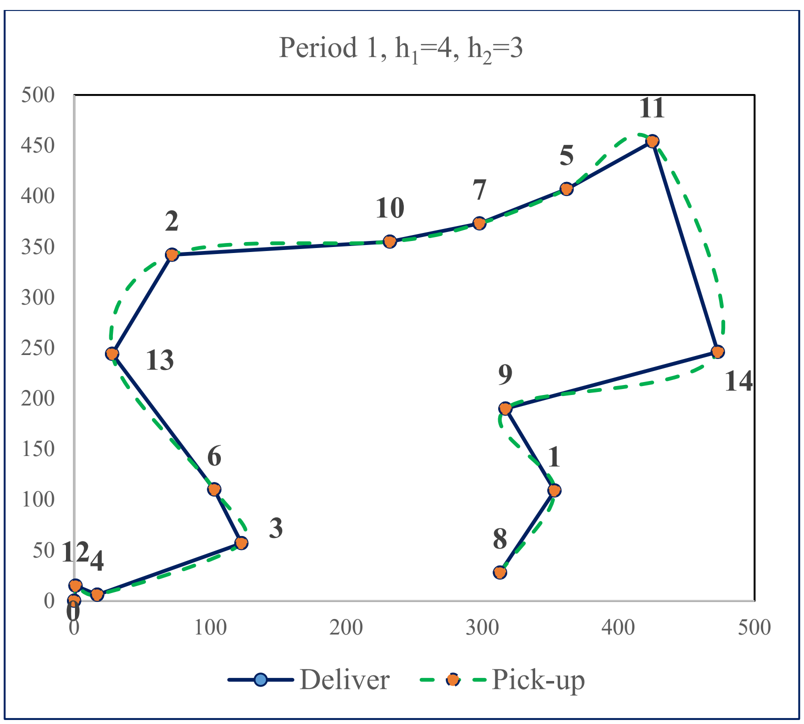
Table 15
CLIRPB results for a single instance considering supplier location at a randomly nodes

				Depot location (231, 71)						Visit Scheduling															Computing time
h_1	h_2	[$]$C_1$	[$]$C_2$	Transportation	[$]$C_3$	[$]$C_4$	Holding	Total cost	Deliver	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
4.00	3.000	9268	9268	18536	288	216	504	19040	D	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9	8.2
									P	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9	
4.00	2.000	9268	7738	17006	288	1548	1836	18842	D	12	12	12	12	12	12	12	12	12	12	12	12	12	11	9	77.34
									P	12	12	11	12	10	11	12	12	11	12	10	12	12	10	9	
4.00	1.000	9463	4444	13907	900	2442	3342	17249	D	12	12	12	12	12	12	12	12	12	12	12	12	12	11	9	657.92
									P	11	10	10	10	10	1	10	11	11	10	10	10	10	11	3	
4.00	0.500	9810	3458	13268	288	2246	2533.5	15802	D	12	12	12	12	12	12	12	12	12	12	12	12	12	12	7	2517.64
									P	7	9	9	9	7	9	7	8	7	4	7	9	9	7	1	
4.00	0.250	9938	3125	13063	288	1267	1554.75	14618	D	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9	10445.49
									P	8	9	7	7	8	3	8	8	8	3	8	7	9	3	3	
4.00	0.125	9810	3458	13268	288	2246	2533.5	15802	D	12	12	12	12	12	12	12	12	12	12	12	12	12	12	7	20231.09
									P	4	6	3	3	1	3	1	8	1	1	4	3	6	4	1	
2.00	1.500	5824	5645	11469	3144	2520	5664	17133	D	11	10	10	10	12	10	12	11	11	10	12	10	10	11	8	29.94
									P	11	10	10	10	12	10	12	11	11	10	12	10	10	11	2	
2.00	1.000	5714	4444	10158	3360	2442	5802	15960	D	11	10	10	10	12	10	12	11	11	10	12	10	10	11	1	21.33
									P	11	10	10	10	10	3	10	11	11	10	10	10	10	11	1	
2.00	0.500	5714	4444	10158	3360	1221	4581	14739	D	11	10	10	10	12	10	12	11	11	10	12	10	10	11	1	167.36
									P	11	10	10	10	10	3	10	11	11	10	10	10	10	11	1	
2.00	0.250	6664	2937	9601	2640	1588	4227.5	13829	D	12	11	11	11	12	11	12	12	10	11	12	11	11	11	5	2627.44
									P	9	8	4	4	10	2	10	9	3	1	10	4	2	6	5	
2.00	0.125	7297	2115	9412	2118	1262	3380.25	12792	D	12	11	11	11	12	11	12	12	10	11	12	11	11	12	7	2492.09
									P	3	2	4	4	5	4	1	9	5	1	5	4	2	3	1	
2.00	0.062	7980	2138	10118	1338	669.4	2007.38	12125	D	12	12	10	10	12	10	12	12	11	12	12	10	12	10	9	1156.53
									P	1	4	5	5	2	3	6	1	6	6	2	5	4	2	6	
1.00	0.750	4444	4444	8888	2442	1832	4273.5	13162	D	11	10	10	10	10	1	10	11	11	10	10	10	10	11	5	99.09
									P	11	10	10	10	10	1	10	11	11	10	10	10	10	11	5	
1.00	0.500	4444	4444	8888	2442	1221	3663	12551	D	11	10	10	10	10	1	10	11	11	10	10	10	10	11	5	28.05
									P	11	10	10	10	10	1	10	11	11	10	10	10	10	11	5	
1.00	0.250	4577	2948	7525	2418	1808	4225.5	11751	D	11	10	10	10	11	10	11	11	11	6	11	10	10	11	6	665.3
									P	11	10	5	5	2	1	6	11	6	6	2	5	10	4	6	
1.00	0.125	4847	2185	7032	2418	1245	3663.38	10695	D	10	11	11	11	10	11	10	10	10	5	10	11	11	10	4	654.61
									P	5	2	6	6	1	6	7	5	3	1	1	6	2	5	4	
1.00	0.062	4867	2082	6949	2418	669.4	3087.38	10036	D	10	11	11	11	10	11	10	10	10	3	10	11	11	10	3	592.84
									P	1	2	4	4	5	6	3	1	5	3	5	4	2	1	3	
1.00	0.031	4867	2082	6949	2418	334.7	2752.7	9702	D	10	11	11	11	10	11	10	10	10	3	10	11	11	10	3	634.72
									P	1	2	4	4	5	6	3	1	5	3	5	4	2	1	3	

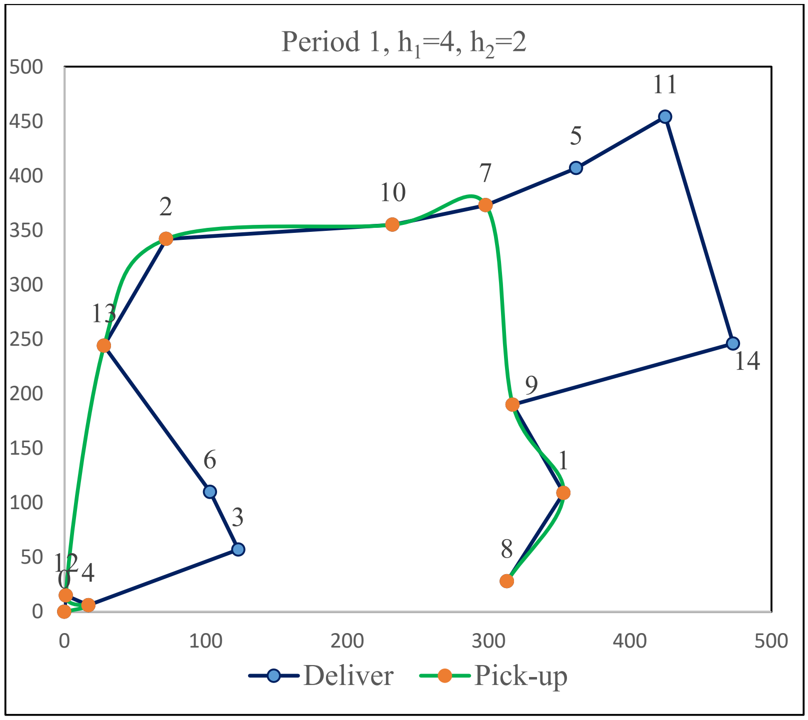
Table 16
Variations related to highest h_1 and h_2 values considering supplier location at (0,0)

