

A new matheuristic approach based on Chu-Beasley genetic approach for the multi-depot electric vehicle routing problem

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ABSTRACT

Operations with Electric Vehicles (EVs) on logistic companies and power utilities are increasingly related due to the charging stations representing the point of standard coupling between transportation and power networks. From this perspective, the Multi-depot Electric Vehicle Routing Problem (MDEVRP) is addressed in this research, considering a novel hybrid matheuristic approach combining exact approaches and a Chu-Beasley Genetic Algorithm. An existing conflict is shown in three objectives handled through the experimentations: routing cost, cost of charging stations, and increased cost due to energy losses. EVs driving range is chosen as the parameter to perform the sensitivity analysis of the proposed MDEVRP. A 25-customer transportation network conforms to a newly designed test instance for methodology validation, spatially combined with a 33 nodes power distribution system.

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Nomenclature

Sets:

V_c	Set of customers
V_d	Set of depots
V	Set containing customers and depots $V = V_c \cup V_d$
K	Set of vehicles

Parameters:

$dist_{ij}$	Distance from i to j
Q	Load capacity of the vehicle
dep_start	Vector of depot nodes
dep_end	Copy vector of depot nodes
seq	Sequence of the nodes visited by a vehicle
Q_{bat}	Battery capacity of the Electric Vehicle (EV)
T	Study timeframe for the operation variables

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AF	Annualization factor
β	Cost per traveled kilometer [USD/km]
Υ	Cost of a charging station [USD]
t_{on}	Operation time of a charging station during the charge of an EV [$hours$]
S_{base}	Base power [kW]
Ω	Cost per kWh of energy [USD]
α	Penalty factor

Variables:

x_{ijk}	Binary decision variable that takes the value of 1 if vehicle k travels from node i to node j ; and 0
Y_{ik}	Binary decision variable that takes the value of 1 if customer i is visited by the vehicle k
d_i	Demand at node i
t_{ijk}	Remaining merchandise to be delivered at arc i, j by vehicle k
Z_{inf}	Value of infeasibility of a solution in a population of the Chu-Beasley Genetic Algorithm (CBGA)
$d_{trav(i)}$	Distance traveled at node i [km]
Z_{new}	Distance traveled by the EV along the sequence described in vector seq [km]
ER_{inst}	Charging stations installed
Δ_{losses}	Increase in energy losses of the power distribution system respect to the benchmark case [kWh]
f_{obi1}	Cost of routing of the EVs [USD]
f_{obi2}	Cost of installed charging stations [USD]
f_{obi3}	Cost of Δ_{losses} [USD]
F_{obi}	Value of the Objective Function (Total Cost) [USD]
Z_{fit}	Value of fitness function [USD]

1. Introduction

Transportation electrification represents a remarkable measure to face global warming. According to the Net Zero Emissions (NZE) scenario explained in (International Energy Agency, 2021), a 1.5°C stabilization in temperature rising is forecasted for 2050 due to zero CO₂ levels released to the atmosphere. This fact is framed into a more electrified vehicle fleet, in which the NZE scenario pledges 60% of the share of Electric Vehicles EVs sales by 2030 and almost 100% for 2050, being an ambitious goal of 250 million and 1600 millions of EVs for 2030 and 2050 respectively. Non-economic barriers such as insufficient recharging infrastructure and unreliable grids affect the EVs deployment. Particularly, the participation of EVs charging infrastructure in developing economies and emerging markets is only 0.3% worldwide. Furthermore, by 2030 the EV investment, considering electric cars and charging infrastructure, must be 25 times the present investment, boosted by clear targets regarding funding improvement and new business models. These are mainly supported by tax incentives and credit lines implemented by green banks and finance institutions.

As an effort to decrease carbon emissions in the supply chain and freight distribution management, logistic companies have been introducing zero-emission and eco-friendly vehicles in their fleets, including the last mile delivery (Zhang et al., 2020, Cataldo-Díaz et al., 2021). The main challenges around replacing freight vehicles with EVs are the high acquisition cost, waiting times for recharging, and short driving range (Juvvala & Sarmah, 2021). This latter induces range anxiety, defined as the EV driver's fear of running out of power before reaching the destination or another charging facility. The range anxiety can be diminished by either increasing EVs battery autonomy or deploying more charging stations (Pevce et al., 2020). Battery autonomy and range performance in EVs are mainly affected by external factors, i.e., driving patterns, climatic conditions, topography, and payload weight in the case of EVs for freight transportation (Christensen et al., 2017). On the other hand, the EVs range from 100 to 400 km, and internal factors affect this feature in EVs, related to battery cell configuration and power and energy density (Sharma et al., 2020). Hence, the EVs are required to refuel more often than the conventional vehicles due to the short distance traversed, especially for goods delivery operations (Juan et al., 2016).

Due to the limited travel autonomy of EVs for freight transportation, it is necessary to provide the appropriate location of charging stations, acting as range extenders of EVs batteries, considering the integration of power distribution and transportation networks. Optimal routing schemes are essential by the end of logistic companies to provide suitable decision-making results. In this regard, the Multi Depot Electric Vehicle Routing Problem (MDEVRP) is an alternative to design proper routes for EVs used in freight transportation. The MDEVRP extends the well-known Capacitated Vehicle Routing Problem CVRP with EVs (Paz et al., 2018), using multiple depots where the vehicles start to complete their routes, along which the goods are delivered to the customers. The electric nature of the vehicles makes them detour to charging stations one or several times during the route, depending on the battery autonomy.

Under the context of power networks, deploying well-planned charging facilities is highly desirable. Improvised charging of EVs at the power distribution system PDS can cause a higher peak load, voltage deviations at nodes, and violations in the

thermal limit of transformers and lines (Arias-Londoño et al., 2020). Random charging station locations could impose capacity issues for the grid operators and compromise network reliability.

In this paper, a model referred to describe the MDEVRP is proposed considering the optimal charging station location, the transportation costs, and the power distribution networks. This paper is an extension of the published works proposed by (Arias-Londoño et al., 2021), (Arias et al., 2018), and (Londoño & Granada-Echeverri, 2019) being the value-added development of a methodology combining the Multi-depot Vehicle Routing Problem MDVRP solution applied with EVs and a metaheuristic technique for charging stations located in the power network. The proposal is split into two stages. In the first stage, an exact technique is used to solve the goods distribution problem, i.e., the MDVRP was given to the customers, and their demands are solved using CPLEX. Once the routes are obtained, a Chu Beasley Genetic Algorithm CBGA is executed for each route by considering the location of charging stations.

The main contributions of this research are the following:

- We consider three objectives under conflict: EVs routing cost, installation of charging stations cost, and the cost of energy losses in the power distribution system.
- A novel hybrid matheuristic approach combining exact techniques and a metaheuristic procedure based on a genetic algorithm is proposed for the logistics and EVS charging stations location subproblems.
- We propose a new instance including the same spatial framework, the transportation, and distribution network, from the specialized literature, for algorithmic validation purposes.

