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Sales mode selection strategic analysis for risk-averse manufacturers under revenue sharing contracts

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A B S T R A C T

This paper considers a sales mode selection problem under revenue sharing contracts between resale and agency modes for risk-averse manufacturers with traditional retail channel, direct selling channel, and e-commerce platform channel. By considering the factors including price competition intensity, market share, revenue sharing ratio, commission rate, and degree of risk aversion, we construct leader-follower game models with manufacturers as leaders and traditional retailers and e-commerce platforms as followers. To obtain optimal solutions, we discuss conditions to ensure the upper and lower models to be convex and then give the optimal strategies for all members in the network. Through numerical experiments, we analyze the involved parameters' impact on sales mode selection strategy and the changing trends of each member's optimal pricing and profit under different sales modes. The numerical results reveal the following revelations: The manufacturer should choose the agency mode when the commission rate is low and the direct selling channel has a large market share. If both the commission rate and degree of risk aversion are high, direct selling channels have a low market share, and price competition intensity is weak, the manufacturer should choose the resale mode. The degree of risk aversion has an effect on each member's optimal decision. Regardless of which sales mode the manufacturer chooses, the optimal price of each member decreases as the degree of risk aversion increases. Under certain conditions, the manufacturer's choice of agency mode can create win-win situations with supply chain members.

1. Introduction

The rapid growth of information technology and the e-commerce economy has resulted in efficient and convenient online shopping, which has attracted an increasing number of customers. Affected by some emergencies like COVID-19, many enterprises encourage employees to work at home so that online shopping demand is gradually increasing. According to the China Internet Network Information Center's 49th Statistical Report on the Development of the Internet in China, the number of netizens in China reached 1.032 billion as of December 2021, with online shopping users accounting for 81.6% of internet users[1]. In the face of such a large consumer group, while maintaining offline and online direct sales, manufacturers take the powerful advantages of e-commerce platforms to attract consumers so as to form an online and offline multi-channel sales structure. In general, the sales modes of e-commerce platforms are divided into resale mode and agency mode. The resale mode means that e-commerce platforms wholesale products to manufacturers and carry out self-management, while the agency mode means that manufacturers enter e-commerce platforms in the form of flagship stores by setting product prices by themselves and e-commerce platforms provide manufacturers with direct contact with consumers by charging commission rates and slotting fees. Thus, each manufacturer has two different sales modes to select. For example, Lenovo chooses the

[1] http://www.cnnic.net.cn/hlwfzyj/hlwxzbg/

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resale mode on Amazon.com, Sony chooses the agency mode on Tmall.com, Haier and Gree choose a mixed mode on Suning.com, and so on.

Due to the diversification of consumer demand, enterprises have to face various risks. Different enterprises have different attitudes towards risks, which are generally divided into risk-averse, risk-neutral, and risk-seeking. In the real world, most decision-makers are risk-averse. Moreover, as an important regulation to coordinate supply chain (SC) members, revenue sharing contracts (RSCs) are gradually becoming more common in the e-commerce and video rental industries. For example, the market share of Blockbuster had increased from 24% to 40% within five years (Warren & Peers, 2002), and profits of the video rental industry had increased by around 10% (Mortimer, 2008). These results suggest that RSCs can motivate members to cooperate and realize SC coordination.

This paper takes risk-averse manufacturers with online and offline sales channels as the research object to study the influence of RSCs and risk aversion behavior on sales mode selection strategy. Our research mainly solves the following problems: What are the conditions for manufacturers to choose resale mode or agency mode? How do RSCs and risk aversion affect manufacturers' sales mode selection? What impact does manufacturers' sales mode selection strategy have on each member in the SC network?

This research is mainly related to sales mode selection strategies on e-commerce platforms. It is worth noting that the existing studies in this field mainly have the following limitations: Firstly, the existing researches mainly focus on single channel and dual channel, but less on multi-channel cases. Secondly, participants are mostly risk-neutral, but most decision-makers are risk-averse under market fluctuations. Thirdly, few scholars consider RSCs in the study of sales mode selection strategy.

This research makes the following significant contributions: (1) The SC system consists of a manufacturer, a traditional retailer, an e-commerce platform, and some consumers. In particular, the manufacturer is risk-averse and has multi-channel online/offline sales modes on the platform. The manufacturer's sales mode selection strategy and the effect of risk aversion on sales mode selection and each member's profit will be analyzed by us. (2) We introduce the RSC, an important rule among SC members, into the SC and explore the influence of the revenue sharing ratio on the manufacturer's sales mode selection under a multi-channel structure.

The remainder of this paper is organized as follows. Relevant literature is reviewed in Section 2. Section 3 briefly describes the main issues of this paper. Section 4 builds leader-follower game models in resale and agency modes, respectively, and analyzes each member's optimal decisions. Section 5 introduces numerical experiments, which include manufacturer's sales mode selection strategy and influence of RSCs and risk aversion. Some conclusions and future research directions are given in Section 6. The appendix contains proofs of all propositions.

2. Literature Review

Our research is closely related to risk-averse SCs, RSCs, and sales mode selection on e-commerce platforms.

2.1 Risk-averse SCs

As is known to us, popular risk measures include *mean-variance* (MV), *value at risk* (VaR), and *conditional value at risk* (CVaR). In particular, the famous MV model was proposed by Markowitz (1959) in portfolio risk management, which uses random return and loss variance to measure risk. Jorion (1996) pointed out that VaR is the maximum possible loss of the portfolio. When the loss exceeds the VaR value, the CVaR proposed by Rockafellar & Uryasev (2000) represents the average loss value.

In risk SC management, the existing literature mainly discusses how risk aversion affects different SCs. For example, Xiao & Yang (2008) considered two SCs containing a risk-neutral manufacturer and a risk-averse retailer, and found that risk sensitivity and product substitutability affect the pricing competition strategy. Xie et al. (2011) built MV models for three different SC structures and studied how quality and pricing decisions are affected by risk tolerance. By comparing with risk-neutral SC, the result was that retailers' risk aversion has no effect on the manufacturer leader's product quality decision. For risk-averse retailers and risk-neutral manufacturers in the SC, Li et al. (2014) used the CVaR and Nash bargaining models to study the effect of risk aversion on decisions and found that risk aversion is harmful to retailers.

Some studies concentrate on the competition among numerous risk-averse retailers. For instance, Li et al. (2013) established an MV model for the fast fashion SC and discussed the effect of information asymmetry, return policies, and risk aversion. An interesting result is that the return policy is beneficial to the SC of multiple risk-averse retailers. Yang et al. (2009) investigated the competitive strategies of price and service level among members, and found that the optimal decision of another retailer may be reduced if one retailer is risk-averse. For a similar SC structure, Hsieh & Lu (2010) established a manufacturer-leader model based on CVaR and investigated the influence of retailers' risk aversion on the manufacturer's return strategy. Cai et al. (2020) established an MV model for the case that competitive retailers share private risk-averse information and they found that the supplier may suffer profit loss due to accepting common risk-averse information when retailers share information.

It should be noted that there is much research on risk aversion under single-channel or dual-channel SCs, and few of them involve multi-channel SCs.

2.2 RSCs

RSCs are significant rules for coordinating SCs and mainly include *wholesale-price contracts with revenue sharing* (WCRS) and *consignment contracts with revenue sharing* (CCRS). WCRS means that manufacturers wholesale products to retailers at a lower price, and retailers share part of their revenue to compensate manufacturers for losses, further promoting collaboration and improving SC efficiency (Cachon, 2003). Furthermore, Cachon & Lariviere (2005) considered the WCRS between one manufacturer and multiple retailers and discovered that it is equivalent to a fixed-price buyback contract. Note that this revenue sharing ratio is fixed and independent of revenue in the WCRS. Subsequently, Palsule-Desai (2013) proposed a revenue-dependent WCRS for economic downturn in the Indian film industry and found that this kind of contract can coordinate SC more effectively than the revenue independent. Different from the distribution based on the proportion of revenue, Katok & Wu (2009) considered a fixed proportion of order quantity, while Wang et al. (2012) extended RSCs to profit sharing contracts and found that RSCs may perform better when system parameters are uncertain.

CCRS means that manufacturers retain ownership of products until they are sold and retailers receive a percentage of the sales revenue from manufacturers. Wang et al. (2004) introduced the consignment contract into the newsvendor model and found that SC members' behaviors are affected by price elasticity and cost sharing ratio. For the revenue sharing ratio of CCRS, it mainly includes non-fixed proportion of product price (Chen et al., 2011), fixed ratio (Hu & Li, 2012), and fixed fees (Cai et al., 2012). Subsequently, Zhang et al. (2019) considered the coordination contract including revenue sharing of logistics services and transfer price of logistics orders in e-commerce logistics SCs.

Based on the above literature on simple RSCs, some scholars design more complex ones. For example, Van Der Rhee et al. (2010) found that the multi-echelon SC can achieve coordination when the most downstream party signs a spanning RSC with the logistics center. Thien et al. (2015) proposed a two-way penalties RSC to coordinate the two-stage SC under asymmetric information. In order to enable manufacturers and retailers to allocate reasonable profits in a financially constrained SC, Xiao et al. (2017) proposed a generalized RSC that combines quantity discount and buyback.

RSCs are widely used in risk-averse SCs. In particular, Wei & Choi (2010) introduced the WCRS into the SC and studied how retailers' risk aversion and information asymmetry affect SC members. Xu et al. (2014) analyzed the effect of risk tolerance on members' pricing decisions and two-way RSCs under the centralized and decentralized models. Shang & Yang (2015) considered the influence of risk aversion on the SC under contract negotiation. Zhao & Zhu (2018) discussed the influence of risk tolerance on optimal decisions of risk-averse remanufacturers and retailers and designed RSCs for the SC.

Although studies on RSCs are relatively mature, most studies focus on SC coordination and few of them consider the influence of RSCs on sales mode selection.