Depot location (0, 0)							
h_1 [$/Unit]	h_2 [$/Unit]	Transportation cost [$]	Holding cost [$]	Transportation Δ Cost [$]	Holding Δ Cost [$]	Transportation Δ Cost [%]	Holding Δ Cost [%]
4.00	3.000	17600	504	0	0	0.00	0.00
4.00	2.000	16058	1836	1542	-1332	8.76	-264.29
4.00	1.000	14075	2439	3525	-1935	20.03	-383.93
2.00	1.500	10606	5757	6994	-5253	39.74	-1,042.26
2.00	1.000	10606	4740	6994	-4236	39.74	-840.48
2.00	0.500	11065	3141	6535	-2637	37.13	-523.21
1.00	0.750	9223	3672.75	8377	-3168.75	47.60	-628.72
1.00	0.500	8947	3388.5	8653	-2884.5	49.16	-572.32
1.00	0.250	8507	3141.8	9093	-2637.8	51.66	-523.36

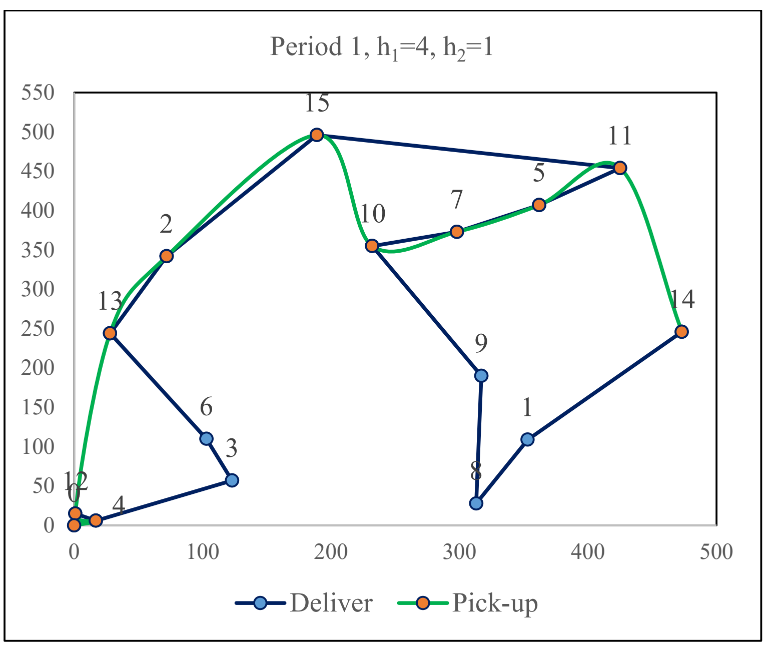
On the other hand, the difference between the cost of handling inventories between final goods and RTIs affects the scheduling of visits (see Table 16). Fig. 7 shows the performed routes for the first period to visit customers for delivery and pick-ups, considering three cost scenarios for maintaining inventory of RTIs in customers (other parameters remain constant). Each route starts from the depot (node 0) and moves along the route marked by the blue line, making all scheduled deliveries. Once the deliveries have been performed, the vehicle begins collecting the empty RTIs, visiting the scheduled customers, and following the green route, as shown in Fig.7. Therefore, when the handling RTIs cost is much lower than the cost of final goods, the visit schedule is modified, making the collection frequency more minor than the frequency of visits in the case of delivery. It can be seen, for example, in Fig. 7, in cases A, B, and C for period 1 of the planning horizon that, when the values of h_1 and h_2 are similar, the delivery and collection visit schedule are the same. However, in the case where h_2 is low about h_1, the delivery schedule changes, making the frequency of visits to customers to collect the RTIs less clearly due to the cost of having the RTIs stored in the customer facilities is very low compared to the cost of visiting the customer. It must also be considered that the vehicle's capacity may become a constraint that forces the frequency of visits to be more significant. Another factor related to the frequency of visits is the proximity of customers to the warehouse. Note that due to the supplier's proximity to customers 4 and 12, their transportation cost is very low, so the visit is scheduled for all periods of the planning horizon. See Fig. 7(a).



a)



b)



c)

Fig. 7. A sequence of visits for period 1 of the planning horizon in the CLIRPB, with 15 customers by considering the supplier location at (0,0) and values of $\boldsymbol{h_1} = \mathbf{4}$ and $\boldsymbol{h_2} = [\mathbf{3\ 2\ 1}]$, cases a, b, c.

Considering the importance of parameters h_1 and h_2 for a scheduled visit, we have calculated the difference between $h_1 - h_2$. Using the concept of elasticity, we have calculated the percentage impact on costs C that generates a percentage increase of 1% of the difference of $h_1 - h_2$. We have performed the proposed approach's two runs or scenarios (A and B). Let be the values of C_{A} and C_{B} as the value of (1) for scenarios A and B. The traditional point elasticity formula calculated is $e =$

$\frac{(h_{1B}-h_{2B})-(h_{1A}-h_{2A})}{(h_{1A}-h_{2A})} / \frac{(C_B-C_A)}{C_A}$, where $(h_{1B}-h_{2B})$ and $(h_{1A}-h_{2A})$ are the differences associated h_1-h_2 for the corresponding model A and B respectively. Note that the value of $e \in [0,1]$. Indeed, varying the difference h_1-h_2 within the range $(h_{1B}-h_{2B})-(h_{1A}-h_{2A})$ (increasing or decreasing) by 1%, then cost C must vary in the same direction (increase or decrease) but by a smaller percentage (inelastic). If $e<0$, it is understood that when h_1-h_2 varies by 1% in a specific direction, C will vary in the opposite direction. Finally, if $|e|>1$, it is said that h_1-h_2 is elastic concerning C or varies by a percentage greater than the percentage variation of h_1-h_2.

Table 17 shows the calculation of the elasticity of the distance h_1-h_2 concerning the Total Cost. For example, the value of $e=-1.57$ is calculated with the values of $h_{15}-h_{25}=3.75$, $h_{16}-h_{26}=3.87$, $C_5=14.233$, $C_6=13.487$, and its meaning is that an increase of 1% in the distance h_1-h_2 , within the range $(h_{16}-h_{26})-(h_{15}-h_{25})$, is associated with a 1.57% reduction in total costs (the value of $e=-1.57$ is shown in the last line of Table 16 – see the value of the last column). Note that the smaller value of $(h_{15}-h_{25})$ and $(h_{16}-h_{26})$ are, the elasticity of h_1-h_2, on total cost decreases, noting a greater impact on costs as the difference of h_1-h_2 is greater.

Table 17
Elasticity of $\boldsymbol{h_1-h_2}$over Total Cost by considering supplier location at (0,0) and $\boldsymbol{h_1=4}$

Run	h_1	h_2	h_1-h_2	Total Cost (1)	Elasticity of h_1-h_2 over Total Cost
1	4.00	3.00	1.00	18104	
2	4.00	2.00	2.00	17894	-0.01
3	4.00	1.00	3.00	16514	-0.15
4	4.00	0.50	3.50	15355	-0.42
5	4.00	0.25	3.75	14233	-1.02
6	4.00	0.13	3.87	13487	**-1.57**

Table 18 shows the elasticities of distance h_1-h_2 on Holding Costs and Transportation Costs, highlighting in bold the highest elasticity values are always associated with cases associated with greater distances h_1-h_2. When considering a different supplier location and another demand condition, the same is observed concerning the elasticity (disturbance) of costs in the case of greater distances h_1-h_2 .