The remainder of this study is organized as follows: Section 2 presents the literature review related to the green vehicle routing problem within multiple depots and the metaheuristics that have been used for solution purposes. The mathematical model for the Multi Depot Vehicle Routing Problem (MDVRP) is described in Section 3. Then, Section 4 described the hybrid methodology proposed for solving the MDEVRP. In Section 5, the CBGA is presented for solving the MDEVRP. The test system used to validate the hybrid methodology and the computational experiments are discussed in Section 6. Concluding remarks are shown in Section 7.

2. Literature Review

The MDEVRP has not been primarily addressed to our knowledge in the specialized literature. Instead, different contributions are found with a green vehicle routing problem with multiple depots. In (Wang et al., 2021a), the authors propose a hybrid methodology using genetic algorithm and column generation to solve the multi-depot EV scheduling problem. In (Paz et al., 2018), the MDEVRP is addressed as an extension of the Vehicle Location Routing Problem with multiple depots considering partial charging, battery swap stations, and time windows constraints. The mathematical models proposed are solved using CPLEX over a set of modified instances. Combined with the MDEVRP, (Zhu et al., 2020) include the demand as two-dimensional weighted items framed within the bin packing problem. Variable Neighborhood Search and Space Saving Heuristic algorithms are combined to solve vehicle routing and packing problems simultaneously. A practical logistic distribution case is considered in the numerical experiments for managerial decision purposes.

The concept of Green Vehicle Routing Problem GVRP (Lin et al., 2014) has been studied as a variation of the VRP, characterized by introducing environmental and economic costs in the objective, via the implementation of effective routes following financial indexes and environmental concerns. Unlike the Electric Vehicle Routing Problem EVRP, the GVRP involves a set of refueling stations and a set of alternative fuel vehicles AFVs, which corresponds to a broader approach (Normasari & Lathifah, 2021).

The Multi-depot Green Vehicle Routing Problem MDGVRP has been addressed by (Jabir et al., 2017). The work proposes and develops mathematical models that minimize economic and emission costs, considering managerial decisions related to the allocation of customers to depots and the assignment of the corresponding routes. An efficient ant colony optimization algorithm ACO is used to solve small and medium-size instances, while large-scale instances are solved using an integrated algorithm between ACO and VNS. Likewise, the proposed model (Li et al., 2019) applies an improved ACO algorithm to solve conflicting objectives, including revenue maximization and cost minimization. This latter encompasses traveling time and reduction of CO₂ emissions. As mentioned above, the ACO algorithm is a suitable option for solving the MDGVRP even though the problem is NP-hard, which is noticeable (Li et al., 2019). The proposal includes a two-stage algorithm, being the first stage the decomposition of the original problem into several GVRP for complexity reduction with the K-means clustering algorithm. The second stage is problem-solving, with an efficient, improved ACO algorithm that uses an adaptive pheromone incremental updating strategy.

Further diversified work is presented in (Wang et al., 2019), where the MDGVRP is solved within a multi-heuristic framework. Heuristics such as Clarke and Wright savings and Sweep algorithm are used together with the Multi-Objective Particle Swarm Optimization Algorithm. In (Wang et al., 2021b), a hybrid evolutionary algorithm is proposed composed of the VNS combined with a crossover operator and a population updating strategy to enhance search diversification. Similarly, the MDGVRP is formulated in (Sadati & Çatay, 2021) as a mixed-integer linear programming model, solved using a hybrid

proposal combining the VNS and Tabu search tested over a dataset from the literature to study the computational performance and provide managerial insights. The MDGVRP is presented in (Fan et al., 2021) and solved via a hybrid genetic algorithm and the VNS to generate an initial solution. The integer programming model is focused on a time-dependent approach with time windows considering temporal-spatial distance based on historical traffic information of the customer's distribution network.

More generally, various mathematical model proposals have been proposed for the MDVRP, and some alternative formulations are suggested by Ramos et al., (2020). The two-commodity flow formulation for the MDVRP extends the CVRP in conjunction with the travelling salesman problem. Two different commodities have to be delivered and collected at each customer node. Conversely, the classical three-index formulation, rendered as an equivalence of the previously mentioned model, involves a binary decision variable that indicates whether a vehicle traverses an arc between two customers.

Subtour elimination procedure has been proposed by several approaches, such as the Dantzig-Fulkerson-Johnson constraint, the Miller-Tucker-Zemlin formulation, transit load constraint, and arrival time constraint. Recent publications have framed the MDVRP from different perspectives, such as transportation efficiency and water distribution. In (Wang et al., 2021a), multiple depots' pick-up and delivery problems are addressed to maximize vehicle utilization and minimize logistics operating costs. The efficiency in logistics transportation is improved by using a customer splitting scheme to balance the spatial demand distribution. A similar context is found in (Medeiros Vieira et al., 2021) to face the problem of water distribution to drought-affected populations at a large scale. Water sources represent the depots, and each set of demand points is assigned to a single water source. The procedure is applied to a real case of water distribution in the Brazilian territory, which includes operational priorities of humanitarian aid for disaster relief scenarios.

3. Mathematical formulation of the MDVRP

The mathematical model for the MDVRP can find the optimal routes of the vehicles with minimum traveled distance. The proposed mathematical model is the first stage of the proposed methodology. The MDVRP is mathematically defined by a complete graph $G = (V, A)$ where $V = \{1, \dots, n + w\}$ is the set of vertices and $A = \{(i, j): i, j \in V, i \neq j\}$ is the set of arcs. The set of vertices V is split into two subsets: $V_c = \{1, \dots, n\}$ and $V_d = \{n + 1, \dots, n + w\}$ that represent the sets of customers and depots respectively. Each depot has a maximum number of available vehicles with maximum load capacity, belonging to set K . Each vertex i belonging to the set of customers V_c has a non-negative demand d_i . It is necessary to distance matrix $dist_{i,j}$ associated with the set of arcs to quantify the objective function, A . In the MDVRP, the performed routes are obtained at minimum cost in such a way that: each route starts and ends in the same depot, each customer is visited by just one vehicle, and the route demand cannot exceed the vehicle capacity. Accordingly, the equations that represent the mathematical model for the MDVRP fitted to the purposes of this research are presented as follows:

$$\min Z = \sum_i \sum_j \sum_k dist_{ij} \cdot x_{ijk} \quad (1)$$

Subject to:

$$\sum_k Y_{ik} = 1 \quad \forall i \in V_c \quad (2)$$

$$\sum_{j \in V_c} x_{ijk} = Y_{ik} \quad \begin{matrix} \forall i \in V_c \\ \forall k \in K \end{matrix} \quad (3)$$

$$\sum_{i \in V_c} \sum_{k \in K} x_{ijk} = 1 \quad \forall j \in V_c, i \neq j \quad (4)$$

$$\sum_{i \in V_c} x_{ihk} - \sum_{j \in V_c} x_{hjk} = 0 \quad \begin{matrix} \forall h \in V \\ \forall k \in K \end{matrix} \quad (5)$$

$$\sum_{i \in V_c} \sum_{j \in V} d_i \cdot x_{ijk} \leq Q \quad \forall k \in K \quad (6)$$