2.3 Sales mode selection on e-commerce platforms

Since platforms can play the role of distributors or third parties, the sales modes include resale mode and agency mode. Existing research mainly focuses on influence of external attributes on sales mode selection strategy in single-channel or dual-channel SCs. For the case of single-channel, Hagiu & Wright (2015) discussed the influence of cost difference, spillover effect, and information asymmetry on sales mode selection strategy on platforms, and found that decision-makers prefer to choose the agency mode. Abhishek et al. (2016) studied sales mode selection under competition among online retailers and analyzed the influence of network spillover effect and competition intensity on sales mode selection. They further found that online retailers choose the agency mode when there is intense price competition. Liu et al. (2021) showed that, under a certain order fulfillment cost, as the competition intensity of e-commerce platforms increases, manufacturers' optimal selection strategy gradually changes to agency mode, hybrid mode, and resale mode. Moreover, some scholars concentrate on competition among manufacturers. In particular, for the SC consisting of competitive manufacturers and one platform, Kwark et al. (2017) investigated the influence of competition intensity and third-party information on selection strategies and found that accuracy of quality information is conducive to resale mode selection, while accuracy of suitable information is conducive to agency mode selection. Tian et al. (2018) studied how order fulfillment costs and price competition affect sales mode selection strategies. Based on the competition between the major supplier with large potential demand and the small supplier with small potential demand, Zennyo (2020) found that suppliers choose agency mode if both commission rate and product substitution rate are low.

For the case of dual-channel, scholars mainly consider the influence of factors such as product vertical differentiation, SC power structure, bundled sales, information asymmetry and so on. In particular, Tan & Carrillo (2017) analyzed the impact of vertically differentiated products on manufacturer's selection of agency, resale, and fixed-price modes, and found that agency mode is better than the others. For different SC power structures, Dennis et al. (2017) found that choosing resale mode under the dominance of retailers is beneficial to manufacturers. By exploring interaction between bundling sales of upstream manufacturers and sales mode selection of downstream e-commerce platforms, Geng et al. (2018) showed that high commission rate is not necessarily beneficial to e-commerce platforms. Shen et al. (2019) established leader-follower game models with e-commerce platforms as leaders, manufacturers as followers, and traditional retailers as weak followers to analyze interaction between channel selection of manufacturers and sales mode selection of e-commerce platforms. They further showed that high slotting fees are beneficial to e-commerce platforms and overall channel efficiency, but not to manufacturers. Zhang and Zhang (2020) discussed the influence of offline entry cost, channel substitution rate, and information uncertainty on sales mode selection. Chen et al. (2020) considered the situation that manufacturers sell products through brick-and-mortar stores during peak-seasons and through e-commerce platforms during off-seasons. Their results showed that downstream competing e-commerce platforms prefer to choose resale mode when the inventory levels are moderate.

Note that the above literatures focus on risk-neutral single-channel or dual-channel SCs, and rarely consider the selection of sales mode for multi-channel SCs under risk aversion.

In this paper, we take a risk-averse manufacturer with multiple sales channels including traditional retail channel, direct selling channel, and e-commerce platform channel, as our research object and introduce RSCs as well to study the effect of risk aversion and RSCs on manufacturer's sales mode selection and each member's profit.

3. Problem formulation

Our research considers a multi-channel SC consisting of a risk-averse manufacturer, a traditional retailer, and an e-commerce platform. We mainly discuss the manufacturer's sales mode selection between resale and agency modes under the traditional retail channel, direct selling channel, e-commerce platform channel and RSCs. The notations related to this problem are described in Table 1.

Table 1
Notations

Notation	Description
I	Set of sales modes $i \in I = \{R, A\}$: R represents the resale mode; A represents the agency mode
J	Set of channels $j \in J = \{T, D, E\}$: T represents the traditional retail channel; D represents the direct selling channel; E represents the e-commerce platform channel
α	Market share of traditional retail channel, $\alpha \in [0,1]$
β	Market share of direct selling channel, $\beta \in [0, 1-\alpha]$
$\delta_j = \delta$	The intensity of price competition, $\delta \in [0,1]$
d	Potential market demand
ϕ	Degree of risk aversion, $\phi > 0$
λ_{iT}	Ratio of revenue that retailer gives to manufacturer in sales mode i, $\lambda_{iT} \in [0,1]$
λ_{RE}	Ratio of revenue that e-commerce platform gives to manufacturer in resale mode, $\lambda_{RE} \in [0,1]$
ρ	Commission rate in agency mode, $\rho \in [0, 0.3]$
k	Slotting fee in agency mode
η	Random error of market demand, $\eta \sim N(0, \sigma^2)$
w_i	Wholesale price in sales mode i
p_{iT}	Price of traditional retail channel in sales mode i
p_{RD}	Price of direct selling channel in resale mode
p_{RE}	Price of e-commerce platform channel in resale mode
$\boldsymbol{p_R} = (p_{RT}, p_{RD}, p_{RE})$	Price vector in resale mode
p_A	Prices of direct selling and e-commerce platform channels in agency mode
D_{ij}	Demand function of channel j in sales mode i
$\pi_{iM}, \pi_{iT}, \pi_{iE}$	Profit function of manufacturer, retailer and platform in sales mode i
$E(\pi_{iM}), E(\pi_{iT}), E(\pi_{iE})$	Expected profit function of manufacturer, retailer and platform in sales mode i
U_{iM}, U_{iT}, U_{iE}	Utility function of manufacturer, retailer and platform in sales mode i

In resale mode, the manufacturer wholesales goods at a low price w_R to the traditional retailer and platform. The traditional retailer and platform give the manufacturer sales revenue in proportion λ_{RT} and λ_{RE}, respectively and sell goods to consumers at prices p_{RT} and p_{RE}, respectively. In direct selling channel, the manufacturer sells consumers its goods at a price p_{RD} through direct selling websites. The structure diagram of the resale mode is shown in Fig. 1.

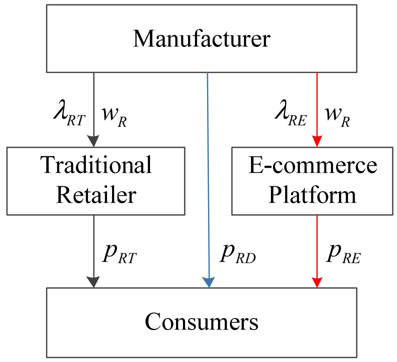


Fig. 1. Resale mode

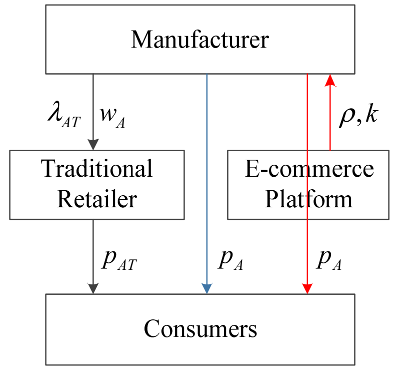


Fig. 2. Agency mode

In agency mode, the manufacturer pays a slotting fee k to the platform and sells its goods at a price p_A, while the platform collects sales revenue from the manufacturer at commission rate ρ. The slotting fee and commission rate are assumed to be exogenous variables, where the commission rate generally does not exceed 30%. In traditional retail channel, the manufacturer gives low price w_A to the traditional retailer, while the retailer gives sales revenue to the manufacturer in a proportion λ_{AT} and sells goods to consumers at price p_{AT}. In direct selling channel, the manufacturer sells goods through direct selling websites at the same price p_A. The structure diagram of agency mode is shown in Fig. 2. As in Petruzzi and Dada (1999), we assume the market demand including a random error. That is, when the manufacturer chooses resale mode, the uncertain demand functions of each channel are

$$D_{RT} = \alpha d - p_{RT} + \delta_D p_{RD} + \delta_E p_{RE} + \eta,$$
$$D_{RD} = \beta d - p_{RD} + \delta_T p_{RT} + \delta_E p_{RE} + \eta,$$
$$D_{RE} = (1-\alpha-\beta)d - p_{RE} + \delta_T p_{RT} + \delta_D p_{RD} + \eta.$$

When the manufacturer chooses agency mode, the uncertain demand functions of each channel are

$$D_{AT} = \alpha d - p_{AT} + \delta_D p_A + \delta_E p_A + \eta,$$
$$D_{AD} = \beta d - p_A + \delta_T p_{AT} + \delta_E p_A + \eta,$$
$$D_{AE} = (1-\alpha-\beta)d - p_A + \delta_T p_{AT} + \delta_D p_A + \eta.$$

Here, d represents the potential market demand, $\alpha \in [0,1]$, $\beta \in [0,1]$ and $1-\alpha-\beta \in [0,1]$ respectively represent the market shares of traditional retail channel, direct selling channel, and e-commerce platform channel. $\delta_j \in [0,1]$ represents the price competition intensity, and η represents random fluctuation and satisfies $\eta \sim N(0,\sigma^2)$. Moreover, similarly as in Lau (1980) and Xie et al. (2011), we employ the MV model to give the risk-averse manufacturer's utility function as

$$U_{iM} = E(\pi_{iM}) - \phi\sqrt{Var(\pi_{iM})}.$$

Here, π_{iM} represents the manufacturer's profit function in sales mode i, $E(\pi_{iM})$ represents the manufacturer's expected profit function in sales mode i, $Var(\pi_{iM}) = E[\pi_{iM} - E(\pi_{iM})]^2$ represents the manufacturer's profit variance function in sales mode i, and $\phi > 0$ means the risk aversion degree of the manufacturer.

4. Models

To simplify models, we suppose that the marginal production cost is zero and the price competition intensity of each channel is δ, namely, $\delta_j = \delta, j \in J = \{T, D, E\}$. We first construct models for resale and agency modes respectively and then analyze their optimal solutions.

