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Minimizing operating expenditures for a manufacturing system featuring quality reassurances, probabilistic failures, overtime, and outsourcing

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A B S T R A C T

Production management operating in recent competitive marketplaces must satisfy the client desired quality and shorter order lead-time and avoid internal fabricating disruption caused by inevitable defects and stochastic equipment failures. Achieving these operational tasks without undesirable quality goods, missing due dates, and fabrication interruption help the management minimize operating expenditures. Motivated by assisting manufacturing firms in the situations mentioned this study explores a manufacturing system that features quality reassurances through reworking or removal of defectives, correction of probabilistic failures, and partial overtime and outsourcing options for reducing uptime. This study finds the function of system operating expenditures through model building, mathematical formulations, optimization approaches, and algorithm proposition, shows its convexity, and derives the optimal batch time for the studied manufacturing model. Finally, this study offers numerical illustrations to confirm our work's applicability and disclose its capability to provide various profound crucial system information that helps the management make strategic operating decisions.

Notation:

t_{1Z} = fabrication uptime/runtime of this study,
T'_Z = batch cycle time,
λ = annual demands,
Q = batch size,
P_{1A} = annual fabrication rate with overtime implementation,
K_A = setup cost with overtime implementation,
C_A = fabrication unit cost with overtime implementation,
P_1 = standard fabrication rate without overtime implementation,
α_1 = the linking parameter between P_{1A} and P_1,
C = standard unit fabrication cost,
K = standard setup cost,
α_2 = the linking parameter between K_A and K,
π = the outsourcing percentage of a lot (where $0 < \pi < 1$),
K_π = setup cost with outsourcing option,
C_π = unit outsourcing cost,
β_1 = the linking variable between K_π and K,

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β_2 = the linking variable between C_π and C,
β = average annual Poisson-distributed equipment failure rate,
t = mean time between the Poisson-distributed failures,
M = equipment repair cost per failure occurrence,
t_r = time allowed to correct a failure,
t'_{2Z} = rework time in situation 1 of this study,
t'_{3Z} = products depleting time in situation 1 of this study,
x = Uniform-distributed nonconforming rate,
d_{1A} = nonconforming items' fabricating rate in t_{1Z}, where $d_{1A} = P_{1A}x$,
θ_1 = the scrap proportion of nonconforming items,
P_{2A} = annual rework rate with overtime implementation,
P_2 = standard reworking rate,
C_{RA} = unit rework cost with overtime implementation,
C_R = standard unit reworking cost,
α_3 = the linking parameter between C_A and C, and C_{RA} and C_R,
θ_2 = the scrap proportion of the reworked products,
d_{2A} = fabrication rate of scrap items during t'_{2Z}, where $d_{2A} = P_{2A}\theta_2$,
C_S = disposal cost per scrapped item,
φ = the overall scrap rate of the nonconforming items,
g = t_r,
h = unit holding cost,
h_1 = reworked item's unit holding cost,
h_3 = unit holding cost of safety stock,
C_1 = unit cost of safety stock,
C_T = unit shipping cost,
T_Z = cycle time in situation 2 of this study,
t_{2Z} = rework time in situation 2,
t_{3Z} = products depleting time in situation 2,
H_0 = stock level when a failure occurs,
H_1 = stock level when uptime ends,
H_2 = stock level when rework ends,
H = stock level after receipt of the outsourced items,
T = cycle time for a system without outsourcing, overtime, nor failure (namely, situation 3),
t_1 = uptime in situation 3,
t_2 = rework time in situation 3,
t_3 = stock depleting time in situation 3,
d_1 = fabrication rate of nonconforming items in situation 3,
d_2 = fabrication rate of scrap items during t_2,
$I(t)$ = stock level at time t,
$I_F(t)$ = safety stock's level at time t,
$I_d(t)$ = nonconforming stock level at time t,
$I_s(t)$ = scrapped stock level at time t,
$TC(t_{1Z})_1$ = total cost per cycle in situation 1,
$TC(t_{1Z})_2$ = total cost per cycle in situation 2,
$E[TC(t_{1Z})_1]$ = the expected total cost per cycle in situation 1,
$E[TC(t_{1Z})_2]$ = the expected total cost per cycle in situation 2,
$E[T'_Z]$ = the expected cycle length in situation 1,
$E[T_Z]$ = the expected cycle time in situation 2,
$\boldsymbol{T_Z}$ = replenishment cycle time of this studied problem,
$E[TCU(t_{1Z})]$ = the expected annual system cost for this studied problem.