$$\sum_{j \in V_c} x_{ijk} \leq 1 \quad \begin{matrix} \forall i \in V_d \\ i \in dep_start(k) \\ \forall k \in K \end{matrix} \quad (7)$$

$$\sum_{i \in V_c} x_{ijk} \leq 1 \quad \begin{array}{l} \forall j \in V_d \\ j \in dep_end(k) \\ \forall k \in K \end{array} \quad (8)$$

$$\sum_{i \in V} x_{jik} = 0 \quad \begin{array}{l} \forall j \in V_d \\ j \notin dep_start(k) \\ \forall k \in K \end{array} \quad (9)$$

$$\sum_{j \in V} x_{jik} = 0 \quad \begin{array}{l} \forall i \in V_d \\ i \notin dep_end(k) \\ \forall k \in K \end{array} \quad (10)$$

$$\sum_{i \in dep_start(k)} \sum_{h \in V_c} x_{ihk} - \sum_{h \in V_c} \sum_{j \in dep_end(k)} x_{hjk} = 0 \quad \forall k \in K \quad (11)$$

$$\sum_{\substack{j \in V \\ j \neq q}} t_{qjk} \leq \sum_{\substack{i \in V \\ i \neq q}} [t_{iqk} - dist_{iq} \cdot x_{iqk}] + Q \cdot \left[1 - \sum_{\substack{i \in V \\ i \neq q}} x_{iqk} \right] \quad \begin{array}{l} \forall q \in V_c \\ \forall k \in K \end{array} \quad (12)$$

$$t_{iqk} \geq 0 \quad \begin{array}{l} \forall i \in V \\ \forall q \in V \\ i \neq q \\ \forall k \in K \end{array} \quad (13)$$

$$t_{iqk} \leq Q \cdot x_{iqk} \quad \begin{array}{l} \forall i \in V \\ \forall q \in V \\ i \neq q \\ \forall k \in K \end{array} \quad (14)$$

$$\sum_{\substack{i \in V_d \\ i = dep_start(k)}} \sum_{q \in V_c} t_{iqk} \leq \sum_{i \in V_c} d_i \quad \forall k \in K \quad (15)$$

Equation (1) represents the objective function, which minimizes the distance traveled by the vehicles. Expressions (2) and (3) guarantee that each customer is visited by one vehicle. In Eq. (4), the number of arcs entering a customer node is equivalent to one. Eqs. (5) guarantee that the number of arcs entering a node equals the number of arcs leaving the same node, either a customer or a depot. Expressions (6) establish that the sum of the customers' demands belonging to a route must be less than the load capacity of the vehicle visiting such route. Eq. (7) and Eq. (8) assure that if a vehicle leaves a determined depot, it returns to the same depot. On the other side, Eq. (9) and Eq. (10) avoid the vehicles not arriving at a different depot from which they depart. Equation (11) guarantees that the number of arcs leaving and entering a depot node is equal. Expressions (12) to (14) keep track of the merchandise flow through the arcs of the route, eliminating the subtours. Lastly, Eq. (15) assures that the total demand of the customers is greater or equal than the sum of the flows through the arcs.

4. Proposed Hybrid Approach

The MDVRP model presented in section 3 is solved by using CPLEX. A CBGA based metaheuristic is implemented to solve the optimal placement of the EVs charging stations, considering the routes provided by the exact method and spatial location of the power distribution nodes. In Figure 1, the two-stage general procedure is presented to solve the MDEVRP, framed within a logistics stage where the route sequence for each EV is found. This latter, together with the power distribution network parameters, represent the input for the electric stage, in which the new sequence for each route is obtained, taking into account the charging stations installed, the new routing cost, and the increase of the energy losses in the power network due to the additional load. The proposed CBGA for this particular problem is explained in the following sections.

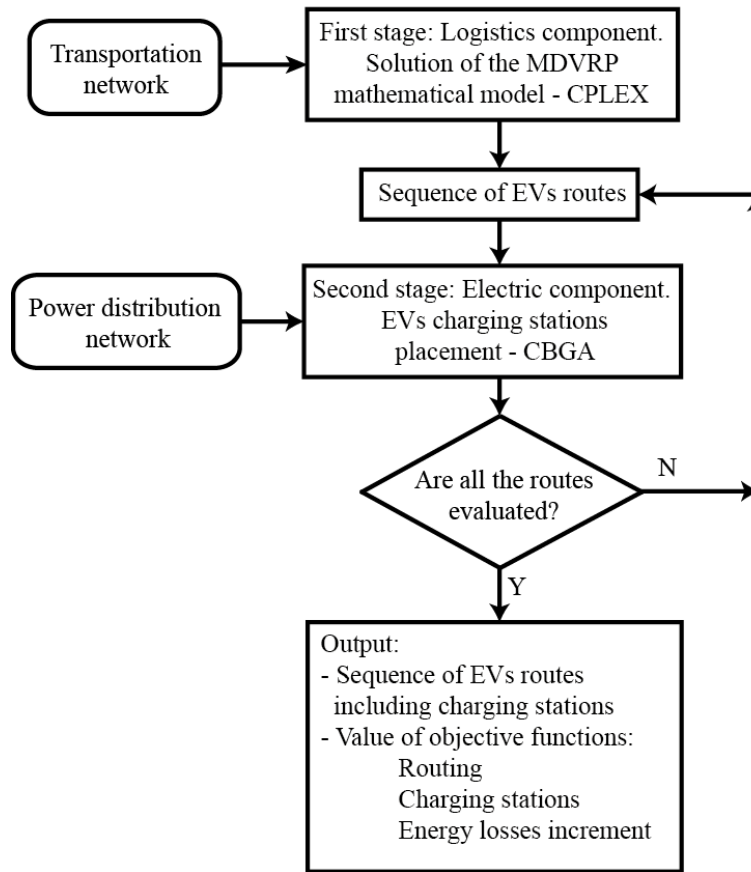


Fig. 1. General procedure to solve the MDEVRP

5. Chu-Beasley Genetic Algorithm CBGA

A genetic algorithm is a solution technique based on the intelligent probabilistic search that simulates the evolution of the species. As the species evolve, better individuals are introduced in the population with improved features. Genetic operators, i.e., selection, crossing, and mutation, are applied at each generation of individuals or solutions. The latter is evaluated according to a fitness measure, also called fitness function. The CBGA was initially designed to solve the generalized assignment problem (Chu & Beasley, 1997), with reports of its adjustment to other types of problems with noticeable results. Unlike the traditional genetic algorithm, the CBGA presents some features that make it competitive to solve large-size problems.

5.1 Problem codification

On the other hand, problem codification plays an important role when implementing the CBGA or any metaheuristic technique. A binary codification represents the solution for this problem, which is explained based on the route sequence depicted in Fig. 2.