4.1 The resale mode

In resale mode, we establish a Stackelberg game model in which the manufacturer is a leader and the retailer and the platform are followers. The manufacturer sets the same wholesale price w_R for the retailer and the platform and its price p_{RD} in the

direct selling channel. The retailer and the platform determine their prices p_{RT} and p_{RE} under the retail channel and the platform channel respectively.
The lower-level models are to maximize the profit functions of both the retailer and the platform, that is,

$$\max_{p_{RT}\geq w_R} \quad \pi_{RT}(p_{RT}) = (1-\lambda_{RT})(p_{RT}-w_R)D_{RT},$$

$$\max_{p_{RE}\geq w_R} \quad \pi_{RE}(p_{RE}) = (1-\lambda_{RE})(p_{RE}-w_R)D_{RE}.$$

When $\lambda_{RT}\in[0,1)$ and $\lambda_{RE}\in[0,1)$, the above two models are convex optimization problems with respect to their own decision variables p_{RT} and p_{RE}. Then, by direct calculations, when $(2+\delta)((1-\delta)w_R-\delta p_{RD})\leq \min\{2\alpha+\delta(1-\alpha-\beta),\ 2(1-\alpha-\beta)+\alpha\delta\}d+(2+\delta)\eta$, the optimal prices of the retailer and platform can be expressed as follows:

$$p_{RT}^*(w_R,p_{RD}) = \frac{2\alpha d+\delta(1-\alpha-\beta)d+(2+\delta)\eta}{4-\delta^2}+\frac{1}{2-\delta}w_R+\frac{\delta}{2-\delta}p_{RD},$$

$$p_{RE}^*(w_R,p_{RD}) = \frac{\alpha\delta d+2(1-\alpha-\beta)d+(2+\delta)\eta}{4-\delta^2}+\frac{1}{2-\delta}w_R+\frac{\delta}{2-\delta}p_{RD}.$$

In the upper-level model, the manufacturer's profit function is

$$\pi_{RM} = w_R D_{RT}+\lambda_{RT}(p_{RT}-w_R)D_{RT}+p_{RD}D_{RD}+w_R D_{RE}+\lambda_{RE}(p_{RE}-w_R)D_{RE}.$$

Its expectation is

$$E(\pi_{RM}) = w_R E(D_{RT})+\lambda_{RT}(p_{RT}-w_R)E(D_{RT})+p_{RD}E(D_{RD})+w_R E(D_{RE})+\lambda_{RE}(p_{RE}-w_R)E(D_{RE})$$

and its variance is

$$Var(\pi_{RM}) = (w_R+\lambda_{RT}(p_{RT}-w_R)+p_{RD}+w_R+\lambda_{RE}(p_{RE}-w_R))^2\sigma^2.$$

In addition, the utility function of the manufacturer is

$$U_{RM} = (w_R+\lambda_{RT}(p_{RT}-w_R))(E(D_{RT})-\phi\sigma)+p_{RD}(E(D_{RD})-\phi\sigma)+(w_R+\lambda_{RE}(p_{RE}-w_R))(E(D_{RE})-\phi\sigma).$$

Therefore, by maximizing the utility function, the manufacturer's model (I) is

$$\max_{w_R,p_{RD}} \quad U_{RM}(w_R,p_{RD}) = \frac{1}{(2-\delta)^2}(A+w_R(A_{11}+0.5A_{12}w_R+A_{13}p_{RD})+p_{RD}(B_{11}+0.5B_{12}p_{RD}))$$

$$\text{s.t.} \quad p_{RD}\geq 0,\ \ w_R\geq 0,$$

$$(2+\delta)((1-\delta)w_R-\delta p_{RD})\leq \min\{2\alpha+\delta(1-\alpha-\beta),\ 2(1-\alpha-\beta)+\alpha\delta\}d+(2+\delta)\eta,$$

where the notations $A, A_{11}, A_{12}, A_{13}, B_{11}, B_{12}$ are given in Table A.1. By calculating the above model, we can attain the equilibrium results.

Proposition 1. In resale mode, under one of the following conditions:
(1) $\delta\in[0,0.5)$, $\lambda_{RT}\in[0,1)$, $\lambda_{RE}\in[0,1)$, $\alpha\in[0,0.5]$, $\beta\in[0,1-2\alpha]$, $\eta\in[\eta_{11},\eta_{12}]$, $\phi\sigma\in(0,\nu_{12}]$;
(2) $\delta\in[0,0.5)$, $\lambda_{RT}\in[0,1)$, $\lambda_{RE}\in[0,1)$, $\alpha\in[0,0.5]$, $\beta\in[0,1-2\alpha]$, $\eta\leq\eta_{11}$, $\phi\sigma\in[\nu_{11},\nu_{13}]$;
(3) $\delta\in[0,0.5)$, $\lambda_{RT}\in[0,1)$, $\lambda_{RE}\in[0,1)$, $\eta\in[\eta_{14},\eta_{12}]$, $\phi\sigma\in(0,\min\{\nu_{12},\nu_{13},\nu_{14}\}]$, $\alpha\in[0,0.5]$, $\beta\in[1-2\alpha,1]$;
(4) $\delta\in[0,0.5)$, $\lambda_{RT}\in[0,1)$, $\lambda_{RE}\in[0,1)$, $\eta\in[\eta_{14},\eta_{12}]$, $\phi\sigma\in(0,\min\{\nu_{12},\nu_{13},\nu_{14}\}]$, $\alpha\in[0.5,1]$, $\beta\in[0,1-\alpha]$,
the optimal decisions of the manufacturer, the retailer, and the platform are

$$w_R^* = \frac{M_2}{M_1}, \qquad p_{RD}^* = \frac{M_3}{M_1},$$

$$p_{RT}^* = \frac{M_1(2\alpha d+\delta(1-\alpha-\beta)d+(2+\delta)\eta)+(2+\delta)M_2+\delta(2+\delta)M_3}{(4-\delta^2)M_1},$$

$$p_{RE}^* = \frac{M_1(\alpha\delta d+2(1-\alpha-\beta)d+(2+\delta)\eta)+(2+\delta)M_2+\delta(2+\delta)M_3}{(4-\delta^2)M_1},$$

where $\eta_{1k}=-\dfrac{a_{1k}}{b_{1k}}(k=1,2,4)$, $\nu_{1k}=\dfrac{a_{1k}+b_{1k}\eta}{-c_{1k}}$, $\nu_{13}=-\dfrac{a_{13}}{b_{13}}$, the notations M_1, M_2, M_3 and $a_{1k}, b_{1k}, c_{1k}, a_{13}, b_{13}$ are given in Tables A.1 and A.2. See the appendix for a proof of the above proposition. It is not difficult to obtain the following result.

Proposition 2. The sensitivity analysis results for the optimal decisions in resale mode are as follows:

$$\frac{\partial w_R^*}{\partial\beta}<0,\ \frac{\partial p_{RD}^*}{\partial\beta}<0,\ \frac{\partial p_{RT}^*}{\partial\beta}<0,\ \frac{\partial p_{RE}^*}{\partial\beta}<0;\ \frac{\partial w_R^*}{\partial\delta}>0,\ \frac{\partial p_{RD}^*}{\partial\delta}>0,\ \frac{\partial p_{RT}^*}{\partial\delta}>0,\ \frac{\partial p_{RE}^*}{\partial\delta}>0;$$

$$\frac{\partial w_R^*}{\partial \alpha}=0,\ \frac{\partial p_{RD}^*}{\partial \alpha}=0,\ \frac{\partial p_{RT}^*}{\partial \alpha}>0,\ \frac{\partial p_{RE}^*}{\partial \alpha}<0;\ \frac{\partial w_R^*}{\partial \lambda_{RT}}>0,\ \frac{\partial p_{RD}^*}{\partial \lambda_{RT}}=0,\ \frac{\partial p_{RT}^*}{\partial \lambda_{RT}}>0,\ \frac{\partial p_{RE}^*}{\partial \lambda_{RT}}>0;$$

$$\frac{\partial w_R^*}{\partial \lambda_{RE}}<0,\ \frac{\partial p_{RD}^*}{\partial \lambda_{RE}}=0,\ \frac{\partial p_{RT}^*}{\partial \lambda_{RE}}<0,\ \frac{\partial p_{RE}^*}{\partial \lambda_{RE}}<0;\ \frac{\partial w_R^*}{\partial \phi}<0,\ \frac{\partial p_{RD}^*}{\partial \phi}<0,\ \frac{\partial p_{RT}^*}{\partial \phi}<0,\ \frac{\partial p_{RE}^*}{\partial \phi}<0.$$

Proposition 2 shows that, as the market share of direct selling channels increases, manufacturers should set lower product prices and wholesale prices, while traditional retailers and e-commerce platforms attract consumers to buy products through their own channels by reducing prices; the increase in price competition intensity is beneficial for SC members to raise prices to seek more profits; manufacturers' prices are not influenced by the traditional retail channels' market share. Moreover, with the increase of traditional retail channels' market share, traditional retailers should take advantage of market share to set higher product prices, while e-commerce platforms should set lower product prices to seek more profits from consumers; manufacturers' direct selling channel prices are not influenced by revenue sharing ratio, but the other prices increase with the increase of the revenue sharing ratio under the traditional retail channel and decrease under the e-commerce platform channel. In addition, increasing risk aversion is not conducive to the members to raise prices to seek more profits.

4.2 The agency mode

In agency mode, the manufacturer and retailer are regarded as the leader and follower, respectively. It is particularly important to note that the platform cannot be a participant because of the exogenous variables, and its profit function is $\pi_{AE}=\rho p_A D_{AE}+k$. The manufacturer determines the wholesale price w_A and the price p_A of the direct selling and e-commerce platform channels. The retailer sets the price p_{AT} for its own channel.

The lower-level model maximizes the profit function of retailer, namely

$$\max_{p_{AT}\geq w_A}\ \pi_{AT}(p_{AT})=(1-\lambda_{AT})(p_{AT}-w_A)D_{AT}.$$

When $\lambda_{AT}\in[0,1)$, the above model is a convex optimization problem with respect to p_{AT}. It is easy to show that, when $w_A\leq \alpha d+\eta+2\delta p_A$, the traditional retailer's optimal price is

$$p_{AT}^*(w_A,p_A)=\frac{\alpha d+\eta}{2}+\frac{1}{2}w_A+\delta p_A.$$

Similarly, the manufacturer's profit function, expected profit function, profit variance function, and utility function can be expressed as

$$\pi_{AM}=w_A D_{AT}+\lambda_{AT}(p_{AT}-w_A)D_{AT}+p_A D_{AD}+(1-\rho)p_A D_{AE}-k,$$
$$E(\pi_{AM})=w_A E(D_{AT})+\lambda_{AT}(p_{AT}-w_A)E(D_{AT})+p_A E(D_{AD})+(1-\rho)p_A E(D_{AE})-k,$$
$$Var(\pi_{AM})=(w_A+\lambda_{AT}(p_{AT}-w_A)+p_A+(1-\rho)p_A)^2\sigma^2,$$
$$U_{AM}=(w_A+\lambda_{AT}(p_{AT}-w_A))(E(D_{AT})-\phi\sigma)+p_A(E(D_{AD})-\phi\sigma)+(1-\rho)p_A(E(D_{AE})-\phi\sigma)-k.$$

Therefore, by maximizing the utility function, the manufacturer's model (II) is

$$\max_{w_A,p_A}\ U_{AM}(w_A,p_A)=B+w_A(A_{21}+0.5A_{22}w_A+A_{23}p_A)+p_A(B_{21}+0.5B_{22}p_A)-k$$
$$\text{s.t.}\quad 0\leq w_A\leq \alpha d+\eta+2\delta p_A,\quad p_A\geq 0,$$

where the notations $B, A_{21}, A_{22}, A_{23}, B_{21}, B_{22}$ are given in Table A.1. We can attain the following equilibrium results for model (II).