1. Introduction

In recent competitive marketplaces, production management must simultaneously satisfy clients' desired quality and short order lead time and avoid internal fabricating disruption caused by inevitable random defects and stochastic equipment failures. Motivating by addressing these real situations in manufacturing planning, this work explores a system featuring quality reassurances through reworking or removal of defectives, correcting probabilistic failures, and partial overtime and outsourcing options to reduce uptime. Managers of manufacturing firms often cautiously monitor the internal fabricating issues such as the quality of production equipment and products. However, it is inevitable to face stochastic equipment breakdowns and random defective products. Rafiee et al. (2011) utilized a cost-minimization mathematical model to explore combined cell formation and stock batch-size problems featuring dynamic routing, capacity and cell-size constraints, operations sequences, equipment failures, and process deterioration. The researchers considered costs of the following:

equipment procurement, cell reconfiguration, material handling, equipment operating, subcontracting, stock holding, defective parts handling, and preventive and corrective repairs. As to multiproduct fabrication, the researchers considered planning multiple processes and each process plan's routing options. Hence, it became a combinatorial issue. Furthermore, considering unreliability conditions and equipment failures made their studied model more practical. The researchers developed an optimized meta-heuristic using the Particle Swarm approach to resolve the model's NP-completeness. Jaehn et al. (2014) explored a single-facility multiproduct sequential batch-size scheduling problem with buffered rework jobs for a real-world car painting case. Random defective items are gathered in a limited-capacity buffer waiting for a later rework by the same facility. There is an independent setup time before its operation for each finished product and a given due date. The aim was to minimize finished products' maximum lateness. The researchers applied a group technology heuristic to analyze this NP-hard problem and justified the scheduling results from their proposed approach using the actual car paint shop's example. Najafi et al. (2018) studied a multiproduct single-machine economic production quantity-based system featuring partial backlogging, scrap, and rework. A constrained nonlinear model was developed to solve the problem with GAMS modeling language and a nonlinear programming solver BARON. Their objectives were to keep the total fabrication- supply system minimum under the capacity and budget constraints, limited warehouse space, and required service level. Lastly, numerical real-world manufacturing examples helped them demonstrate the applicability of their proposed model. Year et al. (2021) examined a two-dimensional Markovian modulated Poisson process – MMPP for system failures featuring dependence of two sequential inter-failure times. The Marshall–Olkin exponential distribution is assumed, and an approximate Bayesian computation algorithm is used to simulate real datasets of consecutive breakdowns from public transportation firms. The researchers estimated system reliability regarding the expected number of failures and their occurring probabilities, time, and distances between failures. Other research (Youssef & ElMaraghy, 2008; Iqbal, 2020; Mabrouk, 2020; Chiu et al., 2021a,b; Dewi et al., 2021; Di Nardo et al., 2021; Patil et al., 2021; Rouhani et al., 2021; Gupta et al., 2022; Kahar et al., 2022; Kaviyarasu and Sivakumar, 2022) studied diverse characteristics of equipment failures' impact and their corresponding actions of product defects on the controlling and managing fabrication systems.