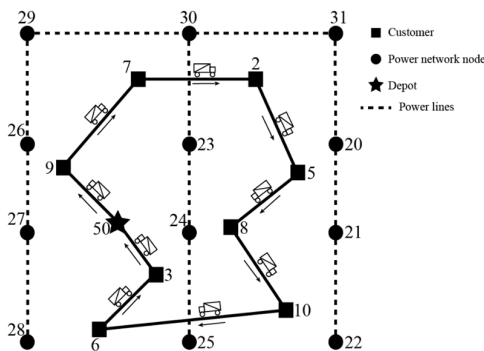


Fig. 2. Sequence of EV route

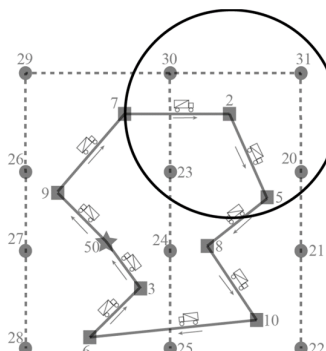


Fig. 3. Coverage radius

According to Fig. 2, the EV route is given by the following sequence: 50 – 9 – 7 – 2 – 5 – 8 – 10 – 6 – 3 – 50, which depot is the node at 50. Dotted lines represent the power distribution circuit, and the candidate nodes for charging stations are the circles. Now let us consider in Fig. 3 a preset coverage radius that provides the candidate nodes for charging stations for each customer node, i.e., once an EV leaves the customer at node 2, the possible charging stations for battery recharge are 20, 23, 30, and 31. Then, notice Fig. 4, an example of the possible charging station nodes that the EV can go to once leaving each customer node, considering the coverage radius.

Customer node	50	50	50	50	9	9	9	9	7	7	...
Power network node	23	25	26	27	23	26	27	29	23	26	...
Solution	0	0	0	1	0	0	0	0	0	0	...
...	7	7	2	2	2	2	5	5	5	5	...
...	29	30	20	23	30	31	20	21	23	24	...
...	1	0	0	0	0	0	0	1	0	0	...
...	5	8	8	8	8	10	10	10	10	6	...
...	31	21	23	24	25	21	22	24	25	24	...
...	0	0	0	1	0	0	0	0	0	0	...
...	6	6	6	3	3	3	3				
...	25	27	28	23	24	25	27				
...	0	0	1	0	0	0	0				

Fig. 4. Solution example for the modified route sequence

The first row in the array presented in Fig. 4 corresponds to the customer's nodes in the route sequence provided by solving the exact technique. The second row shows the power network nodes the EV can recharge once leaving the customer. The third row shows a solution example representing a final solution involving the charging stations. Under this scenario, the modified route is given by the following sequence: 50 – 27 – 9 – 7 – 29 – 2 – 5 – 21 – 8 – 24 – 10 – 6 – 28 – 3 – 50. See Figure 5 that the coverage radius is applied for each customer node, considering that the EV can only go to a recharge station after leaving a customer.

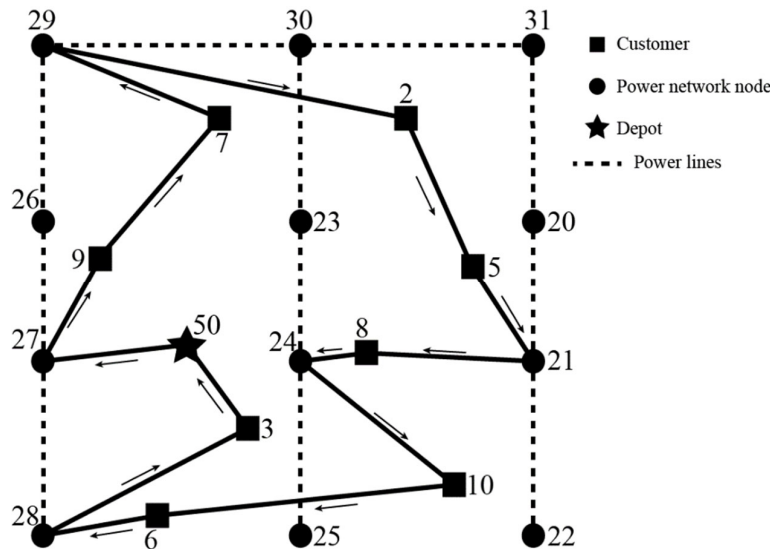


Fig. 5. Modified route sequence considering the coverage radius

5.2 Fitness function evaluation

Search through the solution space is guided by a fitness function that deals with the infeasibility degree (Montoya, et al., 2019). In the CBGA, the objective function and infeasibility are computed for each individual of the population, which are stored separately and used for different purposes. For the MDEVRP proposed in this research, the infeasibility function Z_{inf} of an individual is computed. To evaluate whether a solution is infeasible, the parameter Q_{bat} that stands for battery capacity is known before hand in terms of distance. The battery is fully charged at the depot and starts to decrease as the EV travels through the arcs of the route. Once it reaches a charging station, the battery returns to the full level. The distance traveled

$d_{trav(i)}$ at node i is computed, either customer or charging station. The solution is described by a vector seq that provides the sequence of the nodes traversed by the EV. Then, the value of Z_{inf} is calculated as shown in Eq. (16).

$$Z_{inf} = \alpha \cdot \sum_{i=seq(i)}^{seq(end)} (d_{trav(i)} - Q_{bat}) \quad \text{Only if } d_{trav(i)} > Q_{bat} \quad (16)$$

Each solution of the population has three associated costs, which are calculated by Eq. (17), Eq. (18) and Eq. (19).

$$f_{obj1} = T \cdot AF \cdot Z_{new} \cdot \beta \quad (17)$$

$$f_{obj2} = ER_{inst} \cdot \forall \quad (18)$$

$$f_{obj3} = t_{op} \cdot T \cdot AF \cdot S_{base} \cdot \Delta_{losses} \cdot \Omega \quad (19)$$

Eq. (17) indicates the routing cost, which includes Z_{new} , the distance traversed by the EV along the route provided by the binary solution. Parameter β is the cost per kilometer traveled. Expression in (18) is the cost of the charging stations installed along the EV route. Equation (19) is the cost of increasing the energy losses resulting from the installation of additional electrical loads in the power distribution system. This situation requires running a power flow based on the traditional backward, forward sweep algorithm (Chang, et al., 2007). For f_{obj1} and f_{obj3} , a study timeframe T equivalent to 365 days and the annualization factor AF are considered to stand for the effect throughout the time and compare these two operational costs with the investment in charging stations computed in f_{obj2} . Then the objective F_{obj} and fitness Z_{fit} functions of the solution are given by Eq. (20) and Eq. (21).

$$F_{obj} = f_{obj1} + f_{obj2} + f_{obj3} \quad (20)$$

$$Z_{fit} = F_{obj} + Z_{inf} \quad (21)$$

If $Z_{inf} = 0$, the solution is feasible and $Z_{fit} = F_{obj}$. If $Z_{new} = z$, being z the objective function computed in the MDVRP, there are no charging stations on the solution route.

5.3 Initial population and genetic operators

As aforementioned, a CBGA is applied to each route provided via solving the MDVRP presented in Eq. (1) to Eq. (15). The initial population of individuals is randomly generated, considering that some objective functions are likely to be equal; nevertheless, the binary configuration must be different. Once the initial population is generated, the selection, crossing, and mutation genetic operators are applied.