Proposition 3. In agency mode, under one of the following conditions:

(1) $\delta\in[0,0.5]$, $\lambda_{AT}\in[0,1)$, $\rho\in[0,0.3]$, $\alpha\in[0,1]$, $\beta\in[0,1-\alpha]$, $\eta\in[\eta_{21},\eta_{22}]$, $\phi\sigma\in(0,\nu_{22}]$;
(2) $\delta\in[0,0.5]$, $\lambda_{AT}\in[0,1)$, $\rho\in[0,0.3]$, $\alpha\in[0,1]$, $\beta\in[0,1-\alpha]$, $\eta\in[\eta_{23},\eta_{24}]$, $\phi\sigma\in[\nu_{21},\nu_{23}]$,

the optimal decisions of the manufacturer and the retailer are

$$w_A^*=\frac{N_2}{N_1},\qquad p_A^*=\frac{N_3}{N_1},\qquad p_{AT}^*=\frac{(\alpha d+\eta)N_1+N_2+2\delta N_3}{2N_1},$$

where $\eta_{2k}=-\frac{a_{2k}}{b_{2k}}(k=1,2,3)$, $\eta_{24}=-\frac{b_{21}c_{23}-b_{23}c_{21}}{a_{21}c_{23}-a_{23}c_{21}}$, $\nu_{2k}=-\frac{a_{2k}+b_{2k}\eta}{c_{2k}}$, and the notations N_1,N_2,N_3 and a_{2k},b_{2k},c_{2k} are given in Tables A.1 and A.2.

See the appendix for a proof of the above proposition. It is not difficult to obtain the following result.

Proposition 4. In agency mode, the sensitivity analysis results of the optimal decisions are

$$\frac{\partial w_A^*}{\partial \beta} > 0, \frac{\partial p_A^*}{\partial \beta} > 0, \frac{\partial p_{AT}^*}{\partial \beta} > 0; \quad \frac{\partial w_A^*}{\partial \delta} > 0, \frac{\partial p_A^*}{\partial \delta} > 0, \frac{\partial p_{AT}^*}{\partial \delta} > 0;$$

$$\frac{\partial w_A^*}{\partial \alpha} > 0, \frac{\partial p_A^*}{\partial \alpha} < 0, \frac{\partial p_{AT}^*}{\partial \alpha} > 0; \quad \frac{\partial w_A^*}{\partial \phi} < 0, \frac{\partial p_A^*}{\partial \phi} < 0, \frac{\partial p_{AT}^*}{\partial \phi} < 0.$$

Proposition 4 indicates that, as the market share of direct selling channels increases, the product price and wholesale price should be set higher by manufacturers, and traditional retailers should also set a higher retail price to ensure their interests; increasing price competition intensity is beneficial to SC members to raise prices. Moreover, as the market share of traditional retail channels increases, both wholesale price of manufacturers and price of traditional retailers should be set higher, while the price of direct selling channels should be set lower; the reduction of risk aversion is conducive to the members to raise prices to seek more profits.

5. Numerical analysis

To facilitate analysis, this section considers the case that both retailer and platform are risk-neutral. We mainly discuss which sales mode is better for a risk-averse manufacturer and how the manufacturer's sales mode selection together with RSCs and degree of risk aversion influences each member under different modes. We try to find some relevant management implications from numerical analysis. In our experiments, we set the potential market demand $d = 200$ and the slotting fee $k = 0$. To ensure that each channel's demand function is nonnegative, we set the price competition intensity $\delta \in [0, 0.485]$, market share of traditional retail channel $\alpha \in [0.15, 0.65]$, market share of direct selling channel $\beta \in [0, 0.85]$, commission rate $\rho \in [0, 0.3]$, revenue sharing ratio $\lambda_{RT}, \lambda_{RE}, \lambda_{AT} \in [0,1)$, degree of risk aversion $\phi \in [3,18]$, and the random variable $\eta \sim N(0, \sigma^2)$ with $\sigma = 1$. Note that, in resale mode, the optimal utility functions of the manufacturer, retailer, and platform are respectively

$$U_{RM}^* = E(\frac{(2AM_1^2 + M_2(2A_{11}M_1 + A_{12}M_2 + 2A_{13}M_3) + M_3(2B_{11}M_1 + B_{12}M_3)}{2(2-\delta)^2 M_1^2}),$$

$$U_{RT}^* = E(\pi_{RT}^*) = E(\frac{(1-\lambda_{RT})(M_1(2\alpha d + \delta(1-\alpha-\beta)d + (2+\delta)\eta) + \delta(2+\delta)M_3 + (\delta-1)(2+\delta)M_2)^2}{M_1^2(2-\delta)^2(2+\delta)^2}),$$

$$U_{RE}^* = E(\pi_{RE}^*) = E(\frac{(1-\lambda_{RE})(M_1(\alpha\delta d + 2(1-\alpha-\beta)d + (2+\delta)\eta) + \delta(2+\delta)M_3 + (\delta-1)(2+\delta)M_2)^2}{M_1^2(2-\delta)^2(2+\delta)^2}).$$

In agency mode, the optimal utility functions become

$$U_{AM}^* = E(\frac{2BN_1^2 + N_2(2A_{21}N_1 + A_{22}N_2 + 2A_{23}N_3) + N_3(2B_{21}N_1 + B_{22}N_3)}{2N_1^2}) - k,$$

$$U_{AT}^* = E(\pi_{AT}^*) = E(\frac{(1-\lambda_{AT})(N_1(\alpha d + \eta) + 2\delta N_3 - N_2)^2}{4N_1^2}),$$

$$U_{AE}^* = E(\pi_{AE}^*) = E(\frac{\rho N_3(N_1(\alpha\delta d + 2(1-\alpha-\beta)d + (2+\delta)\eta) + \delta N_2 + 2(\delta^2 + \delta - 1)N_3)}{2N_1^2}) + k.$$

5.1 Manufacturer's sales mode selection analysis

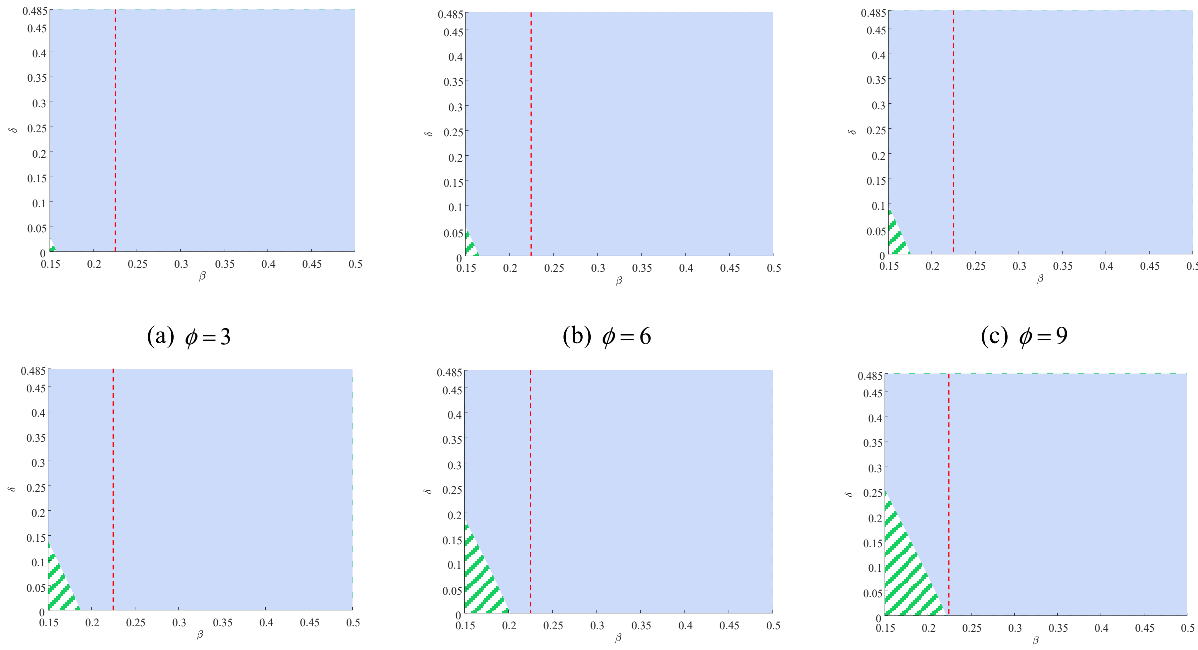
This subsection discusses the effect of each parameter on the manufacturer's sales mode selection by comparing the manufacturer's utility difference between resale and agency modes. Since the parameters α and β are both market share factors and the revenue sharing ratio λ_{RT}, λ_{RE}, λ_{AT} have little effect on manufacturer's sales mode selection, we mainly analyze the impact of price competition intensity δ, market share of direct selling channel β, degree of risk aversion ϕ, and commission rate ρ. Specifically, we set $\alpha = 0.5$, $\lambda_{RT} = \lambda_{RE} = 0.5$, $\lambda_{AT} = 0.1$ and analyze how δ and β affect the manufacturer's sales mode selection by changing the values of ϕ and ρ. Since the commission rate generally does not exceed 30%, we consider three cases: low commission rate with $\rho = 0.1$, moderate commission rate with $\rho = 0.2$, and high commission rate with $\rho = 0.3$.

5.1.1 Low commission rate (ρ = 0.1)

By choosing different values of degree of risk aversion ϕ, we can observe the effect of δ and β on the manufacturer's sales mode selection, as shown in Fig. 3, where the blue region indicates that the manufacturer should choose the agency mode better, and the green slash area indicates that the manufacturer chooses the resale mode better. It can be seen that, when $\phi \in [3,18]$ and $\beta \in [0, 0.225]$, the manufacturer should choose resale mode if the price competition intensity is weak, otherwise choose agency mode. When $\phi \in [3,18]$ and $\beta \in [0.225, 0.5]$, the manufacturer should always choose agency mode, regardless of the intensity of price competition.