Meeting the trend of customers' shorter order due dates, managers of today's manufacturing firms must plan their batch fabrication with the minimum runtime. The practical and often used strategies to reduce fabricating uptime are partially subcontracting and overtime shifts in production. Lusa et al. (2008) believed the annualized working hours could help enterprises adjust their capacity to cope with the unstable demand during the year, consequently reducing the uncertain temporary workforce, inventory expenses, and overtime usage. In this study, the researchers effectively implemented the annualized working hours by breaking them into weekly hours that consider the firms' actual demand and cost data. They proposed two mathematical programming models approach by using adequately selected weekly hours set to build the annual working hours for each worker to meet the firms' service level. Additionally, the study validates that one of the models can serve as a decision-making tool via computational experiments. By spending relatively short computing time, it can derive the optimal solution. Moon (2010) investigated the efficiency, investing efforts, and timing when implementing a partial outsourcing policy under an uncertain market. The researcher explored the potential hidden expenses at the outsourcing preparation stage and evaluated its future profits after implementation using a net present value model focusing on the partial outsourcing option's efforts, timing, and efficiency. Then, the author compared his research results to the existing/traditional approach to indicate what prior studies had underestimated and misled. Lastly, the researcher presented a descriptive scheme to support the partial outsourcing decision-making. Ebrahim and Abdul Rasib (2017) studied the time loss measurement in the assembly lines focusing on the potential unnecessary overtime usage. The researchers considered that hidden time loss affects the productivity of semi-auto and manual assembly lines involving a high degree of product variety. Hence, they believed that when measuring such types of assembly processes' time lost, one should include the unnecessary overtime usage in assembly lines. Thorough literature survey on production operations and their performance measures, the researcher developed the unnecessary overtime structure. Using actual automotive firms' case studies, the researchers validated their proposed excessive overtime structure. They concluded that unnecessary overtime should be included in measuring their hidden time loss for semi-auto and manual assembly lines involving a high degree of product variety. Heydari et al. (2020) examined the effect of outsourcing (quantity flexibility) contracts in a two-echelon stochastic-demand supply chain with a retailer and a manufacturer member. Outsourcing production on the manufacturer's side is based on reducing risks of overstock and overproduction under the pre-warning of demand changes from the retailer. The decision variables of the studied problem, for the retailer, is the order quantity and for the manufacturer, is the outsource/in-house amount. The researchers offered numerical experiments using different contract parameters. The sensitivity analyses results indicate that implementing a partial outsourcing policy could increase the profit of the retailer, manufacturer, and the entire supply chain. Other studies (Reynard, 1998; Zhu, 2015; Ramasubbu et al., 2019; Dekker et al., 2020; Ishida et al., 2020; Ouaddi et al., 2020; Chiu et al., 2021a,b; Çimen et al., 2022; Sung et al., 2022; Waiyawuththanapoom and Jermsittiparsert, 2022) disclosed the impact of overtime and subcontracting's diverse strategies on planning and expediting the fabrication batch time. Few past works focused on minimizing operating expenditures for a manufacturing system with quality reassurances, probabilistic failures, overtime, and outsourcing. This work tries to fill the gap.

2. Problem description and modeling

This work minimizes operating expenditures for a manufacturing system featuring quality reassurances, probabilistic failures, overtime, and outsourcing. The following describes the problem and its relevant mathematical modeling. The proposed batch fabrication system with lot-size Q has to satisfy the annual requirement rate λ. A portion πQ is outsourced to an outside vendor to reduce uptime. To further cut down the fabrication runtime, the system implements an overtime option with a rate of P_{1A} to fabricate in-house the other $(1-\pi)Q$ portion of the lot. Associating with the following subcontracting unit cost and setup costs C_π and K_π:

$$K_\pi = K(1+\beta_1) \tag{1}$$

$$C_\pi = C(1+\beta_2) \tag{2}$$

Associating with the overtime strategy, the fabrication/output rate P_{1A} is α_1 more than the standard rate P_1 (as shown in Eq. (3)). Additionally, the following are relationships between standard costs of the overtime setup cost and unit cost K_A and C_A:

$$P_{1A} = P_1(1+\alpha_1)$$

$$K_A = K(1+\alpha_2)$$

$$C_A = C(1+\alpha_3)$$

The quality reassurance focuses on consequent actions on in-house random x proportion of nonconforming items produced. In each cycle, additional screening identifies and scraps the θ_1 portion (where $0 \le \theta_1 \le 1$) of nonconforming items, and the remaining $(1 - \theta_1)$ portion is reworked immediately at the end of uptime. The following relationships are the rework-relating rate and cost against their corresponding standard parameters:

$$P_{2A} = P_2(1+\alpha_1) \tag{6}$$

$$C_{RA} = C_R(1+\alpha_3) \tag{7}$$

Further screening identifies other θ_2 portion (where $0 \le \theta_2 \le 1$) of scraps from the rework process. Therefore, the overall scrap rate among nonconforming, $\varphi = (\theta_1 + \theta_2(1 - \theta_1))$. Meantime, external sources guarantee the subcontracting products' quality. Their schedule of receipt time is right before the starting time of product depleting time. Further, this study assumes the production machine is subject to a random failure that follows a Poisson distribution with β as the mean failure instances per year. A random equipment breakdown may happen in manufacturing uptime t_{1Z}, so we must examine the following separate situations explicitly:

2.1. *Situation 1: The production equipment fails during the uptime*

This study adopts an abort/resume (AR) stock controlling discipline upon a failure instance to perform an immediate correction. If the production equipment fails during the uptime, that means time to equipment failure $t < t_{1Z}$. After the equipment's restoration, we resume production of the interrupted/unfinished lot. Fig. 1 shows the stock level of the situation-one of our problems compared to the same system but without the uptime-reduction strategies (see thinner lines). Fig. 1 shows that when a failure happens, the stock rises H_0. It stays at H_0 during t_r, the correction time (where t_r is the maximum allowable machine repairing time). If actual repair time exceeds t_r, then a piece of rental/spare equipment is in position to avoid fabrication schedule delay. Upon the repair is completed, the stock accumulates to H_1 when t_{1Z} ends. Then, it upsurges to H_2 when the rework ends. It further climbs to H when receiving the subcontracting items. Finally, it gradually depletes to zero in t'_{3Z}. No shortages are permitted, so $(P_{1A} - d_{1A} - \lambda) > 0$ must hold.

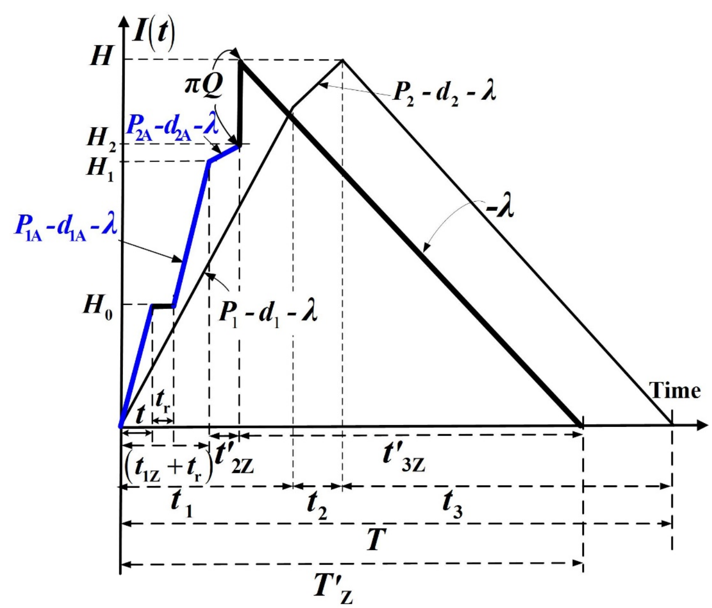

Fig. 1. The stock level of the situation-one of our problem compared to the same system but without the uptime-reduction strategies (in thinner lines)

Fig. 2 shows the situation 1's safety stock level. In t_r, the proposed study uses safety stocks to satisfy the demand.

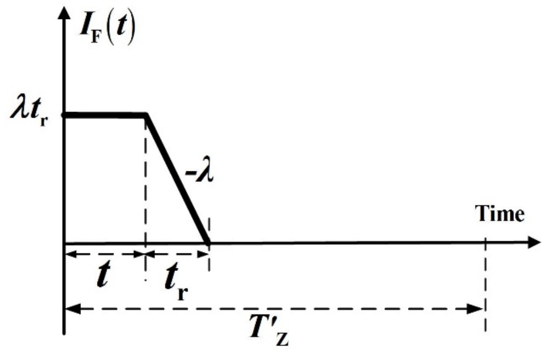

Fig. 2. Safety stock level in situation one

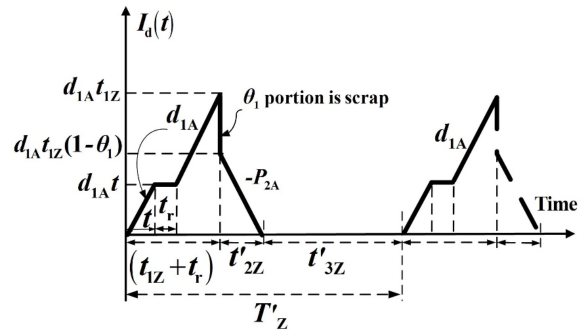

Fig. 3. The level of nonconforming products in situation one of the proposed problem

Fig. 3 depicts the nonconforming-product stocks and Fig. 4 displays scraps in situation one of the proposed problem, respectively.