Table 18
Elasticity of $\boldsymbol{h_1-h_2}$ over Costs considering supplier location at (0,0) and $\boldsymbol{h_1=4}$

Run	$\boldsymbol{h_1}$	$\boldsymbol{h_2}$	$\boldsymbol{h_1-h_2}$	**Elasticity of $\boldsymbol{h_1-h_2}$ on Holding Cost**	**Elasticity of $\boldsymbol{h_1-h_2}$ on Transportation Cost**
1	4.00	3.00	1.00		
2	4.00	2.00	2.00	2.64	-0.08
3	4.00	1.00	3.00	0.65	-0.24
4	4.00	0.50	3.50	0.50	-0.58
5	4.00	0.25	3.75	**-6.11**	0.03
6	4.00	0.13	3.87	-1.76	**-1.54**

Elasticity decomposes the problem analysis performed by the incidence of the holding cost and transportation cost parameters. They not only provide important insight into the performance of the decisions of the inventory routing problem but also point to the parameters where the management should focus. However, for the sensible use of elasticity in real richly problems, additional information related to different types of holding and transportation costs depending on the products and their options for management are required. Such information may indicate that the management should focus on parameters with a large difference between h_1 and h_2 rather than the shorter elasticities. Obtained preliminary results suggest that elasticities are reasonably accurate in predicting even significant changes in the objective function (total cost) due to significant changes in holding and transportation costs. However, more information about the robustness of elasticity in realistic settings is needed, considering effects such as uncertainties in parameter estimates, model structure, covariance, and correlations between h_1 and h_2. However, from practice, when $h_1 >> h_2$ indicates that it is convenient to negotiate with the administration of the RTIs given the low holding costs, it will depend on the customer's storage capacity and the degree of cooperation in the chain. Therefore, the model takes advantage of this fact and reduces collections' frequency accordingly, obtaining significant savings in RTI recovery costs. Regarding the location of the supplier, additional experiments were performed with arbitrary locations for a basic comparison. In all cases studied, it was observed that for a huge value of h_1 concerning the value h_2, the percentage variation of holding, transportation, and total cost is more significant, confirming the above described. In addition, when the impact on the total cost of the different supplier locations has been analyzed, we can see that these changes are not significant due to the delta of the route schedule being related to the edges connecting the supplier with the remaining route.

4.2.2 Base case results with equal demand

In this case, the aim is to inquire about the model's performance when the customer demand is homogeneous (the same for all customers). In this case, when conducting experiments similar to those in Table 17, we get that the elasticity relationship h_2 and the Total Cost is positive for any supplier location. Indeed, when the value of one increases, then the other value increases, and vice versa. The same behavior was observed between h_2 and the Cost of Transportation, with few exceptions, conclusions compatible with the initial case when the demand differed between customers. When comparing the obtained results for the initial case versus the case of equal demand, there are more significant impacts of the greater difference between h_1 and h_2 ($h_1 - h_2$). When the supplier is located at (0,0), there is a more significant difference in transportation costs for all scenarios, specifically when h_1 and h_2 are further away. However, in cases where the supplier is in the center of the graph or elsewhere, the costs do not vary significantly when comparing the two scenarios (Figs. 8-10).

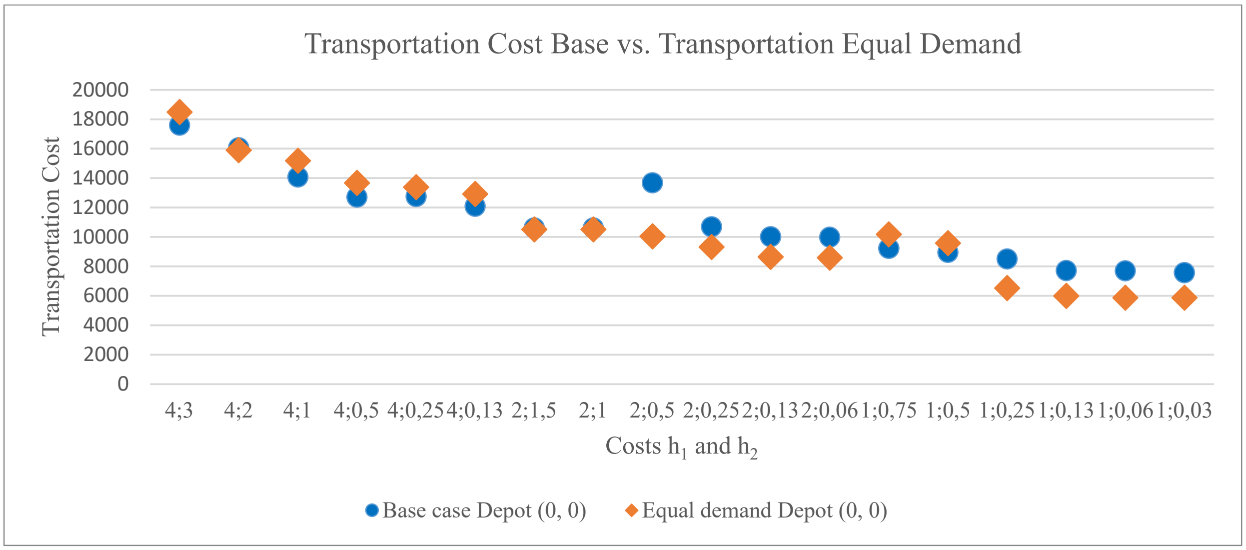


Fig. 8. Transportation cost analysis for base and equal demand cases when the supplier is located at (0,0)

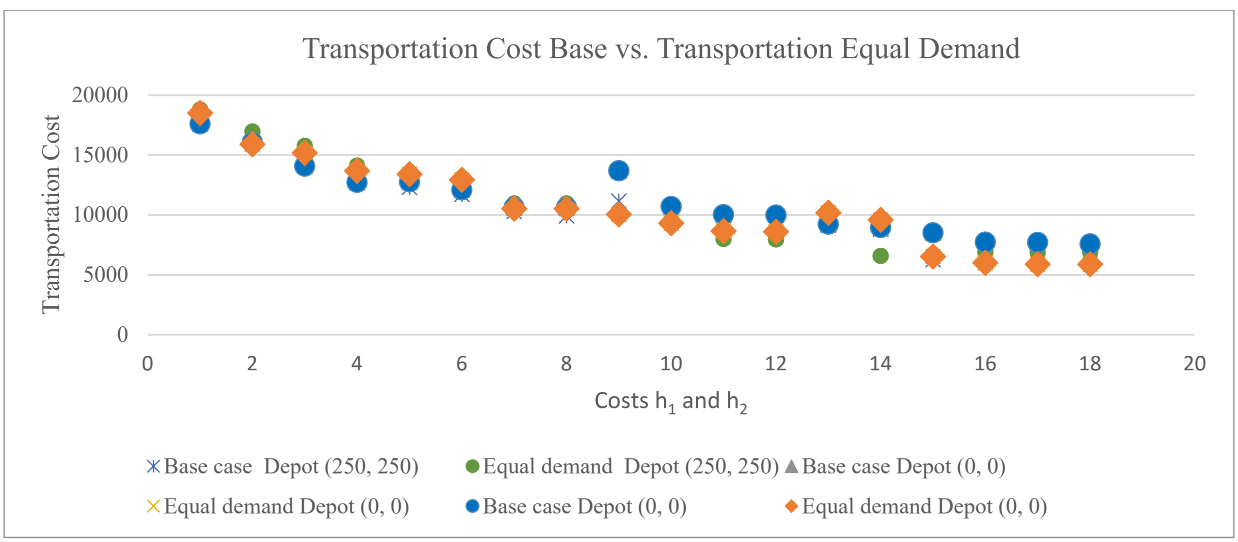


Fig. 9. Transportation cost analysis for base and equal demand cases when the supplier is located at (250,250)

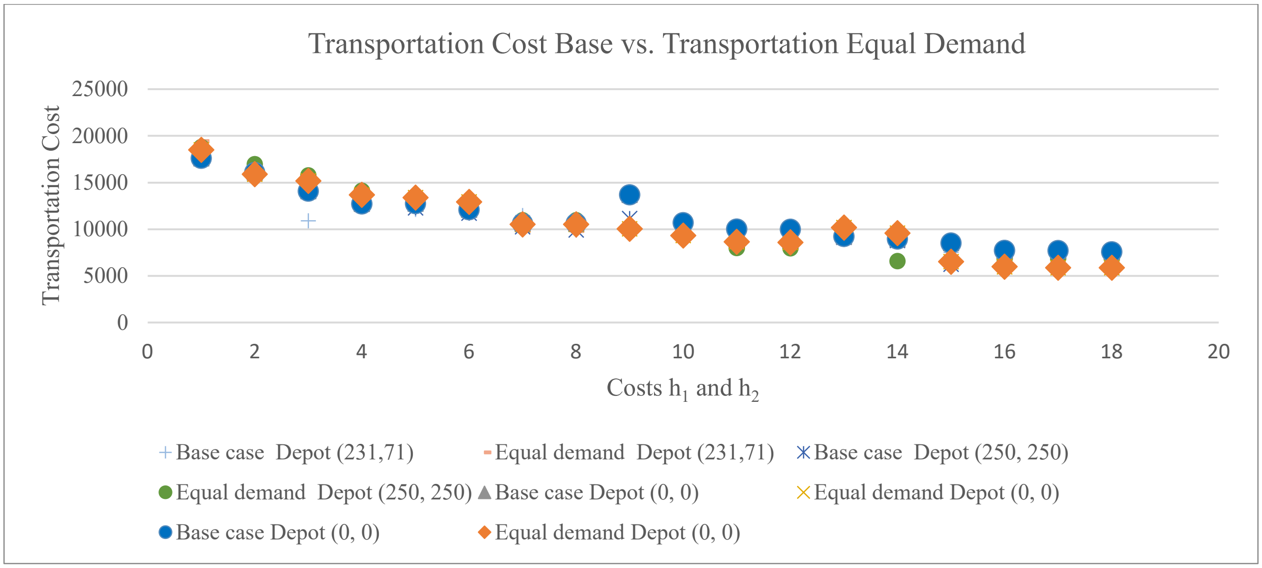


Fig. 10. Transportation cost analysis for base and equal demand cases when the supplier is located at (231,71)

In addition, when the impact on the total cost of the different supplier locations was analyzed, we can see that these changes are insignificant. This fact is due to the route schedule changes related to the arcs connecting the supplier to the remaining route. Finally, Table 19 summarizes the total results for the complete data set.