5.1.2 Moderate commission rate ($\rho = 0.2$)

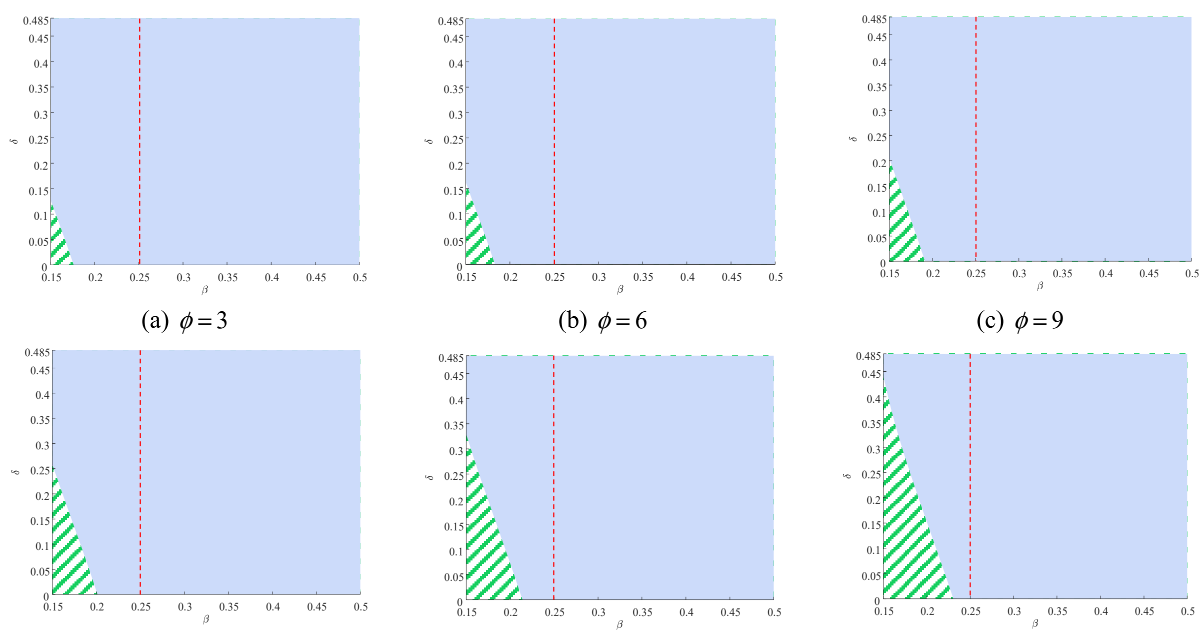
The numerical results for this case are shown in Fig. 4. It can be seen that, when $\phi \in [3,18]$ and $\beta \in [0,0.25]$, the manufacturer chooses resale mode if the price competition intensity is weak, otherwise chooses agency mode. The manufacturer should always choose agency mode when $\phi \in [3,18]$ and $\beta \in [0.25,0.5]$.



(a) $\phi = 3$ (b) $\phi = 6$ (c) $\phi = 9$

(d) $\phi = 12$ (e) $\phi = 15$ (f) $\phi = 18$

Fig. 3. Manufacturer's sales mode selection under low commission rate



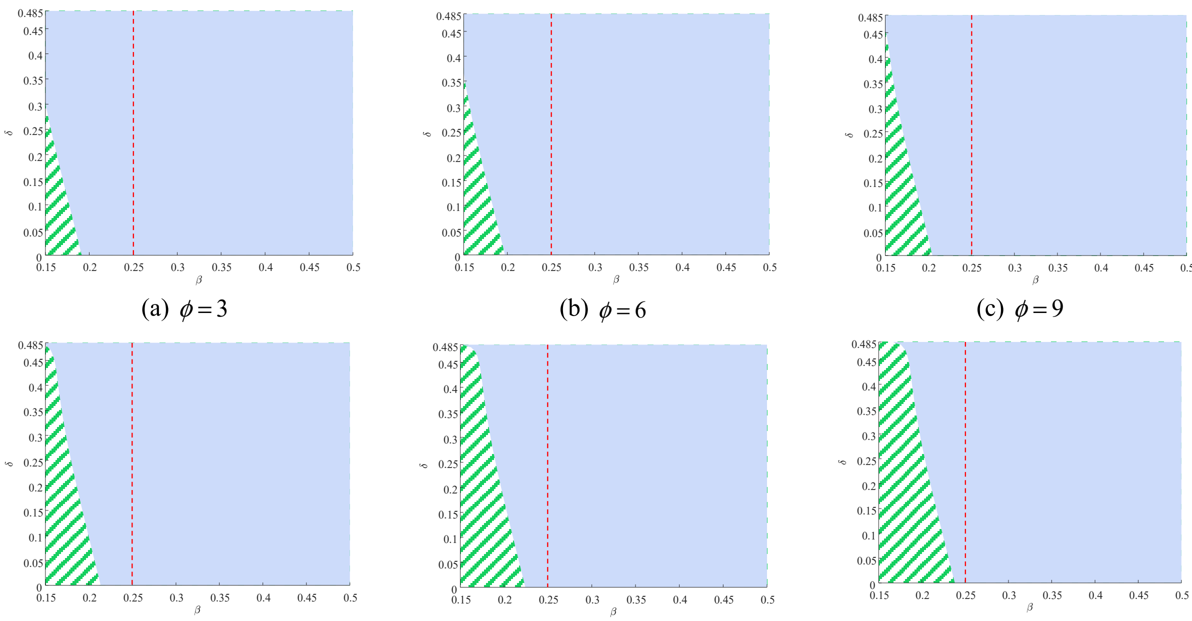
(a) $\phi = 3$ (b) $\phi = 6$ (c) $\phi = 9$

(d) $\phi = 12$ (e) $\phi = 15$ (f) $\phi = 18$

Fig. 4. Manufacturer's sales mode selection under moderate commission rate

5.1.3 High commission rate ($\rho = 0.3$)

The experimental results in this case are shown in Fig. 5. The results reveal that, if $\phi \in [3,12]$ and $\beta \in [0,0.25]$, the manufacturer chooses resale mode if the price competition intensity is weak, otherwise chooses agency mode. The manufacturer should always choose agency mode when $\phi \in [3,12]$ and $\beta \in [0.25, 0.5]$. Moreover, when $\phi \in [15,18]$, the manufacturer chooses resale mode if the direct selling channel's market share is low and agency mode if it is high. In summary, when the direct selling channel has a low market share, if the commission rate is low or moderate, the manufacturer chooses resale mode when the price competition intensity is weak and agency mode when it is strong, and manufacturer's sales mode selection has nothing to do with degree of risk aversion; if the commission rate is high and the degree of risk aversion is low or moderate, the manufacturer chooses resale mode when the price competition intensity is low and agency mode when it is strong; if both the commission rate and degree of risk aversion are high, the manufacturer should always choose resale mode, regardless of the intensity of price competition. Conversely, when the direct selling channel has a high market share, the manufacturer always chooses agency mode, and intensity of price competition, risk aversion, and commission rate do not affect the manufacturer's sales mode selection strategy.



(a) $\phi = 3$ (b) $\phi = 6$ (c) $\phi = 9$

(d) $\phi = 12$ (e) $\phi = 15$ (f) $\phi = 18$

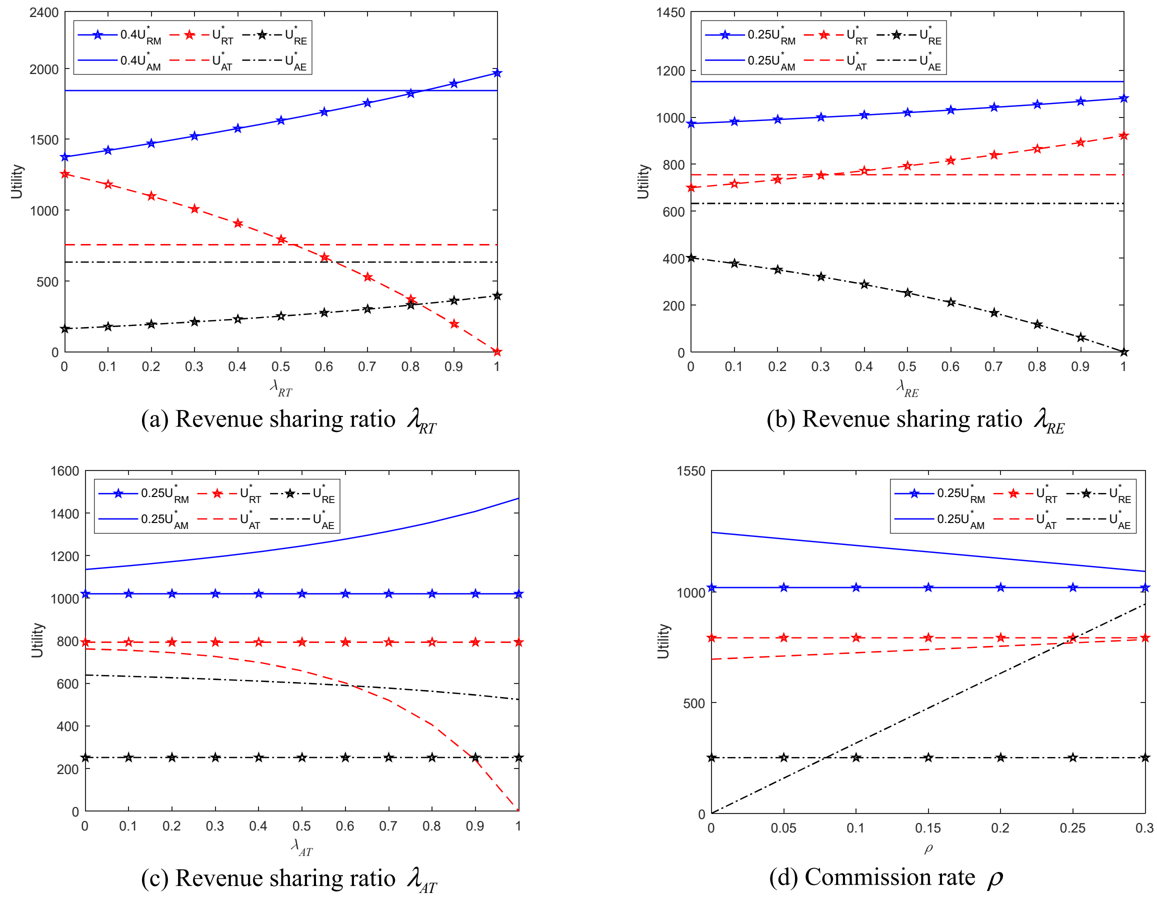
Fig. 5. Manufacturer's sales mode selection under high commission rate

By observing the manufacturer's sales mode selection under the same commission rate, we found that the area to choose resale mode increases with the increase of risk aversion. This indicates that manufacturers tend to choose resale mode when direct selling channels have a low market share, weak price competition intensity, and high degree of risk aversion. By comparing manufacturer's sales mode selection under different commission rates, we found that the area to choose resale mode increases as the commission rate increases, which indicates that the manufacturer's sales mode selection strategy is affected by the commission rate. Substantially, the manufacturer tends to choose agency mode when the commission rate is low and the direct selling channel has a high market share and choose resale mode when both commission rate and degree of risk aversion are high and both market share of direct selling channel and intensity of price competition are low.