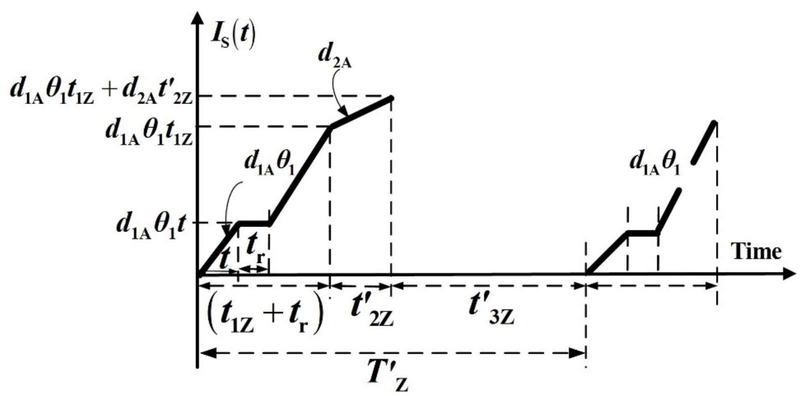

Fig. 4. The level of scraps in situation one

The following formulas can be gained accordingly (refer to Fig. 1 to Fig. 4):

$$H_0 = t\left(P_{1A} - d_{1A} - \lambda\right) \tag{8}$$

$$H_1 = t_{1Z}\left(P_{1A} - d_{1A} - \lambda\right) \tag{9}$$

$$H_2 = H_1 + t'_{2Z}\left(P_{2A} - d_{2A} - \lambda\right) \tag{10}$$

$$H = \pi Q + H_2 \tag{11}$$

$$t_{1Z} = \frac{Q(1-\pi)}{P_{1A}} = \frac{H_1}{P_{1A} - d_{1A} - \lambda} \tag{12}$$

$$t'_{2Z} = \frac{(1-\theta_1)x(1-\pi)Q}{P_{2A}} \tag{13}$$

$$t'_{3Z} = \frac{H}{\lambda} \tag{14}$$

$$T'_Z = t_{1Z} + t_r + t'_{2Z} + t'_{3Z} \tag{15}$$

Formulas (16) and (17) display the situation 1's nonconforming and scrapped products' levels:

$$d_{1A}t_{1Z} = P_{1A}t_{1Z}x = x(1-\pi)Q \tag{16}$$

$$\varphi\left[xQ(1-\pi)\right] = \left[\theta_1 + (1-\theta_1)\theta_2\right]xQ(1-\pi) \tag{17}$$

$TC(t_{1Z})_1$ consists of the following: the fixed and variable costs for outsourcing and in-house fabrication cost, safety stock relating cost (refer to Fig. 2), equipment repair cost, rework and disposal cost (see Fig. 3 and Fig. 4), and holding costs (including the reworked items, perfect and nonconforming products) during T'_Z as exhibited in Eq. (18).

$$TC(t_{1Z})_1 = K_\pi + C_\pi Q\pi + K_A + C_A Q(1-\pi) + C_1(\lambda t_r) + C_T(\lambda t_r) + \left(t + \frac{t_r}{2}\right) h_3(\lambda t_r)$$
$$+ M + C_{RA} Q(1-\pi) x (1-\theta_1) + C_S Q(1-\pi)\varphi x + h_1 \frac{P_{2A}}{2}(t'_{2Z})^2 \tag{18}$$
$$+ \left[\frac{H_1 + d_{1A} t_{1Z}}{2}(t_{1Z}) + \frac{H_1 + H_2}{2}(t'_{2Z}) + \frac{H}{2}(t'_{3Z}) + (H_0 t_r) + (d_{1A} t) t_r\right] h$$

Substitute Eq. (16) and Eqs. (1) to (7) in Eq. (18), we have $TC(t_{1Z})_1$ as follows:

$$TC(t_{1Z})_1 = K(1+\beta_1) + C\pi Q(1+\beta_2) + K(1+\alpha_2) + C(1+\alpha_3)(1-\pi)Q + C_1(\lambda t_r)$$
$$+ C_T(\lambda t_r) + M + \left(t + \frac{t_r}{2}\right)(\lambda t_r) h_3 + C_R x Q(1-\pi)(1-\theta_1)(1+\alpha_3)$$
$$+ C_S x Q(1-\pi)\varphi + \frac{\left[(1+\alpha_1)P_2\right]h_1}{2}(t'_{2Z})^2 \tag{19}$$
$$+ h\left[(t_{1Z})\frac{H_1 + x(1+\alpha_1)P_1 t_{1Z}}{2} + (t'_{2Z})\frac{H_1 + H_2}{2} + (t'_{3Z})\frac{H}{2} + x(1+\alpha_1)P_1(t)t_r + (H_0 t_r)\right]$$