Figs. 11-13 show the computing times for each of the instances (10 to 30) for the different values of h_1 and h_2. Figures 11 to 13 show the computing times for instances (customers ranging from 10 to 30; and depot located on the corner, the middle, and random node) for the different values of h1 and h2. These figures show that the computing times have a similar behavior pattern, except where the depot is located on the corner of the grid (0, 0). For the instance of 30 customers and when the deposit is located randomly (396,172) for the instance of 25 customers, the computing times become pretty high. Generally, the instances of 25 customers have the highest computational times regardless of the type of depot location. However, even if these results are considered as such since it is a periodic decision that can be used during several future time horizons, these times can be considered acceptable.

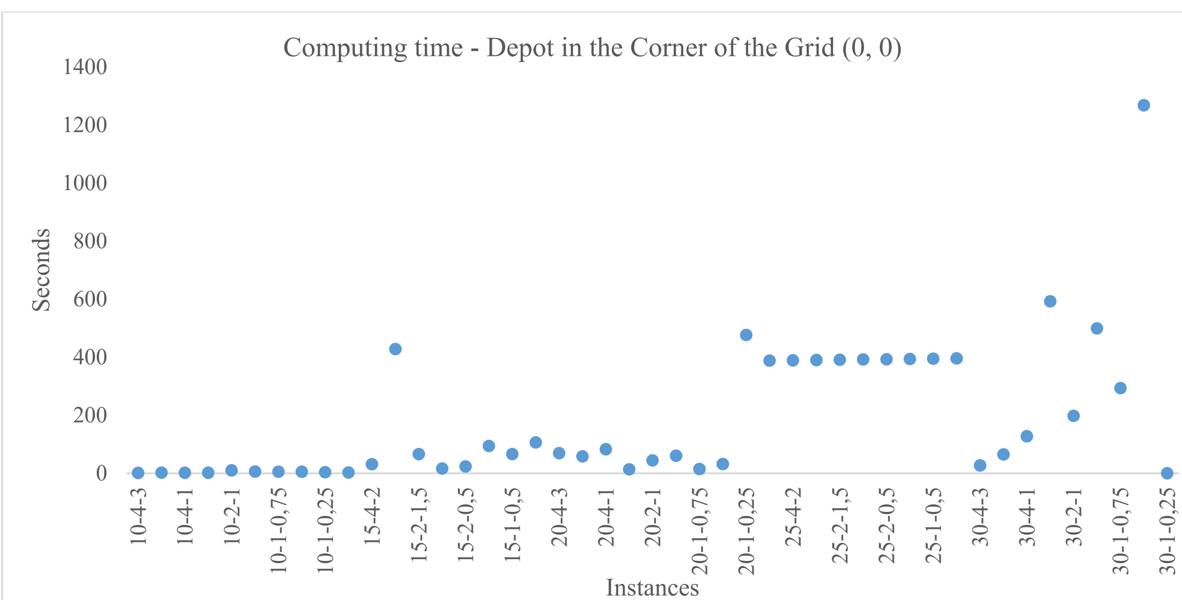


Fig. 11. Computing time when the supplier is located at (0,0)

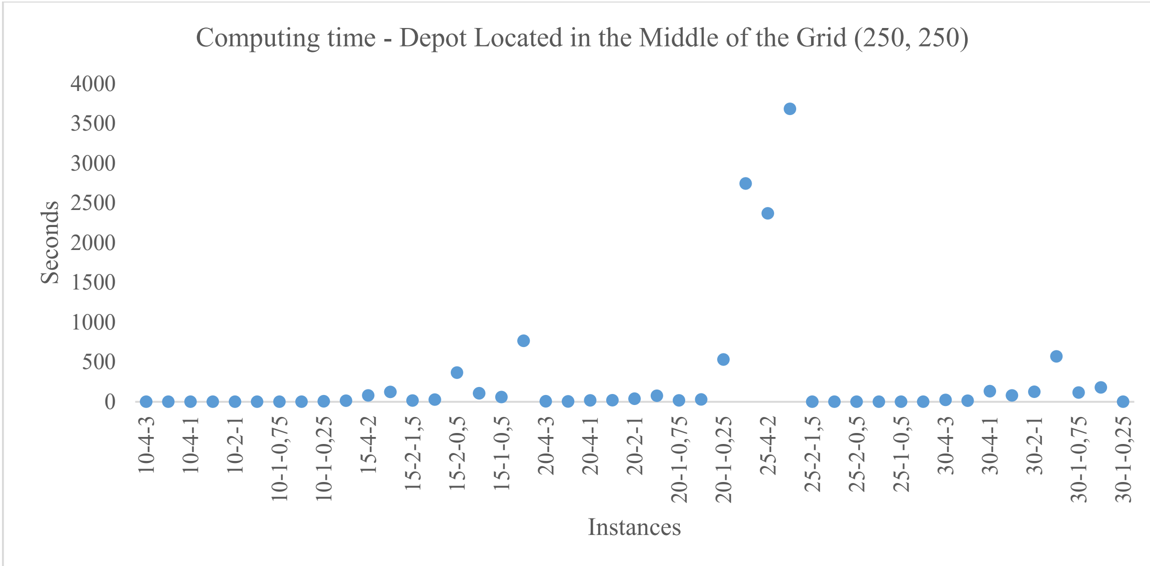


Fig. 12. Computing time when the supplier is located at (250,250)