5.2 Impact of RSC

This subsection mainly discusses the influence of revenue sharing ratios λ_{RT} and λ_{RE} in resale mode, and revenue sharing ratio λ_{AT} and commission rate ρ in agency mode on optimal utility of each member. The basic parameters were set as $\delta = 0.3$, $\alpha = 0.5$, $\beta = 0.2$, $\phi = 10$, $\lambda_{RT} = \lambda_{RE} = 0.5$, $\lambda_{AT} = 0.1$, $\rho = 0.2$.

Firstly, the effect of the revenue sharing ratio λ_{RT} on the optimal utility of each member is analyzed. The experimental results are shown in Fig. 6(a). It can be observed that, with the increase of λ_{RT}, the utilities of the manufacturer and platform in resale mode increase, while the utility of retailers decreases, but each member's utility in agency mode is not affected. When λ_{RT} is low, the utility of manufacturer in agency mode is higher than resale mode and the utility of retailer in resale mode is higher than agency mode. No matter how λ_{RT} changes, the utility of the platform in agency mode is always higher than resale mode. Therefore, when λ_{RT} is low, choosing agency mode is beneficial to the platform but not to retailers. When λ_{RT} is very high, choosing resale mode is not conducive to retailer and platform. When λ_{RT} is relatively high, choosing agency mode is beneficial to all members.



(a) Revenue sharing ratio λ_{RT}

(b) Revenue sharing ratio λ_{RE}

(c) Revenue sharing ratio λ_{AT}

(d) Commission rate ρ

Fig. 6. Impact of RSC on optimal utility

Secondly, we analyze the influence of the revenue sharing ratio λ_{RE} on each member's optimal utility. The experimental results in Fig. 6 (b) show that, as the λ_{RE} increases, the utilities of manufacturer and retailer increase in resale mode, while the utility of platform decreases, but the utilities of each member in agency mode are not affected. The utility of retailers in agency mode is higher than resale mode when λ_{RE} is low. Regardless of how λ_{RE} changes, the utilities of manufacturer and platform in agency mode are always higher than resale mode. Therefore, it is beneficial to both retailer and platform if the manufacturer chooses agency mode when λ_{RE} is low. When λ_{RE} is moderate or high, choosing agency mode is beneficial to the platform but not to retailers.

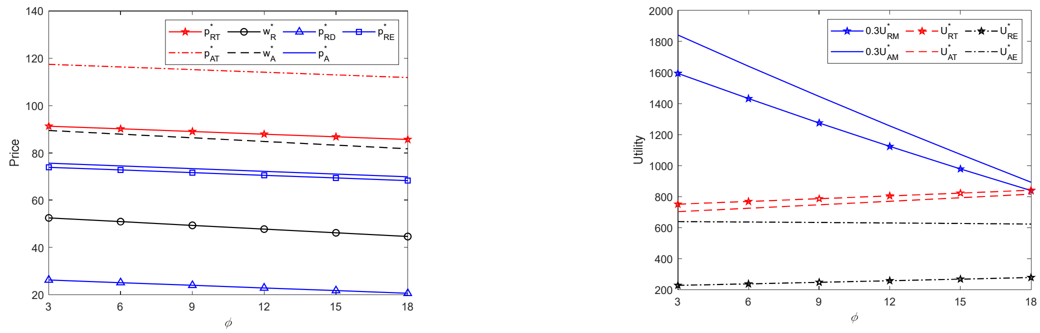
Thirdly, the impact of the revenue sharing ratio λ_{AT} is analyzed. Fig. 6(c) depicts the experimental results. It is clear that, with the increase of λ_{AT}, the utility of manufacturer in agency mode increases, the utilities of retailer and platform decrease, but each member's utility in resale mode are not affected. No matter how λ_{AT} changes, the utilities of the manufacturer and platform in resale mode are always lower than agency mode, while the utility of retailers in resale mode is always higher than agency mode. Therefore, choosing agency mode is beneficial to the platform but not to retailers.

Finally, we report the experimental results about the impact of commission rate on the utility of each member in Fig. 6(d). It can be observed that, with the increase of ρ, the utility of manufacturer in agency mode decreases, while the utilities of retailer and platform increase, but each member's utility in resale mode are not affected. The utility of the platform in resale mode is higher than agency mode when the commission rate is low. The manufacturer's utility in agency mode is always higher than resale mode, while the retailer's utility is opposite. Therefore, choosing agency mode is not conducive for the retailer and platform to obtain more utility when the commission rate is low. The selection of agency mode under a high commission rate is beneficial to the platform but not to the retailer.

5.3 Impact of risk aversion

In this subsection, we investigate the effect of risk aversion on the optimal price and utility of each member. The experiment results are shown in Fig. 7. It is clear from Fig. 7(a) that, with the increase of ϕ, all optimal prices gradually decrease, regardless of which sales mode manufacturer chooses, which indicates that optimal price of each member should be set low as degree of risk aversion increases. No matter how degree of risk aversion changes, there always holds $p^*_{AT} > p^*_{RT}$, $w^*_A > w^*_R$,

$p_A^* > p_{RE}^* > p_{RD}^*$, which means that degree of risk aversion has no effect on price changes under different sales modes in same channel.



(a) Optimal price (b) Optimal utility

Fig. 7. Impact of risk aversion on optimal price and optimal utility

From Fig. 7(b), with the increase of ϕ, the manufacturer's utility decreases, but retailer's utility increases in both resale and agency modes, while the utility of platform increases in resale mode and decreases in agency mode. Therefore, if the manufacturer chooses agency mode, it is conducive to e-commerce platforms but not to traditional retailers. In summary, when market share of traditional retail channel, intensity of price competition, degree of risk aversion, and commission rate are all moderate and both market share of direct selling channel and λ_{AT} are low, manufacturer will choose the agency mode as long as λ_{RT} is high and is low, which is conducive to the retailer and platform seeking more utility and further realizing a win-win situation.

6. Conclusions

We studied a sales mode selection problem between resale mode and agency mode on an e-commerce platform for risk-averse manufacturers with traditional retail channel, direct selling channel, and e-commerce platform channel. By adopting RSCs among SC members, we constructed two leader-follower game models with manufacturer as a leader and traditional retailer and e-commerce platform as followers. We discussed the convexity conditions for upper and lower models and obtained optimal decisions for each member. Through numerical experiments, we analyzed the impact of price competition intensity, market share, ratio of revenue sharing, commission rate, and degree of risk aversion on sales mode selection and each member's optimal pricing and utility under different sales modes. The numerical results reveal the following revelations: Firstly, the manufacturer's sales mode selection strategy is mainly influenced by intensity of price competition, market share of direct selling channels, commission rate, and degree of risk aversion. When commission rate is low or moderate, manufacturer should choose agency mode if direct selling channel has a high market share or if intensity of price competition is high and direct selling channel has a low market share; Otherwise, manufacturer should choose resale mode if both intensity of price competition and market share of direct selling channel are low. When commission rate is high, manufacturers should choose agency mode if market share of direct selling channel is high and choose resale mode if degree of risk aversion is high while direct selling channel has a low market share. Secondly, the degree of risk aversion of the manufacturer can affect the optimal decision of each member. No matter which sales mode manufacturer chooses, optimal pricing of each member gradually decreases with the increase of risk aversion, which implies that optimal price of each member should be set low as degree of risk aversion increases. Thirdly, when market share of traditional retail channel, price competition intensity, degree of risk aversion, and commission rate are moderate and both market share of direct reselling channel and λ_{AT} are low, manufacturer should choose agency mode when λ_{RT} is high and λ_{ET} is low, which is able to achieve win-win situations. As a future work, we will consider the general case with multiple manufacturers and multiple retailers. However, these participants may be risk-averse, risk-neutral, and risk-seeking. Obviously, the models for this general case are more complicated and, especially, the number of parameters will greatly increase, which may cause some difficulties in solving the models and corresponding sensitivity analysis. How to deal with these difficulties will be our next topic.

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Appendix

Table A.1
Related notations in models and propositions

Notation	Definition
A	$\begin{pmatrix} (2\alpha d+\delta(1-\alpha-\beta)d+(2+\delta)\eta)(2\alpha d+\delta(1-\alpha-\beta)d+(\delta^2-4)\phi\sigma)\lambda_{RT} \\ +(\alpha\delta d+2(1-\alpha-\beta)d+(2+\delta)\eta)(\alpha\delta d+2(1-\alpha-\beta)d+(\delta^2-4)\phi\sigma)\lambda_{RE} \end{pmatrix}/(2+\delta)^2$
A_{11}	$\begin{pmatrix} (\delta^2-4)(d(\beta-1)-2(\delta-2)\phi\sigma)+(\delta-1)(-2d(\alpha(\delta-2)+(\beta-1)\delta) \\ +(2+\delta)(\eta+(\delta-2)\phi\sigma))\lambda_{RT}+(\delta-1)(2d(2-2\beta+\alpha(\delta-2)) \\ +(2+\delta)(\eta+(\delta-2)\phi\sigma))\lambda_{RE} \end{pmatrix}/(2+\delta)$
A_{12}	$2(\delta-1)(2(2-\delta)+(\delta-1)(\lambda_{RT}+\lambda_{RE}))$
A_{13}	$2\delta(2(2-\delta)+(\delta-1)(\lambda_{RT}+\lambda_{RE}))$
B_{11}	$\begin{pmatrix} (\delta^2-4)(2\beta(\delta-1)+(2-\delta)\phi\sigma-\delta)+\delta(-2d(\alpha(\delta-2)+(\beta-1)\delta) \\ +(2+\delta)(\eta+(\delta-2)\phi\sigma))\lambda_{RT}+\delta(2d(2-2\beta+\alpha(\delta-2)) \\ +(2+\delta)(\eta+(\delta-2)\phi\sigma))\lambda_{RE} \end{pmatrix}/(2+\delta)$
B_{12}	$2(2-\delta)(2\delta^2+\delta-2)+2\delta^2(\lambda_{RT}+\lambda_{RE})$
B	$0.25\lambda_{AT}(\alpha d+\eta)(\alpha d-2\phi\delta)$
A_{21}	$((2-\lambda_{AT})(\alpha d-2\phi\sigma)-\lambda_{AT}(\alpha d+\eta))/4$
A_{22},A_{23}	$0.5(\lambda_{AT}-2),\ 0.5\delta(4-2\lambda_{AT}-\rho)$
B_{21}	$d+d(\beta-1)\rho+\alpha d(\delta+\rho-0.5\delta\rho-1)+(\rho-2)\phi\sigma+0.5\delta(2\alpha d+\eta-2\phi\sigma)\lambda_{AT}$
B_{22}	$2((2-\rho)(\delta^2+\delta-1)+\delta^2\lambda_{AT})$
M_1,M_2,M_3	$A_{12}B_{12}-A_{13}^2,\ \ A_{13}B_{11}-A_{11}B_{12},\ \ A_{11}A_{13}-A_{12}B_{11}$
N_1,N_2,N_3	$A_{22}B_{22}-A_{23}^2,\ \ A_{23}B_{21}-A_{21}B_{22},\ \ A_{21}A_{23}-A_{22}B_{21}$