For the MDEVRP, tournament selection is applied. Two tournaments are performed. In each of them, individuals of the current population are chosen to participate. The parameter is usually within 2 to 4. The process is as follows: individuals of the current population are randomly chosen; their objective functions are compared, and the best fitness function is stored as the first parent. Then the process is done again to find the second parent, conditional to both parents being different.

After selecting the two parents, their binary codifications are interchanged by a one-point crossing process. This change results in two offspring, composed of part of the first parent's information and part of the information of the second parent. Only one offspring can pass to the next stage of the algorithm, which corresponds to mutation; the other offspring are randomly discarded. It is worth mentioning that the two offspring from the crossing process are likely to have more than one charging station activated for one customer node. In this case, the remaining charging stations assigned to the customer are randomly discarded, as shown in Fig. 6.

Customer node	50	50	50	9	9	9	9	7	7	•••
Power network node	25	26	27	23	26	27	29	23	26	•••
Solution	0	0	1	0	1	1	1	0	0	•••

Two of them are discarded

Fig. 6. Random discard of additional charging stations assigned to a customer

Once the crossing process is executed, mutation over the selected offspring is performed. This step involves a parameter in terms of a solution vector size percentage. This aspect provides the number of cells for mutation and includes changing from zero to one or from one to zero. Once again, more than one charging station is likely to be activated after the solution vector mutation process, and the same procedure shown in Fig. 6 must be performed.

5.4 Criteria for population modification

In the CBGA, only one individual or solution is replaced at each generational cycle. The offspring resulting from the mutation process has to replace the worst quality element of the population, as long as the offspring are better quality and the diversity criteria have complied. This procedure means that the offspring is different from the individuals of the current population. Otherwise, the genetic operators must be applied again to find another offspring until the diversity criteria are fulfilled. The following situations can be found to introduce the offspring in the current population:

- If the offspring is infeasible, the current population is revised to look for infeasible parents. If the offspring's infeasibility is less than the most infeasible parent, then the offspring is introduced in the population to replace that parent. Otherwise, the offspring is discarded, and the procedure is repeated from selection to find another offspring. If there are no infeasible parents, the offspring is discarded.
- If the offspring is feasible, the current population is revised to look for infeasible parents. The offspring replaces the most infeasible parent. If all the parents are feasible and if the offspring has an objective function better than the worst objective function of the current population, then the individual is introduced or discarded otherwise.

In the MDEVRP, the stopping criterion is set as a maximum number of iterations or generations.

6. Test System and Experimental Results

This section presents the transportation network and power distribution system information employed to validate the hybrid methodology proposed in this study. The MDVRP that corresponds to the first stage of the methodology is solved using the GAMS package, particularly the CPLEX solver (GAMS, 2021). The second stage, directed to solve the charging station location and minimize the energy losses, is solved using the CBGA implemented in MATLAB. Complete details of these test systems are presented as follows.

The transportation network that contains the spatial location of the customers involves an instance containing 25 customers from the literature, which is part of a set of instances widely used to validate transportation mathematical models found in Networking and Emerging Optimization (2013). The instance contains four depots and four vehicles. Each vehicle has a load capacity of 200. For coding purposes, the depots are tagged from 26 to 29 and their copies from 30 to 33. The power distribution system is a 33-nodes radial test feeder with a rated voltage of 12.66 kV (Grisales-Noreña, et al., 2018). The loads' total active and reactive power demanded are 3715 kW and 2300 kVar, respectively. In the benchmark case, i.e., the power losses feeder is 210.97 kW when no charging stations are installed. Voltage and power base values are 12.66kV and 100 kW, respectively.

The nodes of transportation and power distribution networks are presented in Appendix A.1. The first column is the node ID. The second and third columns are the X and Y coordinates, and the last column corresponds to the demand for goods. This field is zero if the node belongs to the power distribution system or a depot. Fig. 7 presents both networks in a unified instance to validate the hybrid methodology. Notice that the power distribution system is numbered from 34 to 66, node 34 the substation node or slack node. Appendix A.2 shows the configuration of the 33 nodes radial distribution network.

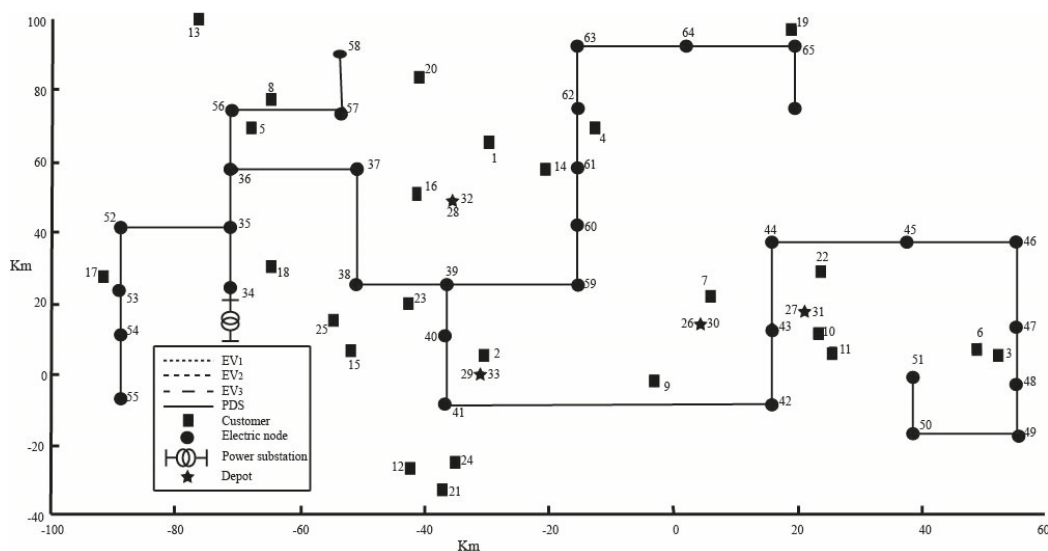


Fig. 7. Unified test system: transportation and power distribution networks

We have defined independent parameters whose values must be determined by extensive computational experiments for the proposed approach. These parameters are considered candidate settings for any given factor. Since the performance of the proposed approach depends on the value for each of the below described parameters, a calibration process has been carefully done. This procedure is iteratively performed by considering every single factor (variable) and finds its “best value”, giving the lower objective function. In this sense, initial values of some parameters and particular conditions of the EVs obtained from previous works are chosen for the tests. As explained in Hydro Quebec (2015), DC fast charging operation involves a 40 kW of instantaneous power applied to charge an EV battery in a t_{op} of 25 minutes to pursue 75 km of driving autonomy. Under this context, it is assumed that routing time is not affected by charging time. The additional load represented by the EVs charging in the power network is introduced as a constant power load in the electric nodes. According to Nicholas (2019) the cost for installing a DC fast charging station is 22000 USD, which includes materials, labor, permits, data, connectivity, and network upgrades if needed. Concerning EV operation, a cost of 2.34 USD is taken as a reference to travel 75 km, considering an energy price of 0.15 USD/kWh. This latter is also applied for the energy losses. For EV maintenance purposes, it is estimated a cost of 100 USD to travel 5000 km (Arias-Londoño, et al., 2021).