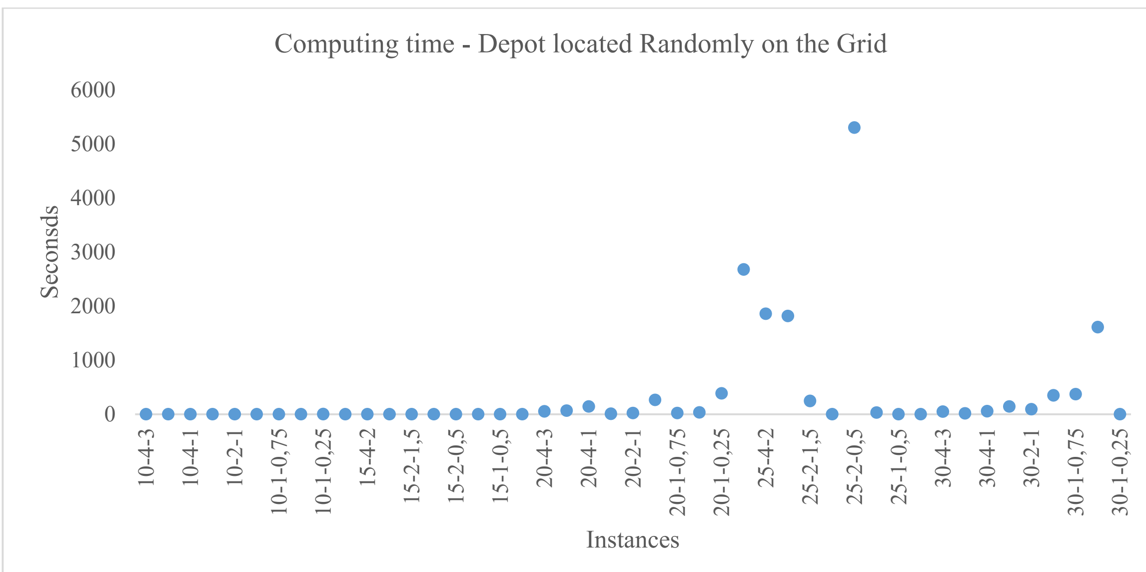


Fig. 13. Computing time when the supplier is located at random grid

Table 19
Final results for the complete synthetic data set

Depot Location	Customers	[\$/unit]h_1	[\$/unit]h_2	[\$]C_1	[\$]C_2	Transportation cost [\$]	[\$]C_3	[\$]C_4	Holding cost [\$]	Total cost [\$]	Time
Grid Corner (0, 0)	10	4.00	3.00	6363	6363	12726	924	693	1617	14343	0.94
	10	4.00	2.00	5601	4971	10572	1812	1470	3282	13854	1.76
	10	4.00	1.00	6609	3612	10221	672	1581	2253	12474	1.28
	10	2.00	1.50	4971	4641	9612	1470	1377	2847	12459	1.24
	10	2.00	1.00	3612	3612	7224	3162	1581	4743	11967	9.53
	10	2.00	0.50	5043	3019	8062	1470	1360.5	2830.5	10892.5	5.81
	10	1.00	0.75	3612	3612	7224	1581	1185.8	2766.8	9990.8	5.12
	10	1.00	0.50	3612	3612	7224	1581	790.5	2371.5	9595.5	4.92
	10	1.00	0.25	3612	3612	7224	1581	395.2	1976.2	9200.2	3.62

Table 19
Final results for the complete synthetic data set (Continue)

Depot Location	Customers	[$/unit]$h_1$	[$/unit]$h_2$	[$]$C_1$	[$]$C_2$	Transportation cost [$]	[$]$C_3$	[$]$C_4$	Holding cost [$]	Total cost [$]	Time
Grid Middle (250,250)	10	4.00	3.00	6504	5451	11955	252	1143	1395	13350	1.16
	10	4.00	2.00	6504	4557	11061	252	1470	1722	12783	1.16
	10	4.00	1.00	6504	3411	9915	252	1581	1833	11748	2.88
	10	2.00	1.50	4557	4227	8784	1470	1377	2847	11631	2.24
	10	2.00	1.00	4557	3918	8475	1470	1140	2610	11085	3.94
	10	2.00	0.50	4557	3111	7668	1470	1090.5	2560.5	10228.5	9.17
	10	1.00	0.75	2756	2756	5512	2214	1660.5	3874.5	9386.5	8.38
	10	1.00	0.50	2797	2562	5359	2214	1233	3447	8806	6.59
	10	1.00	0.25	2756	1989	4745	2214	1021.5	3235.5	7980.5	7.34
Grid Middle (107, 430)	10	4.00	3.00	4461	4461	8922	2412	1809	4221	13143	1.42
	10	4.00	2.00	4461	4167	8628	2412	1470	3882	12510	1.56
	10	4.00	1.00	4461	4167	8628	2412	1470	3882	12510	3.09
	10	2.00	1.50	4461	3264	7725	2412	1398	3810	11535	1.41
	10	2.00	1.00	4167	4167	8334	1470	1102.5	2572.5	10906.5	0.81
	10	2.00	0.50	4167	3264	7431	1470	1398	2868	10299	2.72
	10	1.00	0.75	3264	3264	6528	1398	1048.5	2446.5	8974.5	4.28
	10	1.00	0.50	2989	2891	5880	1716	924	2640	8520	4.62
	10	1.00	0.25	3167	1944	5111	1716	1130.2	2846.2	7957.2	3.22
Grid Corner (0,0)	15	4.00	3.00	8800	8800	17600	288	216	504	18104	2.49
	15	4.00	2.00	8800	7258	16058	288	1548	1836	17894	31.19
	15	4.00	1.00	9205	4870	14075	288	2151	2439	16514	428.05
	15	2.00	1.50	5936	4670	10606	2706	3051	5757	16363	66.2
	15	2.00	1.00	5936	4670	10606	2706	2034	4740	15346	16.06
	15	2.00	0.50	6841	4224	11065	1554	1587	3141	14206	23.28
	15	1.00	0.75	4634	4634	9268	2073	1554.75	3627.75	12895.75	94.11
	15	1.00	0.50	4551	4396	8947	2187	1201.5	3388.5	12335.5	66.08
	15	1.00	0.25	4577	3930	8507	2226	915.75	3141.75	11648.75	106.02
Grid Middle (250,250)	15	4.00	3.00	9016	9016	18032	1056	792	1848	19880	13.67
	15	4.00	2.00	9016	7198	16214	1056	2058	3114	19328	79.55
	15	4.00	1.00	9792	4801	14593	444	2286	2730	17323	124.84
	15	2.00	1.50	5139	5139	10278	4074	3055	7129	17407	15.03
	15	2.00	1.00	5139	4775	9914	4074	2319	6393	16307	27.03
	15	2.00	0.50	7251	3894	11145	2052	1848	3900	15045	367.17
	15	1.00	0.75	4634	4510	9144	2439	1946.25	4385.25	13529.25	107.78
	15	1.00	0.50	4634	4165	8799	2439	1593	4032	12831	60.09
	15	1.00	0.25	4256	2005	6261	3162	2223	5385	11646	766.56
Grid Middle (231, 71)	15	4.00	3.00	9268	9268	18536	288	216	504	19040	8.2
	15	4.00	2.00	9268	7738	17006	288	1548	1836	18842	77.34
	15	4.00	1.00	9463	4444	13907	900	2442	3342	17249	657.92
	15	2.00	1.50	5824	5645	11469	3144	2520	5664	17133	29.94
	15	2.00	1.00	5714	4444	10158	3360	2442	5802	15960	21.33
	15	2.00	0.50	5714	4444	10158	3360	1221	4581	14739	167.36
	15	1.00	0.75	4444	4444	8888	2442	1831.5	4273.5	13161.5	99.09
	15	1.00	0.50	4444	4444	8888	2442	1221	3663	12551	28.05
	15	1.00	0.25	4577	2948	7525	2418	1807.5	4225.5	11750.5	665.3
Grid Corner (0,0)	20	4.00	3.00	10761	10761	21522	636	477	1113	22635	69.01
	20	4.00	2.00	10815	8424	19239	636	2244	2880	22119	58.38
	20	4.00	1.00	11103	5577	16680	636	3123	3759	20439	82.49
	20	2.00	1.50	8391	7290	15681	2244	2596.5	4840.5	20521.5	13.33
	20	2.00	1.00	9009	5577	14586	1860	3123	4983	19569	43.86
	20	2.00	0.50	9009	5577	14586	1860	1561.5	3421.5	18007.5	60.01
	20	1.00	0.75	5577	5577	11154	3123	2342.25	5465.25	16619.25	14.25
	20	1.00	0.50	5577	5577	11154	3123	1561.5	4684.5	15838.5	31.77
	20	1.00	0.25	5495	4846	10341	3243	1365.75	4608.75	14949.75	476.61
Grid Middle (250,250)	20	4.00	3.00	10650	10650	21300	636	477	1113	22413	7.14
	20	4.00	2.00	10650	10650	21300	636	477	1113	22413	4.17
	20	4.00	1.00	11391	5673	17064	120	2880	3000	20064	18.11
	20	2.00	1.50	7830	7755	15585	2646	2052	4698	20283	19.3
	20	2.00	1.00	8049	5673	13722	2646	2880	5526	19248	37.83
	20	2.00	0.50	7993	4323	12316	2646	2599.5	5245.5	17561.5	77.33
	20	1.00	0.75	5673	5439	11112	2880	2342.25	5222.25	16334.25	16.92
	20	1.00	0.50	5673	4981	10654	2880	1981.5	4861.5	15515.5	29.22
	20	1.00	0.25	5746	3482	9228	3000	1911.75	4911.75	14139.75	532.5

Table 19
Final results for the complete synthetic data set (Continue)