Table A.2
Related notations in propositions

Notation	Definition
a_{11}	$-(\delta-2)d(4\alpha(\delta-2)+(\beta-1)(-2+3\delta))+(\delta-1)2d(-1+2\alpha+\beta)(\delta-2)\lambda_{RE}$
b_{11}	$4(\delta^2-4)-(-2+\delta+\delta^2)(\lambda_{RT}+\lambda_{RE})$
c_{11}	$-2(\delta-2)^2(2+\delta)+(-2+\delta+\delta^2)(\delta-2)(\lambda_{RT}+\lambda_{RE})$
a_{12}	$\begin{pmatrix} (\delta^2-4)(4\beta(\delta-1)\delta-2\delta^2-d(\beta-1)(-2+\delta+2\delta^2))+d(\beta-1)\delta(4-4\delta-5\delta^2 \\ +4\delta^3)\lambda_{RT}+d(\beta-1)(8-12\delta-10\delta^2+9\delta^3)\lambda_{RE}+2d\alpha(\delta-2)^2(-1+\delta+2\delta^2) \\ (\lambda_{RT}-\lambda_{RE})-(2+\delta)(2\beta(\delta-1)^2\delta+\delta^2-\delta^3)(\lambda_{RT}+\lambda_{RE}) \end{pmatrix}$
b_{12}	$-(2+\delta)(2-3\delta-3\delta^2+2\delta^3)(\lambda_{RT}+\lambda_{RE})$
c_{12}	$4(\delta^2-4)(2-\delta-2\delta^2+\delta^3)-(2+\delta)(-4+6\delta+2\delta^2-6\delta^3+2\delta^4)(\lambda_{RT}+\lambda_{RE})$
a_{13}	$(4-2\delta+(\delta-1)\lambda_{RT}+(\delta-1)\lambda_{RE})(\delta+d\delta-\delta^2+\beta(2-(4+d)\delta+2\delta^2))$

b_{13}	$(4-2\delta+(\delta-1)\lambda_{RT}+(\delta-1)\lambda_{RE})(-2-\delta+\delta^2)$
a_{14}	$(\delta-2)d(-(\beta-1)(\delta-6)-4\alpha(\delta-2))+(\delta-1)2d(-1+2\alpha+\beta)(\delta-2)\lambda_{RT}$
b_{14}	$4(\delta^2-4)+(-2+\delta+\delta^2)(\lambda_{RT}+\lambda_{RE})$
c_{14}	$2(\delta-2)^2(2+\delta)-(-2+\delta+\delta^2)(\delta-2)(\lambda_{RT}+\lambda_{RE})$
a_{21}	$d(0.5\delta\rho((1-\beta)\rho-1)+\alpha(-2+\rho+\delta(2-0.5\rho-0.5\rho^2)+\delta^2(4-3\rho+0.5\rho^2)))$
b_{21}	$-4+4\delta+8\delta^2+2\rho-2\delta\rho-4\delta^2\rho+0.25\delta^2\rho^2+0.5(1-\delta-2\delta^2)(2-\rho)\lambda_{AT}$
c_{21}	$-4+4\delta+8\delta^2+2\rho-\delta\rho-3\delta^2\rho-0.5\delta\rho^2+(1-\delta-2\delta^2)(2-\rho)\lambda_{AT}$
a_{22}	$\begin{pmatrix} d(\alpha(-1+\delta+\delta^2)(\rho-2)+(2-0.5\rho)(\delta+\delta\rho(\beta-1)+\alpha\delta(-1+\delta+\rho-0.5\delta\rho)) \\ +(-0.5\alpha\delta^2\rho+\alpha(-2+\delta(3-2\rho)+\delta^2(2-0.5\rho)+\rho)+\delta(-1+\rho-\beta\rho))\lambda_{AT}) \end{pmatrix}$
b_{22}	$(\delta^2+0.5(1-\delta-\delta^2)(\rho-2)-0.25\delta^2\rho)\lambda_{AT}$
c_{22}	$(2\delta+2(1-\delta-\delta^2)-0.5\delta\rho)(\rho-2)+(2-2\delta^2-\rho+1.5\delta^2\rho)\lambda_{AT}$
a_{23}	$\begin{pmatrix} d+d\alpha\delta+d(\beta-1)\rho+d\alpha(-1+\delta+\rho-0.75\delta\rho)+(d(-0.5+0.5\rho-0.5\beta\rho \\ +\alpha(0.5-\delta-0.5\rho+0.5\delta\rho)))\lambda_{AT} \end{pmatrix}$
b_{23}	$0.125\delta\rho\lambda_{AT}$
c_{23}	$(-2\delta+(\rho-2)+0.5\delta\rho)+(1+\delta-0.5\rho-0.25\delta\rho)\lambda_{AT}$

Proof of Proposition 1: The Hessian matrix of the utility function U_{RM} in model (I) is

$$\nabla^2 U_{RM}(w_R, p_{RD}) = \frac{1}{(2-\delta)^2}\begin{pmatrix} A_{12} & A_{13} \\ A_{13} & B_{12} \end{pmatrix}.$$

It is easy to see that, when $\delta\in[0,0.5]$, $\lambda_{RT}\in[0,1)$, $\lambda_{RE}\in[0,1)$, the above matrix is negative semidefinite and hence the utility function U_{RM} is concave with respect to (w_R, p_{RD}). Since the constraints are all linear, model (I) is a convex optimization problem. Solving $\nabla_{(w_R, p_{RD})}U_{RM}=0$ yields

$$w_R^* = \frac{M_2}{M_1}, \quad p_{RD}^* = \frac{M_3}{M_1},$$

where M_1, M_2, M_3 are defined in Table A.1. In order to satisfy the constraints, it requires that $M_2\ge 0$, $M_3\ge 0$, and $(2+\delta)((1-\delta)M_2-\delta M_3)\le(\min\{2\alpha+\delta(1-\alpha-\beta), 2(1-\alpha-\beta)+\alpha\delta\}d+(2+\delta)\eta)M_1$. These conditions can be divided into two cases: $\alpha<1-\alpha-\beta$ and $\alpha>1-\alpha-\beta$. Denote by

$$Z_{11}=(2+\delta)((1-\delta)M_2-\delta M_3)-(2\alpha d+\delta(1-\alpha-\beta)d+(2+\delta)\eta)M_1,$$
$$Z_{12}=(2+\delta)((1-\delta)M_2-\delta M_3)-(2(1-\alpha-\beta)d+\alpha\delta d+(2+\delta)\eta)M_1.$$

Case 1: $\alpha<1-\alpha-\beta$. Let $Z_{11}(\eta,\varphi\sigma)=a_{11}+b_{11}\eta+c_{11}\varphi\sigma$, $M_2(\eta,\varphi\sigma)=a_{12}+b_{12}\eta+c_{12}\varphi\sigma$, and $M_3(\eta,\varphi\sigma)=a_{13}+b_{13}\varphi\sigma$, where $a_{11},b_{11},c_{11},a_{12},b_{12},c_{12},a_{13},b_{13}$ are defined in Table A.2. When $\delta\in[0,0.5)$, $\lambda_{RT}\in[0,1)$, $\lambda_{RE}\in[0,1)$, $\alpha\in[0,0.5]$, and $\beta\in[0,1-2\alpha]$, we have $c_{11},c_{12},b_{13},b_{11},b_{12}<0$ and hence $Z_{11}(\eta,\varphi\sigma)$, $M_2(\eta,\varphi\sigma)$, $M_3(\eta,\varphi\sigma)$ are decreasing in $\varphi\sigma>0$, while $Z_{11}(\eta,0)$, $M_2(\eta,0)$ are decreasing in η. Moreover, when $\eta\ge-\frac{a_{11}}{b_{11}}$, $\varphi\sigma>0$ or $\eta\le-\frac{a_{11}}{b_{11}}$, $\varphi\sigma\ge\frac{a_{11}+b_{11}\eta}{-c_{11}}$, we have $Z_{11}(\eta,\varphi\sigma)\le 0$; when $\eta\le-\frac{a_{12}}{b_{12}}$, $\varphi\sigma\in(0,\frac{a_{12}+b_{12}\eta}{-c_{12}}]$, we have $0\le M_2(\eta,\varphi\sigma)\le M_2(\eta,0)$; when $\varphi\sigma\in(0,-\frac{a_{13}}{b_{13}}]$, we have $0\le M_3(\eta,\varphi\sigma)\le M_3(\eta,0)$. Due to $(-\frac{a_{11}}{b_{11}})<(-\frac{a_{12}b_{13}-a_{13}c_{12}}{b_{12}b_{13}})<(-\frac{a_{12}}{b_{12}})$, we have $\eta\in[-\frac{a_{11}}{b_{11}},-\frac{a_{12}}{b_{12}}]$, $\varphi\sigma\in(0,-\frac{a_{12}+b_{12}\eta}{c_{12}}]$ or $\eta\le-\frac{a_{11}}{b_{11}}$, $\varphi\sigma\in[-\frac{a_{11}+b_{11}\eta}{c_{11}},-\frac{a_{13}}{b_{13}}]$. Therefore, under one of the following conditions:

(1) $\delta\in[0,0.5)$, $\lambda_{RT}\in[0,1)$, $\lambda_{RE}\in[0,1)$, $\alpha\in[0,0.5]$, $\beta\in[0,1-2\alpha]$, $\eta\in[\eta_{11},\eta_{12}]$, $\phi\sigma\in(0,\nu_{12}]$;
(2) $\delta\in[0,0.5)$, $\lambda_{RT}\in[0,1)$, $\lambda_{RE}\in[0,1)$, $\alpha\in[0,0.5]$, $\beta\in[0,1-2\alpha]$, $\eta\in[-\infty,\eta_{11}]$, $\phi\sigma\in[\nu_{11},\nu_{13}]$;

where $\eta_{11}=-\frac{a_{11}}{b_{11}}$, $\eta_{12}=-\frac{a_{12}}{b_{12}}$, $\nu_{11}=-\frac{a_{11}+b_{11}\eta}{c_{11}}$, $\nu_{12}=-\frac{a_{12}+b_{12}\eta}{c_{12}}$, $\nu_{13}=-\frac{a_{13}}{b_{13}}$, then (w_R^*, p_{RD}^*) is a global optimal solution of model (I).