Computational experiments are shown in Table 1 for different values of EV battery capacity. Columns 2 to 5 are the costs for routing f_{obj1} , EVs charging stations installation f_{obj2} , increase in energy losses f_{obj3} , and cost of objective function F_{obj} , respectively. Columns 6 to 8 are the details of the routes, the numbers in bold are the charging stations installed in each case. The experiments were run on a Malaysian manufactured Intel Core i5 2.1 GHz processor, a 64-bit operative system with 8.00 GB of RAM. The MDVRP was solved with a CPLEX solver, and the CBGA was implemented in MATLAB.

Table 1
Computational experiments of the hybrid methodology

Battery capacity [km]	f_{obj1} [USD]	f_{obj2} [USD]	f_{obj3} [USD]	F_{obj} [USD]	$k = 1$	$k = 2$	$k = 3$
60	85990	176000	13742	275732	26-7-22- 43 -3-6- 51 -11-10- 43 -9-30	28-14-1-4- 62 -19- 64 -20- 58 -13-8- 56 -5-16-32	29- 41 -12-21-24- 41 -15-25- 34 -17- 34 -18-23-2-33
60*	103480	198000	14533	316013	26-7- 44 -22-3-6- 47 -11-47-10- 44 -9-30	28-14-1-4- 62 -19- 64 -20- 57 -13- 56 -8-5-16-32	29- 41 -12-21-24- 41 -15-25- 38 -17- 52 -18- 38 -23-2-33
70	85063	132000	8539	225601	26-7-22- 43 -3-6-11-10- 43 -9-30	28-14-1-4- 64 -19- 64 -20- 58 -13-8- 56 -5-16-32	29-12-21-24- 41 -15-25-17- 53 -18-23-2-33
80	85063	132000	8539	225601	26-7-22- 43 -3-6-11-10- 43 -9-30	28-14-1-4- 64 -19- 64 -20- 58 -13-8- 56 -5-16-32	29-12-21-24- 41 -15-25-17- 53 -18-23-2-33
90	82810	132000	8984	223794	26-7-22-3-6- 51 -11-10-9-30	28-14-1-4- 64 -19- 64 -20- 58 -13-8- 56 -5-16-32	29-12-21-24- 41 -15-25-17- 34 -18-23-2-33
100	81883	110000	8342	200225	26-7-22-3-6- 51 -11-10-9-30	28-14-1-4- 64 -19-20- 58 -13-8-5-16-32	29- 41 -12-21-24-15-25- 34 -17-18-23-2-33
110	81220	88000	5685	174906	26-7-22-3-6-11-10- 43 -9-30	28-14-1-4-19- 64 -20- 58 -13-8-5-16-32	29-12-21-24-15-25- 34 -17-18-23-2-33
140	81618	88000	5685	175303	26-7-22-3-6-11-10- 43 -9-30	28-14-1-4- 64 -19-20- 58 -13-8-5-16-32	29-12-21-24-15-25- 34 -17-18-23-2-33
140*	91555	88000	7396	186952	26-7-22-3-6- 51 -11-10-9-30	28-14-1-4- 62 -19-20- 62 -13-8-5-16-32	29- 41 -12-21-24-15-25-17- 54 -18-23-2-33
150	80955	66000	3236	150191	26-7-22-3-6-11-10-9-30	28-14-1-4-19- 64 -20- 58 -13-8-5-16-32	29-12-21-24-15-25- 34 -17-18-23-2-33
160	80823	44000	2310	127133	26-7-22-3-6-11-10-9-30	28-14-1-4-19- 64 -20-13-8-5-16-32	29-12-21-24-15-25- 34 -17-18-23-2-33
170	80955	44000	915	125870	26-7-22-3-6-11-10-9-30	28-14-1-4-19-20- 58 -13-8-5-16-32	29-12-21-24-15-25- 34 -17-18-23-2-33
180	80955	44000	915	125870	26-7-22-3-6-11-10-9-30	28-14-1-4-19-20- 58 -13-8-5-16-32	29-12-21-24-15-25- 34 -17-18-23-2-33
190	82280	44000	812	127093	26-7-22-3-6-11-10-9-30	28-14-1-4-19-20- 57 -13-8-5-16-32	29-12-21-24-15-25-17- 34 -18-23-2-33
200	80955	44000	915	125870	26-7-22-3-6-11-10-9-30	28-14-1-4-19-20- 58 -13-8-5-16-32	29-12-21-24-15-25- 34 -17-18-23-2-33
230	80690	22000	915	103605	26-7-22-3-6-11-10-9-30	28-14-1-4-19-20- 58 -13-8-5-16-32	29-12-21-24-15-25-17-18-23-2-33
250	80690	22000	736	103426	26-7-22-3-6-11-10-9-30	28-14-1-4-19-20-13-8-5- 37 -16-32	29-12-21-24-15-25-17-18-23-2-33
260	80558	0	0	80558	26-7-22-3-6-11-10-9-30	28-14-1-4-19-20-13-8-5-16-32	29-12-21-24-15-25-17-18-23-2-33

Results shown in Table 1 present the behavior of the analyzed costs as the battery capacity is changed in terms of a sensitivity analysis. Although the instance involves at the beginning four vehicles, the results provided by the CPLEX show that only three vehicles were used to perform the routing. The decrease of the objective function F_{obj} is notorious with the increment in the battery capacity within 60 km and 260 km. The last value corresponds to the results for the benchmark case; this means that the battery capacity is large enough that no charging stations are installed, and the increase in the energy losses is null. For values of driving range less than 60 km, the CBGA resulted in infeasible solutions. It becomes almost constant concerning routing costs from a driving range of 100 km. This behavior, which is kept until the benchmark case, is in response to the little

variation of the route with the increase in battery capacity, being less likely to perform detours, as fewer charging stations are necessary.

Fig. 8 presents the routes performed by the EVs for a battery capacity of 60 km. Notice that several charging stations are visited due to the low battery capacity, being necessary to visit some charging stations in multiple times. This situation represents for the CBGA a less expensive alternative, rather than to install additional charging stations at other electric nodes. In this particular case, the charging stations' cost is almost 13 times the cost of the increase in energy losses, according to Table 1. Due to this, it is more suitable for the CBGA to install charging stations in the nodes at the end of the power distribution feeder, despite this could significantly increase the energy losses and the corresponding cost.