Depot	Customers	[$/unit]$h_1$	[$/unit]$h_2$	[$]$C_1$	[$]$C_2$	Transportation cost [$]	[$]$C_3$	[$]$C_4$	Holding cost [$]	Total cost [$]	Time
Grid Random (96, 49)	20	4.00	3.00	10761	10761	21522	636	477	1113	22635	55.61
	20	4.00	2.00	10815	8424	19239	636	2244	2880	22119	66.94
	20	4.00	1.00	11103	5505	16608	636	3123	3759	20367	143.56
	20	2.00	1.50	8391	7290	15681	2244	2596.5	4840.5	20521.5	10.91
	20	2.00	1.00	9009	5505	14514	1860	3123	4983	19497	23.58
	20	2.00	0.50	9246	5152	14398	1548	1981.5	3529.5	17927.5	265.83
	20	1.00	0.75	5505	5505	11010	3123	2342.25	5465.25	16475.25	24.05
	20	1.00	0.50	5505	5505	11010	3123	1561.5	4684.5	15694.5	34.2
	20	1.00	0.25	5423	4622	10045	3243	1365.75	4608.75	14653.75	387.01
Grid Corner (0, 0)	25	4.00	3.00	11460	11460	22920	3244	0	0	22920	388.01
	25	4.00	2.00	11241	10902	22143	3245	288	402	22545	389.01
	25	4.00	1.00	11862	6531	18393	3246	0	3462	21855	390.01
	25	2.00	1.50	9465	9063	18528	3247	2010	1863	20391	391.01
	25	2.00	1.00	9993	6936	16929	3248	1566	3135	20064	392.01
	25	2.00	0.50	9993	6408	16401	3249	1566	1860	18261	393.01
	25	1.00	0.75	6531	6531	13062	3250	3462	2596.5	15658.5	394.01
	25	1.00	0.50	6531	6531	13062	3251	3462	1731	14793	395.01
	25	1.00	0.25	6883	5102	11985	3252	3228	1902.8	13887.8	396.01
Grid Middle (250, 250)	25	4.00	3.00	12549	10905	23454	0	1557	1903.8	25357.8	2745
	25	4.00	2.00	12549	10089	22638	0	1644	1904.8	24542.8	2370
	25	4.00	1.00	12549	6606	19155	0	3531	1905.8	21060.8	3683
	25	2.00	1.50	9684	9072	18756	2088	2151	1906.8	20662.8	1733.4
	25	2.00	1.00	10089	6606	16695	1644	3531	1907.8	18602.8	2002.2
	25	2.00	0.50	10089	6002	16091	1644	2169	1908.8	17999.8	9549.2
	25	1.00	0.75	6240	6240	12480	3918	2938.5	1909.8	14389.8	42.2
	25	1.00	0.50	6606	6002	12608	3531	2169	1910.8	14518.8	43.1
	25	1.00	0.25	6428	3677	10105	4086	2635.5	1911.8	12016.8	59.4
Grid Random (396, 172)	25	4.00	3.00	11568	11568	23136	0	0	1912.8	25048.8	2677
	25	4.00	2.00	11349	11010	22359	288	402	1913.8	24272.8	1859
	25	4.00	1.00	11970	6273	18243	0	3660	1914.8	20157.8	1818
	25	2.00	1.50	9573	9171	18744	2010	1863	1915.8	20659.8	248
	25	2.00	1.00	10893	6789	17682	528	3498	1916.8	19598.8	7212.2
	25	2.00	0.50	10101	6150	16251	1566	1959	1917.8	18168.8	5303.2
	25	1.00	0.75	6273	6273	12546	3660	2745	1918.8	14464.8	31
	25	1.00	0.50	6162	6162	12324	3789	1894.5	1919.8	14243.8	33.0
	25	1.00	0.25	6640	4825	11465	3426	2022	1920.8	13385.8	570.0
Grid Corner (0, 0)	30	4.00	3.00	12804	12804	25608	720	540	1921.8	27529.8	26.94
	30	4.00	2.00	12804	12201	25005	720	882	1922.8	26927.8	64.99
	30	4.00	1.00	12804	8558	21362	720	3096	1923.8	23285.8	127.44
	30	2.00	1.50	11793	11007	22800	1278	1629	1924.8	24724.8	592.86
	30	2.00	1.00	11941	8558	20499	1146	3096	1925.8	22424.8	197.39
	30	2.00	0.50	12405	6589	18994	696	2760	1926.8	20920.8	499.52
	30	1.00	0.75	8083	7360	15443	3618	3323.25	1927.8	17370.8	293.41
	30	1.00	0.50	8083	6992	15075	3618	2523	1928.8	17003.8	1268.39
	30	1.00	0.25	8083	6026	14109	3618	2004.75	1929.8	16038.8	33680.88
Grid Middle (250, 250)	30	4.00	3.00	12711	12711	25422	876	657	1930.8	27352.8	23.14
	30	4.00	2.00	12574	12574	25148	1044	522	1931.8	27079.8	13.53
	30	4.00	1.00	13101	7592	20693	492	3900	1932.8	22625.8	133.12
	30	2.00	1.50	11455	10577	22032	1710	2101.5	1933.8	23965.8	80.47
	30	2.00	1.00	11455	7592	19047	1710	3900	1934.8	20981.8	126.24
	30	2.00	0.50	11455	6356	17811	1710	2641.5	1935.8	19746.8	571.58
	30	1.00	0.75	7548	6356	13904	3942	3962.25	1936.8	15840.8	115.84
	30	1.00	0.50	7548	6356	13904	3942	2641.5	1937.8	15841.8	180.64
	30	1.00	0.25	7137	4844	11981	4473	2432.25	1938.8	13919.8	10698.84
Grid Random (152, 34)	30	4.00	3.00	12432	12432	24864	876	657	1939.8	26803.8	48.92
	30	4.00	2.00	12295	12295	24590	1044	522	1940.8	26530.8	19.09
	30	4.00	1.00	12432	8177	20609	876	3249	1941.8	22550.8	58
	30	2.00	1.50	12024	10740	22764	834	1687.5	1942.8	24706.8	142.47
	30	2.00	1.00	12295	8177	20472	522	3249	1943.8	22415.8	94.34
	30	2.00	0.50	12342	6173	18515	522	2838	1944.8	20459.8	349.5
	30	1.00	0.75	7816	6944	14760	3657	3440.25	1945.8	16705.8	371.81
	30	1.00	0.50	7816	6576	14392	3657	2601	1946.8	16338.8	1612.56
	30	1.00	0.25	7243	5368	12611	4521	2036.25	1947.8	14558.8	27505.17

5. Concluding Remarks

This paper introduces the IRP by considering not only the forward delivery but also the use of RTIs in the distribution strategy considering inventory routing decisions with RTIs collection (backhaul customers) within a Closed-Loop Supply Chain within a short-term planning horizon. We consider a distribution system with a supplier and a set of N customers that must be served to meet their known demand on a discrete planning horizon of T periods (days within a week). We proposed a mathematical model to represent and solve the considered problem. The supply schedule must decide the period to visit the customer, the quantity of product to be delivered and collected for each visit, and the vehicle's route.

We have analyzed the elasticity of the results (schedule visits) by considering the transportation and holding cost changes. The results show the more significant impacts of the more significant difference between h_1 and h_2 $(h_1 - h_2)$ when comparing the obtained results for the initial case versus the case of equal demand. The location of the supplier has a more significant difference in transportation costs, specifically when h_1 and h_2 are further away from each other. The obtained results show the efficiency of the proposed optimization scheme for solving the combined IRP with RTIs, which could be extended to different real application problems.

For future research, we suggest the following aspects to develop.

- Consider many patterns and corresponding input values to feed the proposed model. The delivery and collection patterns are intended to simulate possible basic patterns evaluated in the proposed approach without being exhaustive. The delivery and collection patterns are intended to simulate possible basic patterns evaluated in the proposed approach without being exhaustive. The mathematical model is flexible, and other patterns and relative input parameters could be explored in future works. Indeed, any set of patterns could be chosen. The proposed model is intended to be generic and flexible, exposing analyses that do not seek to depend on the selected patterns.
- Extend the problem by considering various functions (environmental emissions (Mahjoob et al., 2021), financial risk (Chen & Lin, 2009), and perishable aspects (Shamsiet et al., 2014)). Besides, obtaining a multiobjective problem using some strategy to generate a Pareto frontier with two or three objective functions (Escobar, 2017; Polo et al., 2019; Tordecilla-Madera et al., 2018; Escobar et al., 2020).
- Use the optimization of heuristics techniques to solve the model in large instances (Cárdenas-Barrón & Melo, 2021; Alvarez et al., 2020; Ramadhan & Imran, 2020), and also extend the heuristic such as granular tabu search based on similar problems (Puenayán et al., 2014; Escobar et al., 2022; Escobar et al., 2014a; Linfati et al., 2014a; Linfati et al., 2014b; Escobar, 2014; Bernal et al., 2017, and Bernal-Moyano et al., 2017) or genetic algorithms (Bolaños et al., 2018 and Escobar-Falcón et al., 2021). Besides, the problem's solution is by considering efficient approaches based on Granular Tabu Search with the combination of Variable Neighborhood Search (VNS) or Iterated Local Search (ILS) such as Escobar et al. (2014b) and Escobar et al. (2015).
- Solving the proposed model for large instances uses advanced mathematical optimization techniques, such as branch-and-cut and branch-and-price (Bard & Nananukul, 2010; Grønhaug et al., 2010).
- Solving the multiperiod problem by considering a homogeneous and heterogeneous fleet of vehicles. Besides, solving the stochastic version of this problem and the CLIRPB.