2.2. Situation 2: No equipment failure during the uptime

No equipment failure during the uptime means time to equipment failure $t > t_{1Z}$. Fig. 5 shows situation two's inventory level. When t_{1Z} ends, the level surges to H_1, and when rework ends, it upsurges to H_2. Upon receiving the subcontracting goods, the level arrives at H. Finally, the inventory level gradually depletes down to zero during t_{3Z}, before the next replenishing cycle initiates. Fig. 6 displays the safety stock level in situation 2. Since there are no equipment failures, its status is unchanged throughout T_Z. The level of nonconforming and scrap products in situation 2 are the same as those shown in Figs. 3 and 4, excluding t_r. Similarly, we can observe the following straightforward formulas for situation 2:

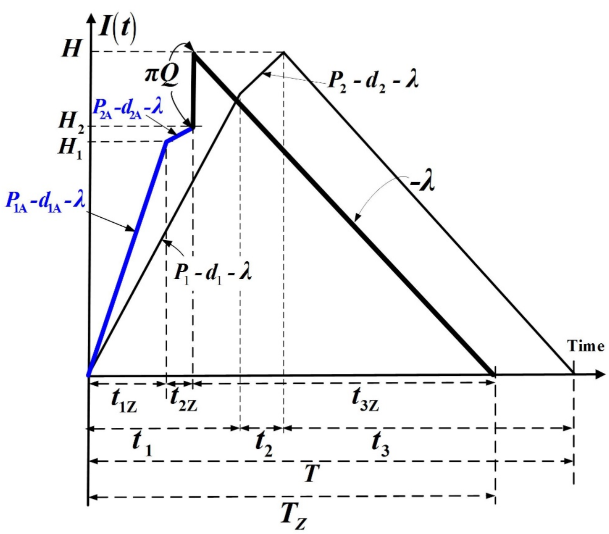

Fig. 5. The situation two's stock level compared to the same system with only the quality reassurances (in thinner lines)

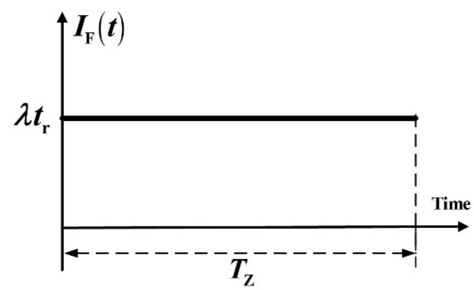

Fig. 6. The safety stock in situation two

$$H_1 = t_{1Z}(P_{1A} - d_{1A} - \lambda) \tag{20}$$

$$H_2 = H_1 + t_{2Z}(P_{2A} - d_{2A} - \lambda) \tag{21}$$

$$H = \pi Q + H_2 \tag{22}$$

$$t_{1Z} = \frac{Q(1-\pi)}{P_{1A}} = \frac{H_1}{P_{1A} - d_{1A} - \lambda} \tag{23}$$

$$t_{2Z} = \frac{(1-\theta_1)\left[xQ(1-\pi)\right]}{P_{2A}} \tag{24}$$

$$t_{3Z} = \frac{H}{\lambda} \tag{25}$$

$$T_Z = t_{1Z} + t_{2Z} + t_{3Z} \tag{26}$$

$TC(t_{1Z})_2$ consists of the following: variable and fixed subcontracting and in-house fabricating costs, safety stock relating cost (refer to Fig. 6), rework and disposal cost, and overall holding costs (containing perfect, reworked, and nonconforming items) in T_Z as displayed in Eq. (27).

$$TC\left(t_{1Z}\right)_2 = K_\pi + C_\pi\left(\pi Q\right) + K_A + C_A Q\left(1-\pi\right) + T_Z\left(\lambda t_r\right)h_3 + C_{RA}Qx\left(1-\pi\right)\left(1-\theta_1\right) + C_S Qx\left(1-\pi\right)\varphi + \frac{P_{2A}h_1}{2}\left(t_{2Z}\right)^2 + \left[\left(t_{1Z}\right)\frac{H_1 + d_{1A}t_{1Z}}{2} + \left(t_{2Z}\right)\frac{H_1 + H_2}{2} + \left(t_{3Z}\right)\frac{H}{2}\right]h \tag{27}$$