Case 2: $\alpha>1-\alpha-\beta$. Let $Z_{12}(\eta,\varphi\sigma)=a_{14}+b_{14}\eta+c_{14}\varphi\sigma$, where a_{14},b_{14},c_{14} are defined in Table A.2. When $\delta\in[0,0.5)$,

$\lambda_{RT} \in [0,1)$, $\lambda_{RE} \in [0,1)$ together with $\alpha \in [0,0.5]$, $\beta \in [1-2\alpha,1]$ or $\alpha \in [0.5,1]$, $\beta \in [0,1-\alpha]$, we have $c_{12}, b_{13}, b_{12}, b_{14} < 0$ and $c_{14} > 0$. Therefore, $Z_{12}(\eta, \varphi\sigma)$ is increasing in $\varphi\sigma > 0$ and $Z_{12}(\eta, 0)$ is decreasing in η . Therefore, $M_2(\eta, \varphi\sigma)$ and $M_3(\eta, \varphi\sigma)$ have the same properties as Case 1. Moreover, when $\eta \geq -\frac{a_{14}}{b_{14}}$, $\varphi\sigma \in (0, -\frac{a_{14}+b_{14}\eta}{c_{14}}]$, we have $Z_{12}(\eta, \varphi\sigma) \leq 0$; when $\eta \leq -\frac{a_{12}}{b_{12}}$, $\varphi\sigma \in (0, \frac{a_{12}+b_{12}\eta}{-c_{12}}]$, we have $0 \leq M_2(\eta, \varphi\sigma) \leq M_2(\eta, 0)$; when $\varphi\sigma \in (0, -\frac{a_{13}}{b_{13}}]$, we have $0 \leq M_3(\eta, \varphi\sigma) \leq M_3(\eta, 0)$. Due to $(-\frac{a_{14}}{b_{14}}) < (-\frac{a_{14}b_{13}-a_{13}c_{14}}{b_{14}b_{13}}) < (-\frac{a_{14}c_{12}-a_{12}c_{14}}{b_{14}c_{12}-b_{12}c_{14}}) < (-\frac{a_{12}b_{13}-a_{12}c_{14}}{b_{12}b_{13}}) < (-\frac{a_{12}}{b_{12}})$, we have $\eta \in [-\frac{a_{14}}{b_{14}}, -\frac{a_{12}}{b_{12}}]$, $\varphi\sigma \in (0, \min\{-\frac{a_{12}+b_{12}\eta}{c_{12}}, -\frac{a_{13}}{b_{13}}, -\frac{a_{14}+b_{14}\eta}{c_{14}}\}]$. Therefore, under one of the following conditions:

(1) $\delta \in [0,0.5)$, $\lambda_{RT} \in [0,1)$, $\lambda_{RE} \in [0,1)$, $\eta \in [\eta_{14}, \eta_{12}]$, $\phi\sigma \in (0, \min\{\nu_{12}, \nu_{13}, \nu_{14}\}]$, $\alpha \in [0,0.5]$, $\beta \in [1-2\alpha,1]$;

(2) $\delta \in [0,0.5)$, $\lambda_{RT} \in [0,1)$, $\lambda_{RE} \in [0,1)$, $\eta \in [\eta_{14}, \eta_{12}]$, $\phi\sigma \in (0, \min\{\nu_{12}, \nu_{13}, \nu_{14}\}]$, $\alpha \in [0.5,1]$, $\beta \in [0,1-\alpha]$,

where $\eta_{14} = -\frac{a_{14}}{b_{14}}$, $\nu_{14} = -\frac{a_{14}+b_{14}\eta}{c_{14}}$, then (w_R^*, p_{RD}^*) is a global optimal solution of model (I).

Combining the above two cases, we can attain the optimal solution conditions for model (I). Substituting (w_R^*, p_{RD}^*) into $p_{RT}^*(w_R, p_{RD})$ and $p_{RE}^*(w_R, p_{RD})$, we can get p_{RT}^* and p_{RE}^* . This completes the proof.

Proof of Proposition 3: The Hessian matrix of the utility function U_{AM} in model (II) is

$$\nabla^2 U_{AM}(w_A, p_A) = \begin{pmatrix} A_{22} & A_{23} \\ A_{23} & B_{22} \end{pmatrix}.$$

It is easy to see that, when $\delta \in [0,0.5]$, $\lambda_{AT} \in [0,1)$, and $\rho \in [0,0.3]$, the matrix is negative semidefinite and hence the utility function U_{AM} is concave in (w_A, p_A) . Since the constraints are all linear, model (II) is a convex optimization problem. By solving $\nabla_{(w_A, p_A)} U_{AM} = 0$, we can get

$$w_A^* = \frac{N_2}{N_1}, \quad p_A^* = \frac{N_3}{N_1},$$

where N_1, N_2, N_3 are defined in Table A.1. In order to ensure the constraints, it requires that $N_2 \geq 0$, $N_3 \geq 0$, and $N_2 - \alpha d - \eta - 2\delta N_3 \leq 0$.

Let $Z_{21}(\eta, \varphi\sigma) = a_{21} + b_{21}\eta + c_{21}\varphi\sigma$, $N_2(\eta, \varphi\sigma) = a_{22} + b_{22}\eta + c_{22}\varphi\sigma$, $N_3(\eta, \varphi\sigma) = a_{23} + b_{23}\eta + c_{23}\varphi\sigma$, where $a_{21}, b_{21}, c_{21}, a_{22}, b_{22}, c_{22}, a_{23}, b_{23}, c_{23}$ are defined in Table A.2. When $\delta \in [0,0.5]$, $\alpha \in [0,1]$, $\beta \in [0,1-\alpha]$, $\lambda_{AT} \in [0,1)$, $\rho \in [0,0.3]$, we have $b_{21}, b_{22}, c_{21}, c_{22}, c_{23} < 0$ and $b_{23} > 0$. Therefore, $Z_{21}(\eta, \varphi\sigma)$, $N_2(\eta, \varphi\sigma)$, $N_3(\eta, \varphi\sigma)$ are decreasing in $\varphi\sigma > 0$ and $Z_{21}(\eta, 0)$, $N_2(\eta, 0)$ are decreasing in η , while $N_3(\eta, 0)$ is increasing in η . Moreover, when $\eta \geq -\frac{a_{21}}{b_{21}}$, $\varphi\sigma > 0$ or $\eta \leq -\frac{a_{21}}{b_{21}}$, $\varphi\sigma \geq -\frac{a_{21}+b_{21}\eta}{c_{21}}$, we have $Z_{21}(\eta, \varphi\sigma) \leq 0$; when $\eta \leq -\frac{a_{22}}{b_{22}}$, $\varphi\sigma \in (0, -\frac{a_{22}+b_{22}\eta}{c_{22}}]$, we have $0 \leq N_2(\eta, \varphi\sigma) \leq N_2(\eta, 0)$; when $\eta \geq -\frac{a_{23}}{b_{23}}$, $\varphi\sigma \in (0, -\frac{a_{23}+b_{23}\eta}{c_{23}}]$, we have $0 \leq N_3(\eta, \varphi\sigma) \leq N_3(\eta, 0)$. Due to $(-\frac{a_{23}}{b_{23}}) < (-\frac{b_{21}c_{23}-b_{23}c_{21}}{a_{21}c_{23}-a_{23}c_{21}}) < (-\frac{a_{21}}{b_{21}}) < (-\frac{a_{22}}{b_{22}})$, we have $\eta \in [-\frac{a_{21}}{b_{21}}, -\frac{a_{22}}{b_{22}}]$, $\varphi\sigma \in (0, -\frac{a_{22}+b_{22}\eta}{c_{22}}]$ or $\eta \in [-\frac{a_{23}}{b_{23}}, -\frac{b_{21}c_{23}-b_{23}c_{21}}{a_{21}c_{23}-a_{23}c_{21}}]$, $\varphi\sigma \in [-\frac{a_{21}+b_{21}\eta}{c_{21}}, -\frac{a_{23}+b_{23}\eta}{c_{23}}]$.

In summary, under one of the following conditions:

(1) $\delta \in [0,0.5]$, $\lambda_{AT} \in [0,1)$, $\rho \in [0,0.3]$, $\alpha \in [0,1]$, $\beta \in [0,1-\alpha]$, $\eta \in [\eta_{21}, \eta_{22}]$, $\phi\sigma \in (0, \nu_{22}]$;

(2) $\delta \in [0,0.5]$, $\lambda_{AT} \in [0,1)$, $\rho \in [0,0.3]$, $\alpha \in [0,1]$, $\beta \in [0,1-\alpha]$, $\eta \in [\eta_{23}, \eta_{24}]$, $\phi\sigma \in [\nu_{21}, \nu_{23}]$,

where $\eta_{21} = -\frac{a_{21}}{b_{21}}$, $\eta_{22} = -\frac{a_{22}}{b_{22}}$, $\eta_{23} = -\frac{a_{23}}{b_{23}}$, $\eta_{24} = -\frac{b_{21}c_{23}-b_{23}c_{21}}{a_{21}c_{23}-a_{23}c_{21}}$, $\nu_{21} = -\frac{a_{21}+b_{21}\eta}{c_{21}}$, $\nu_{22} = -\frac{a_{22}+b_{22}\eta}{c_{22}}$, $\nu_{23} = -\frac{a_{23}+b_{23}\eta}{c_{23}}$, then the global optimal solution of model (II) is (w_A^*, p_A^*) . Substituting (w_A^*, p_A^*) into $p_{AT}^*(w_A, p_A)$, we can get p_{AT}^* . And that completes the proof.