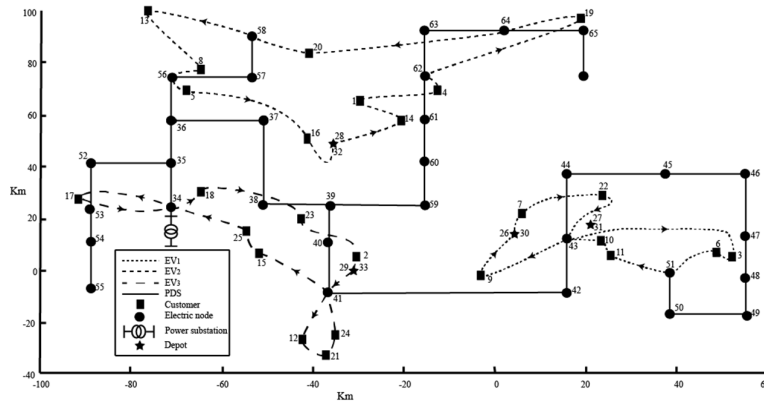


Fig. 8. Routes performed by the EVs for a 60 km driving range

In Fig. 9, the results are presented for a battery capacity of 60 km, considering a restriction of charging stations installed in specific nodes of the power distribution system. Electric nodes restricted for charging station installation are shaded. This situation is identified in Table 1 in the first column with an asterisk. Notice that for EV1, nodes 43 and 51 were used as charging points before posing the restriction. By purpose, these nodes were restricted, making the CBGA install so that the routing cost and charging stations cost is increased as little as possible. After the restriction, node 47 was chosen to install a charging station with two stops, making this node a critical charging station to meet the merchandise demand of customers 3, 6, 10, and 11. EV2 routing is lightly affected by the restriction, as the current charging stations installed are essential charging points for routing completion. If several nodes along this route are restricted, the solution becomes infeasible due to the routing length. EV3 has a similar route before and after the restriction. A charging station was installed at node 34, which corresponds to the substation of the power distribution network.

Additionally, the route includes two stops at this node. In terms of energy losses, installing at the power substation implies that the energy losses are not affected by the additional load that represents the installation of the charging station in this particular node. The load due to the operation of this charging station does not provide additional energy losses, as no matter what quantity of load is connected at the substation, no further current flows are produced through the power lines. Once the restriction is applied, especially for nodes 34 and 53, the CBGA opts to install two other charging stations at two different nodes: 38 and 52, considering that other candidates may result in fewer quality solutions.

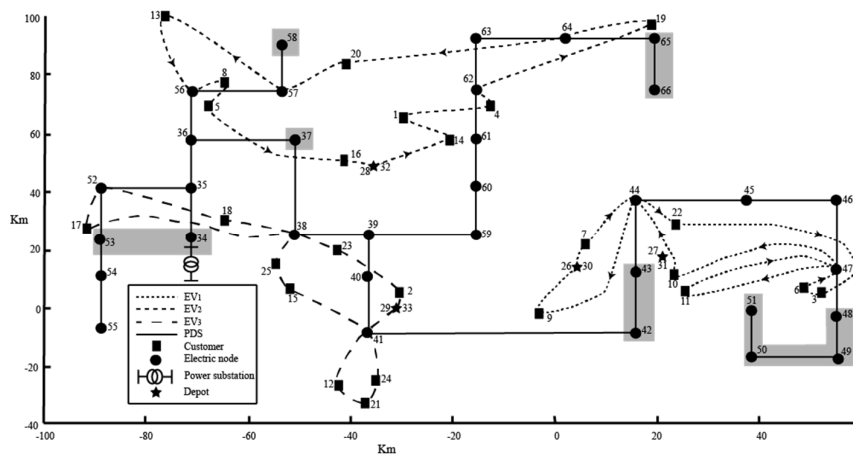


Fig. 9. EVs routes for a 60 km of driving range with power nodes restricted for charging stations installation

In Fig. 10, the EVs routes are depicted for a battery capacity of 140 km. Considering that the driving range is more significant than double the value mentioned above, the number of charging stations is decreased up to half, and no multiple visits to charging points are presented. As proceeded above with the 60 km of driving range, some of the power distribution nodes are restricted for charging stations installation, which is represented in Figure 11 considering the same 140 km of driving range. Compared with the case of 140 km of driving range without restrictions, in this case, the number of charging stations is the same. Furthermore, the routing cost and change in energy losses are increased. Under these restrictions, notice that for the CBGA, it is more suitable to perform longer routes by visiting a charging station multiple times. Before the restriction, EV1 used to charge at node 43, then this node was restricted, including some of its neighbors, except the node 51. After running CBGA, a charging station was installed at node 51, which was expected due to the coverage radius described in Fig. 3. If node 51 is also restricted, the CBGA cannot find a feasible solution for EV1 unless the coverage radius increases. For EV2, it can be observed that node 62 becomes a critical point of recharge for this route after the restriction is imposed. By also restricting this node for charging station installation, the solution gets infeasible. The number of charging stations for this route in the restricted scenario is less than that found in the non-restricted scenario, which is translated into a lower cost of energy losses, as the only charging station installed, i.e., node 62, is not located as close from the end of the feeder, as with nodes 58 and 64 in the non-restricted scenario.