Conflict of Interest

The authors declare that we don't have any conflict or competing interests.

References

Agra, A., Requejo, C., & Rodrigues, F. (2018). A hybrid heuristic for a stochastic production-inventory-routing problem. *Electronic Notes in Discrete Mathematics, 64*, 345-354.

Aksen, D., Kaya, O., Salman, F. S., & Akça, Y. (2012). Selective and periodic inventory routing problem for waste vegetable oil collection. *Optimization Letters, 6*(6), 1063-1080.

Aksen, D., Kaya, O., Salman, F. S., & Tüncel, Ö. (2014). An adaptive large neighborhood search algorithm for a selective and periodic inventory routing problem. *European Journal of Operational Research, 239*(2), 413-426.

Al Shamsi, A., Al Raisi, A., & Aftab, M. (2014). Pollution-inventory routing problem with perishable goods. In Logistics operations, supply chain management and sustainability (pp. 585-596). Springer, Cham.

Alvarez, A., Munari, P., & Morabito, R. (2018). Iterated local search and simulated annealing algorithms for the inventory routing problem. *International Transactions in Operational Research, 25*(6), 1785-1809.

Alvarez, A., Cordeau, J. F., Jans, R., Munari, P., & Morabito, R. (2020). Formulations, branch-and-cut and a hybrid heuristic algorithm for an inventory routing problem with perishable products. *European Journal of Operational Research, 283*(2), 511-529.

Alvarez, A., Cordeau, J. F., Jans, R., Munari, P., & Morabito, R. (2021). Inventory routing under stochastic supply and demand. *Omega, 102*, 102304.
Arab, R., Ghaderi, S. F., & Tavakkoli-Moghaddam, R. (2018). Bi-objective inventory routing problem with backhauls under transportation risks: two meta-heuristics. *Transportation Letters, 12*(2), 1-17.
Archetti, C., Bertazzi, L., Laporte, G., & Speranza, M. G. (2007). A branch-and-cut algorithm for a vendor-managed inventory-routing problem. *Transportation Science, 41*(3), 382-391.
Archetti, C., Desaulniers, G., & Speranza, M. G. (2017). Minimizing the logistic ratio in the inventory routing problem. *EURO Journal on Transportation and Logistics, 6*(4), 289-306.
Archetti, C., Coelho, L. C., & Speranza, M. G. (2019). An exact algorithm for the inventory routing problem with logistic ratio. *Transportation Research Part E: Logistics and Transportation Review, 131*, 96-107.
Azadeh, A., Elahi, S., Farahani, M. H., & Nasirian, B. (2017). A genetic algorithm-Taguchi based approach to inventory routing problem of a single perishable product with transshipment. *Computers & Industrial Engineering, 104*, 124-133.
Bard, J. F., & Nananukul, N. (2009). Heuristics for a multiperiod inventory routing problem with production decisions. *Computers & Industrial Engineering, 57*(3), 713-723.
Bard, J. F., & Nananukul, N. (2010). A branch-and-price algorithm for an integrated production and inventory routing problem. *Computers & Operations Research, 37*(12), 2202-2217.
Benoist, T., Gardi, F., Jeanjean, A., Estellon, B. (2011). Randomized local search for real-life inventory routing. *Transportation Science, 45*(3), 381–398.
Bernal, J., Escobar, J. W., & Linfati, R. (2017). A granular tabu search algorithm for a real case study of a vehicle routing problem with a heterogeneous fleet and time windows. *Journal of Industrial Engineering and Management, 10*(4), 646-662.
Bernal-Moyano, J. A., Escobar, J. W., Marín-Moreno, C., Linfati, R., & Gatica, G. (2017). A comparison of trajectory granular based algorithms for the location-routing problem with heterogeneous fleet (LRPH). *Dyna, 84*(200), 193–201.
Bertazzi, L., Coelho, L. C., De Maio, A., & Laganà, D. (2019). A matheuristic algorithm for the multi-depot inventory routing problem. *Transportation Research Part E: Logistics and Transportation Review, 122*, 524-544.
Bolaños, R., Escobar, J., & Echeverri, M. (2018). A metaheuristic algorithm for the multi-depot vehicle routing problem with heterogeneous fleet. *International Journal of Industrial Engineering Computations, 9*(4), 461–478.
Campbell, A., Clarke, L., Kleywegt, A., & Savelsbergh, M. (1998). The inventory routing problem. In Fleet management and logistics (pp. 95-113). Springer, Boston, MA.
Campbell, A. M., & Savelsbergh, M. W. (2004). A decomposition approach for the inventory-routing problem. *Transportation Science, 38*(4), 488-502.
Cárdenas-Barrón, L. E., & Melo, R. A. (2021). A fast and effective MIP-based heuristic for a selective and periodic inventory routing problem in reverse logistics. *Omega, 103*, 102394.
Cárdenas-Barrón, L. E., González-Velarde, J. L., Treviño-Garza, G., & Garza-Nuñez, D. (2019). Heuristic algorithm based on reduce and optimize approach for a selective and periodic inventory routing problem in a waste vegetable oil collection environment. *International Journal of Production Economics, 211*, 44-59.
Carrasco-Gallego, R., Ponce-Cueto, E., & Dekker, R. (2012). Closed-loop supply chains of reusable articles: a typology grounded on case studies. *International Journal of Production Research, 50*(19), 5582-5596.
Chen, Y. M., & Lin, C. T. (2009). A coordinated approach to hedge the risks in stochastic inventory-routing problem. *Computers & Industrial Engineering, 56*(3), 1095-1112.
Coelho, L. C., Cordeau, J. F., & Laporte, G. (2012). The inventory-routing problem with transshipment. *Computers & Operations Research, 39*(11), 2537-2548.
Coelho, L. C., Cordeau, J. F., & Laporte, G. (2014a). Thirty years of inventory routing. *Transportation Science, 48*(1), 1-19.
Coelho, L. C., Cordeau, J. F., & Laporte, G. (2014b). Heuristics for dynamic and stochastic inventory-routing. *Computers & Operations Research, 52*, 55-67.
Escobar, J. W. (2014). Heuristic algorithms for the capacitated location-routing problem and the multi-depot vehicle routing problem. *4OR, 12*, 99 – 100.
Escobar, J. W., Linfati, R., Toth, P., & Baldoquin, M. G. (2014a). A hybrid granular tabu search algorithm for the multi-depot vehicle routing problem. *Journal of Heuristics, 20*(5), 483–509.
Escobar, J. W., Linfati, R., Baldoquin, M. G., & Toth, P. (2014b). A Granular Variable Tabu Neighborhood Search for the capacitated location-routing problem. *Transportation Research Part B: Methodological, 67*, 344–356.
Escobar, J. W., Linfati, R., & Adarme-Jaimes, W. (2015). A hybrid metaheuristic algorithm for the capacitated location routing problem. *Dyna, 82*(189), 243-251.
Escobar, J. W. (2017). Supply chain optimization with variable demand by considering financial criteria and scenarios. *Revista Facultad de Ingeniería, 26*(44), 23-34.
Escobar, J. W., Marin, A. A., & Lince, J. D. (2020). Multi-objective mathematical model for the redesign of supply chains considering financial criteria optimisation and scenarios. *International Journal of Mathematics in Operational Research, 16*(2), 238-256.
Escobar, J.W, Duque, J., & García-Cáceres, R. (2022). A granular tabu search for the refrigerated vehicle routing problem with homogeneous fleet. *International Journal of Industrial Engineering Computations, 13*(1), 135-150.