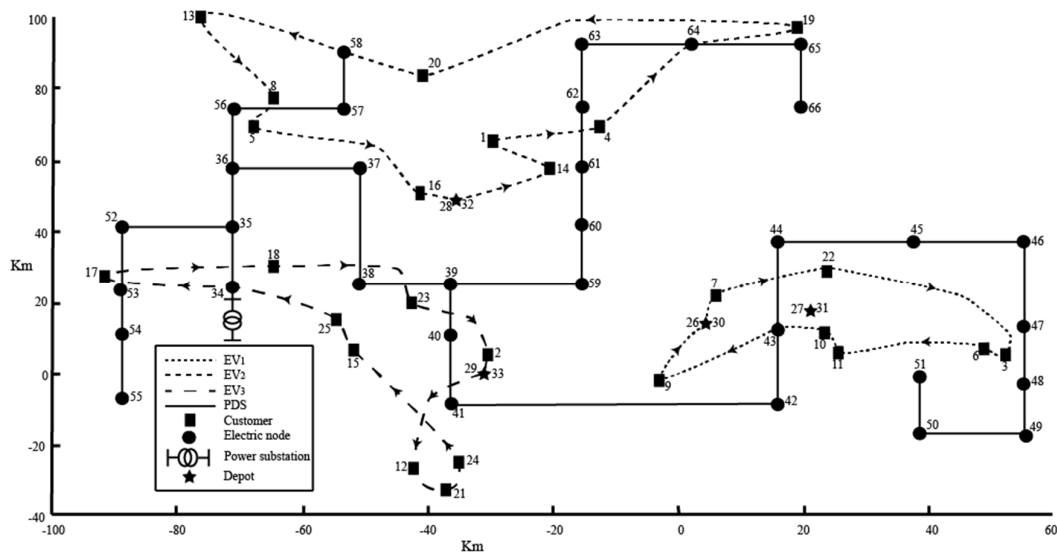


Fig. 10. EVs routes for a 140 km of driving range

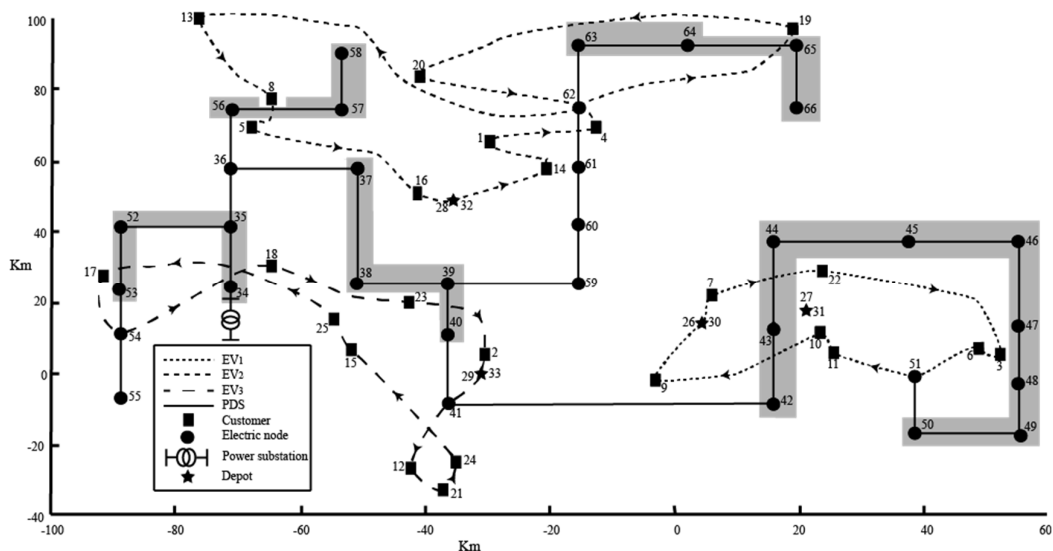


Fig. 11. EVs routes for a 140 km of driving range with power nodes restricted for charging stations installation

7. Conclusions and future works

This paper proposes a hybrid methodology of an exact technique and a metaheuristic to solve the Multi Depot Electric Vehicle Routing Problem. The proposal considered the logistic subproblem that addressed the conventional MDVRP and the electric

subproblem, encompassing the optimal location of the charging stations and the operation of the power distribution system. Three objectives under a conflict that involve the interests of logistics companies and power utilities were considered throughout the experimentations: routing cost, charging stations installation, and increase in energy losses. In order to validate the proposal, battery autonomy resulted in a convenient study of the model sensitivity when the driving range is changed. A remarkable influence is noticed in the three objectives under study when addressing charging stations placement. Accordingly, a power nodes restriction scenario is shown as an alternative to improve the value in the objective function if the vehicles are assessed independently. Although routing cost is increased, the number of charging stations and the energy losses are decreased, as these are related to each other in the power network operation. CPLEX and CBGA were suitable options to deal with the first and second stages of the MDEVRP, respectively.

Remarkably, the routing solution, including detours to charging stations, was represented by a binary codification, which resulted in inappropriate time to implement the genetic operators. Finally, using a metaheuristic technique for the second stage was helpful regarding the combinatorial explosion identified in the test system. Considering an average of 3 options of charging stations that each customer can go to, for the second route that involves EV2 that contained nine customers, the number of possible solutions to evaluate is $2^{(9*3)} = 134.2 \times 10^6$, taking into account the binary codification.

Future research must be focused on proposing other instances that integrate more significant transportation and power distribution networks from the specialized literature and introducing other routing patterns, such as the shortest path problem related to the last-mile delivery problem. Additionally, the CBGA implementation considers all the EVs routes within the same binary codification for efficiency comparison concerning the current proposal. Finally, algorithms based on granular search trajectory algorithms proposed for similar problems such as proposed by Escobar & Linfati (2012), Puenayán et al. (2014), Linfati et al. (2014), Bernal et al. (2017), Bernal et al. (2018), Bernal et al. (2021), Escobar et al. (2022).

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Appendix A.1.

Table A.1. Spatial location and goods demand at nodes for both networks

Node	X	Y	Demand	Node	X	Y	Demand
1	-29.7	64.1	12	34	-71.5	24.6	0
2	-30.7	5.5	8	35	-71.7	41.5	0
3	51.6	5.5	16	36	-71.5	57.7	0
4	-13.2	69.3	5	37	-51.0	57.7	0
5	-67.4	68.3	12	38	-51.1	24.9	0
6	48.9	6.3	5	39	-36.7	24.9	0
7	5.2	22.3	13	40	-36.8	10.9	0
8	-65.0	77.2	20	41	-36.7	-8.5	0
9	-4.2	-1.6	13	42	15.8	-8.3	0
10	23.0	11.6	18	43	15.8	13.1	0
11	25.5	6.3	7	44	15.8	36.8	0
12	-42.6	-26.4	6	45	37.6	36.8	0
13	-76.7	99.3	9	46	55.2	36.9	0
14	-20.7	57.9	9	47	55.2	13.1	0
15	-52.0	6.6	4	48	55.2	-2.5	0
16	-41.4	50.8	25	49	55.2	-16.8	0
17	-91.9	27.6	5	50	38.5	-16.7	0
18	-65.1	30.2	17	51	38.4	-0.4	0
19	18.6	96.7	3	52	-89.3	41.3	0
20	-40.9	83.2	16	53	-89.3	23.8	0
21	-37.8	-33.3	25	54	-89.4	11.2	0
22	23.8	29.1	21	55	-89.3	-6.9	0
23	-43.0	20.5	14	56	-71.5	73.9	0
24	-35.3	-24.9	19	57	-53.8	74.0	0
25	-54.8	14.4	14	58	-53.7	90.2	0
26	4.2	13.6	0	59	-15.9	24.9	0
27	21.4	17.1	0	60	-15.9	41.4	0
28	-36.1	49.1	0	61	-15.9	58.0	0
29	-31.2	0.2	0	62	-15.9	74.6	0
30	4.2	13.6	0	63	-15.9	91.2	0
31	21.4	17.1	0	64	1.7	91.4	0
32	-36.1	49.1	0	65	19.3	91.6	0
33	-31.2	0.2	0	66	19.3	75.0	0

Appendix A.2.

Table A.2

Power distribution system configuration and parameters

Send node	Receive node	R [Ohm]	X [Ohm]	P [kW]	Q [kVar]	Send node	Receive node	R [Ohm]	X [Ohm]	P [kW]	Q [kVar]
34	35	0.0922	0.0477	100	60	50	51	0.732	0.574	90	40
35	36	0.493	0.2511	90	40	35	52	0.164	0.1565	90	40
36	37	0.366	0.1864	120	80	52	53	1.5042	1.3554	90	40
37	38	0.3811	0.1941	60	30	53	54	0.4095	0.4784	90	40
38	39	0.819	0.707	60	20	54	55	0.7089	0.9373	90	40
39	40	0.1872	0.6188	200	100	36	56	0.4512	0.3083	90	50
40	41	1.7114	1.2351	200	100	56	57	0.898	0.7091	420	200
41	42	1.03	0.74	60	20	57	58	0.89	0.7011	420	200
42	43	1.04	0.74	60	20	39	59	0.203	0.1034	60	25
43	44	0.1966	0.065	45	30	59	60	0.2842	0.1447	60	25
44	45	0.3744	0.1238	60	35	60	61	1.059	0.9337	60	20
45	46	1.468	1.155	60	35	61	62	0.8042	0.7006	120	70
46	47	0.5416	0.7129	120	80	62	63	0.5075	0.2585	200	600
47	48	0.591	0.526	60	10	63	64	0.9744	0.963	150	70
48	49	0.7463	0.545	60	20	64	65	0.3105	0.3619	210	100
49	50	1.289	1.721	60	20	65	66	0.341	0.5302	60	40



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