Escobar-Falcón, L., Álvarez-Martínez, D., Wilmer-Escobar, J., & Granada-Echeverri, M. (2021). A specialized genetic algorithm for the fuel consumption heterogeneous fleet vehicle routing problem with bidimensional packing constraints. *International Journal of Industrial Engineering Computations, 12*(2), 191–204.

Farias, K., Hadj-Hamou, K., & Yugma, C. (2019). Mathematical formulations for a two-echelon inventory routing problem. *IFAC-PapersOnLine, 52*(13), 1996-2001.

Fukunaga, T., Nikzad, A., & Ravi, R. (2014). Deliver or hold: approximation algorithms for the periodic inventory routing problem. *In Approximation, Randomization, and Combinatorial Optimization. Algorithms and Techniques (APPROX/RANDOM 2014).*

Gaur, V., & Fisher, M. L. (2004). A periodic inventory routing problem at a supermarket chain. *Operations Research, 52*(6), 813-822.

Gavish, B., & Graves, S. C. (1978). The Travelling Salesman Problem and Related Problems.

Ghiami, Y., Demir, E., Van Woensel, T., Christiansen, M., & Laporte, G. (2019). A deteriorating inventory routing problem for an inland liquefied natural gas distribution network. *Transportation Research Part B: Methodological, 126*, 45-67.

Glock, C. H., & Kim, T. (2014). Container management in a single-vendor-multiple-buyer supply chain. *Logistics Research, 7*(1), 112.

Grønhaug, R., Christiansen, M., Desaulniers, G., & Desrosiers, J. (2010). A branch-and-price method for a liquefied natural gas inventory routing problem. *Transportation Science, 44*(3), 400-415.

Hariga, M., Glock, C. H., & Kim, T. (2016). Integrated product and container inventory model for a single-vendor single-buyer supply chain with owned and rented returnable transport items. *International Journal of Production Research, 54*(7), 1964-1979.

Hellström, D., & Johansson, O. (2010). The impact of control strategies on the management of returnable transport items. *Transportation Research Part E: Logistics and Transportation Review, 46*(6), 1128-1139.

Johansson, O., & Hellström, D. (2007). The effect of asset visibility on managing returnable transport items. *International Journal of Physical Distribution & Logistics Management, 37*(10), 799-815.

Juan, A. A., Grasman, S. E., Caceres-Cruz, J., & Bektaş, T. (2014). A simheuristic algorithm for the single-period stochastic inventory-routing problem with stock-outs. *Simulation Modelling Practice and Theory, 46*, 40-52.

Kim, T., Glock, C. H., & Kwon, Y. (2014). A closed-loop supply chain for deteriorating products under stochastic container return times. *Omega, 43*, 30-40.

Kleywegt, A. J., Nori, V. S., & Savelsbergh, M. W. (2002). The stochastic inventory routing problem with direct deliveries. *Transportation Science, 36*(1), 94-118.

Koç, Ç., & Laporte, G. (2018). Vehicle routing with backhauls: Review and research perspectives. *Computers & Operations Research, 91*, 79-91.

Kroon, L., & Vrijens, G. (1995). Returnable containers: an example of reverse logistics. *International Journal of Physical Distribution & Logistics Management, 25*(2), 56-68.

Lefever, W., Aghezzaf, E. H., Hadj-Hamou, K., & Penz, B. (2018). Analysis of an improved branch-and-cut formulation for the Inventory-Routing Problem with Transshipment. *Computers & Operations Research, 98*, 137-148.

Linfati, R., Escobar, J. W., & Gatica, G. (2014a). Un algoritmo metaheurístico para el problema de localización y ruteo con flota heterogénea. *Ingeniería y Ciencia, 10*(19), 55–76.

Linfati, R., Escobar, J. W., & Cuevas, B. (2014b). An algorithm based on granular tabu search for the problem of balancing public bikes by using multiple vehicles. *Dyna, 81*(186), 284-294.

Liu, S. C., Lu, M. C., & Chung, C. H. (2016). A hybrid heuristic method for the periodic inventory routing problem. *The International Journal of Advanced Manufacturing Technology, 85*(9-12), 2345-2352.

Londoño, J. C., Tordecilla, R. D., Martins, L. D. C., & Juan, A. A. (2021a). A biased-randomized iterated local search for the vehicle routing problem with optional backhauls. *TOP, 29*(2), 387-416.

Londoño J.C, Bravo J.J., Escobar J.W, (2021b). Inventory Routing Problem categorization considering pickup decisions in closed loop supply chains. *International Journal of Integrated Supply Management, forthcoming.*

Mahjoob, M., Fazeli, S. S., Tavassoli, L. S., Mirmozaffari, M., & Milanlouei, S. (2021). A green multi-period inventory routing problem with pickup and split delivery: A case study in flour industry. *Sustainable Operations and Computers, 2*, 64-70.

Malladi, K. T., & Sowlati, T. (2018). Sustainability aspects in Inventory Routing Problem: A review of new trends in the literature. *Journal of Cleaner Production, 197*, 804-814.

Markov, I., Bierlaire, M., Cordeau, J. F., Maknoon, Y., & Varone, S. (2020). Waste collection inventory routing with non-stationary stochastic demands. *Computers & Operations Research, 113*, 104798.

Nikzad, E., Bashiri, M., & Oliveira, F. (2019). Two-stage stochastic programming approach for the medical drug inventory routing problem under uncertainty. *Computers & Industrial Engineering, 128*, 358-370.

Onggo, B. S., Panadero, J., Corlu, C. G., & Juan, A. A. (2019). Agri-food supply chains with stochastic demands: A multi-period inventory routing problem with perishable products. *Simulation Modelling Practice and Theory, 97*, 101970.

Polo, A., Peña, N., Muñoz, D., Cañón, A., & Escobar, J. W. (2019). Robust design of a closed-loop supply chain under uncertainty conditions integrating financial criteria. *Omega, 88*, 110-132.

Qin, L., Miao, L., Ruan, Q., & Zhang, Y. (2014). A local search method for periodic inventory routing problem. *Expert Systems with Applications, 41*(2), 765-778.

Puenayán, D. E., Londoño, J. C., Escobar, J. W., & Linfati, R. (2014). Un algoritmo basado en búsqueda tabú granular para la solución de un problema de ruteo de vehículos considerando flota heterogénea. *Revista Ingenierías Universidad de Medellín, 13*(25), 81-98.

Ramadhan, F., & Imran, A. (2020). Metaheuristic Algorithms for the Many-to-one IRP with Dynamic Velocity. *In Journal of Physics: Conference Series, 1501*(1), 012009. IOP Publishing.

Rau, H., Budiman, S. D., & Widyadana, G. A. (2018). Optimization of the multi-objective green cyclical inventory routing problem using discrete multi-swarm PSO method. *Transportation Research Part E: Logistics and Transportation Review, 120*, 51-75.

Sarkar, B., Ullah, M., & Kim, N. (2017). Environmental and economic assessment of closed-loop supply chain with remanufacturing and returnable transport items. *Computers & Industrial Engineering, 111,* 148-163.

Singh, T., Arbogast, J. E., & Neagu, N. (2015). An incremental approach using local-search heuristic for inventory routing problem in industrial gases. *Computers & Chemical Engineering, 80*, 199-210.

Thinkaran, N., Jayaprakash, J., & Elanchezhian, C. (2019). Optimization of Total Cost in Inventory Routing Problem with Homogenous type of Vehicles using Metaheuristic algorithm–A Review. *Materials Today: Proceedings, 16,* 1043-1047.

Tordecilla-Madera, R., Roa, A. P., Escobar, J. W., & Buriticá, N. C. (2018). A mathematical model for collecting and distributing perishable products by considering costs minimisation and CO2 emissions. *International Journal of Services and Operations Management, 31*(2), 207-234.

Wei, C., Gao, W. W., Hu, Z. H., Yin, Y. Q., & Pan, S. D. (2019). Assigning customer-dependent travel time limits to routes in a cold-chain inventory routing problem. *Computers & Industrial Engineering, 133*, 275-291.

Yadollahi, E., Aghezzaf, E. H., Walraevens, J., & Raa, B. (2018). Optimizing Storage Capacity of Retailers in Stochastic Periodic Inventory Routing Problem. In ICORES (pp. 217-223).

Yadollahi, E., Aghezzaf, E. H., Walraevens, J., Raa, B., & Claeys, D. (2019). Evaluating approximate solution models for the stochastic periodic inventory routing problem. *Journal of Manufacturing Systems, 50*, 25-35.