Substitute Eq. (1) to Eq. (7), and Eq. (16) in Eq. (27), we have $TC(t_{1Z})_2$ as follows:

$$\begin{aligned} TC\left(t_{1Z}\right)_2 &= \left(\pi Q\right)C\left(1+\beta_2\right) + K\left(1+\beta_1\right) + C\left(1-\pi\right)Q\left(1+\alpha_3\right) + K\left(1+\alpha_2\right) + \left(\lambda t_r\right)T_Z h_3 \\ &+ C_R x\left(1-\pi\right)Q\left(1-\theta_1\right)\left(1+\alpha_3\right) + C_S\varphi x\left(1-\pi\right)Q + \frac{P_2\left(1+\alpha_1\right)h_1}{2}\left(t_{2Z}\right)^2 \\ &+ h\left[\left(t_{1Z}\right)\frac{H_1 + xP_1t_{1Z}\left(1+\alpha_1\right)}{2} + \left(t_{2Z}\right)\frac{H_1+H_2}{2} + \left(t_{3Z}\right)\frac{H}{2}\right] \end{aligned} \tag{28}$$

2.3 Integration of situations one and two

The Poisson-distributed breakdown assumption makes the time to failures obey an Exponential distribution. Its functions of cumulative density and density are $F(t) = (1 - e^{-\beta t})$ and $f(t) = \beta e^{-\beta t}$. Because of random scrap assumption, the cycle length is variable. Applying the renewal reward theorem, one obtains the following $E[TCU(t_{1Z})]$:

$$E\left[TCU\left(t_{1Z}\right)\right] = \frac{\left\{\int_0^{t_{1Z}} E\left[TC\left(t_{1Z}\right)_1\right]f(t)\,dt + \int_{t_{1Z}}^{\infty} E\left[TC\left(t_{1Z}\right)_2\right]f(t)\,dt\right\}}{E[\boldsymbol{T}_Z]} \tag{29}$$

where $E[\boldsymbol{T}_Z]$, $E[T'_Z]$, and $E[T_Z]$ are the following:

$$E[T'_Z] = \frac{\lambda t_r + \left[1-\varphi E[x](1-\pi)\right]Q}{\lambda} = \frac{\lambda t_r + t_{1Z}\left[\frac{1}{(1-\pi)} - \varphi E[x]\right]P_{1A}}{\lambda} \tag{30}$$

$$E[T_Z] = \frac{\left[1-(1-\pi)\varphi E[x]\right]Q}{\lambda} = \frac{t_{1Z}\left[\frac{1}{(1-\pi)} - \varphi E[x]\right]P_{1A}}{\lambda} \tag{31}$$

$$E[\boldsymbol{T}_Z] = \int_{t_{1Z}}^{\infty} E[T_Z]f(t)\,dt + \int_0^{t_{1Z}} E[T'_Z]f(t)\,dt \tag{32}$$

We further apply $E[x]$ (the expected values) to formulas (19) and (28) to deal with random nonconforming rates. Substitute Eq. (30), Eq. (19), and Eq. (28) in Eq. (29), with further deriving efforts, $E[TCU(t_{1Z})]$ becomes (see Appendix A):

$$E\left[TCU\left(t_{1Z}\right)\right] = \frac{\lambda}{v_0 + \frac{\left(1-e^{-\beta t_{1Z}}\right)\lambda g}{\left(1+\alpha_1\right)t_{1Z}P_1}}\left\{\begin{matrix}\frac{G_0}{t_{1Z}}\left(1-e^{-\beta t_{1Z}}\right) - G_1\left(e^{-\beta t_{1Z}}\right) + \frac{G_2}{t_{1Z}}\left(1-e^{-\beta t_{1Z}}\right) + \\ G_3\left(1-e^{-\beta t_{1Z}}\right) + \frac{W_1}{t_{1Z}} + W_2 + W_3\left(t_{1Z}\right)\end{matrix}\right\} \tag{33}$$

2.4. Solution processes

Applying $E[TCU(t_{1Z})]$'s 1st and 2nd derivatives (see formulas (A-5) and (A-6) in Appendix A) and demonstrating $E[TCU(t_{1Z})]$'s convexity, if $\delta(t_{1Z}) > t_{1Z} > 0$ holds (refer to Eq. (A-7)). After confirming Eq. (A-7) holds, we can resolve $t_{1Z}*$ through letting the 1st-derivative of $E[TCU(t_{1Z})] = 0$ (see Eq. (A-5)). Because the 1st term of Eq. (A-5) RHS is positive, so we have:

$$\left\{\begin{aligned} &\left[W_3\left[P_1v_0\left(1+\alpha_1\right) - \lambda g\beta e^{-\beta t_{1Z}}\right] + \left(G_1+G_3\right)\left(\beta P_1v_0\left(1+\alpha_1\right)e^{-\beta t_{1Z}}\right)\right]\left(t_{1Z}\right)^2 \\ &+\left[\begin{aligned}&-W_2g\lambda\left(\beta e^{-\beta t_{1Z}}\right) + W_3\left[2g\lambda\left(1-e^{-\beta t_{1Z}}\right)\right] \\ &+\left(G_2+G_0\right)P_1v_0\left(1+\alpha_1\right)\left(\beta e^{-\beta t_{1Z}}\right) + \left(\lambda e^{-\beta t_{1Z}}g\beta\right)G_1\end{aligned}\right]\left(t_{1Z}\right) \\ &+\left[\begin{aligned}&-\left[v_0P_1\left(1+\alpha_1\right) + \lambda\beta ge^{-\beta t_{1Z}}\right]W_1 + \left(G_0+G_2\right)P_1\left(1+\alpha_1\right)v_0\left(e^{-\beta t_{1Z}}-1\right) \\ &+gG_3\lambda + \left(G_1+G_3\right)g\lambda e^{-2\beta t_{1Z}} - gW_2\lambda\left(e^{-\beta t_{1Z}}-1\right) - \left(g\lambda e^{-\beta t_{1Z}}\right)\left(2G_3+G_1\right)\end{aligned}\right]\end{aligned}\right\} = 0 \tag{34}$$

Let w_2, w_1, and w_0 be:

$$w_0 = \begin{bmatrix} -\left[v_0 P_1 (1+\alpha_1) + \lambda\beta g e^{-\beta t_{1Z}}\right] W_1 + (G_0 + G_2) P_1 (1+\alpha_1) v_0 \left(e^{-\beta t_{1Z}} - 1\right) \\ +gG_3\lambda + (G_1 + G_3) g\lambda e^{-2\beta t_{1Z}} - gW_2\lambda\left(e^{-\beta t_{1Z}} - 1\right) - \left(g\lambda e^{-\beta t_{1Z}}\right)(2G_3 + G_1) \end{bmatrix}$$

$$w_1 = -W_2 g\lambda\left(\beta e^{-\beta t_{1Z}}\right) + W_3\left[2g\lambda\left(1 - e^{-\beta t_{1Z}}\right)\right] + (G_2 + G_0) P_1 v_0 (1+\alpha_1)\left(\beta e^{-\beta t_{1Z}}\right) + \left(\lambda e^{-\beta t_{1Z}} g\beta\right) G_1$$

$$w_2 = \left[W_3\left[P_1 v_0 (1+\alpha_1) - \lambda g\beta e^{-\beta t_{1Z}}\right] + (G_1 + G_3)\left(\beta P_1 v_0 (1+\alpha_1) e^{-\beta t_{1Z}}\right)\right]$$

We rearrange Eq. (34) as follows:

$$w_2 (t_{1Z})^2 + w_1 (t_{1Z}) + w_0 = 0 \tag{35}$$

Using the square-roots approach, t_{1Z}* becomes:

$$t_{1Z}{}^* = \frac{-w_1 \pm \sqrt{w_1^2 - 4w_2 w_0}}{2w_2} \tag{36}$$

Since the range of $F(t_{1Z}) = (1 - e^{-\beta t1Z})$ is [0, 1], so does $e^{-\beta t1Z}$. By rearranging Eq. (34), we have $e^{-\beta t1Z}$ as follows:

$$e^{-\beta t_{1Z}} = \frac{-\begin{bmatrix} W_3 P_1 v_0 (1+\alpha_1)(t_{1Z})^2 + 2W_3 g\lambda(t_{1Z}) - W_1 P_1 v_0 (1+\alpha_1) + W_2 g\lambda \\ +gG_3\lambda - P_1 v_0 (1+\alpha_1)(G_2 + G_0) + g\lambda e^{-2\beta t_{1Z}} (G_3 + G_1) \end{bmatrix}}{\begin{Bmatrix} P_1 v_0 (1+\alpha_1)(G_1 + G_3)(t_{1Z})^2 \beta - W_2 g\beta\lambda(t_{1Z}) - W_3 g\beta\lambda(t_{1Z})^2 \\ +P_1 v_0 (1+\alpha_1)\beta(G_2 + G_0)(t_{1Z}) - 2W_3 g\lambda(t_{1Z}) + G_1(g\beta\lambda)(t_{1Z}) \\ -(\lambda)(G_1 + 2G_3) g - W_1\beta\lambda g - W_2 g\lambda + P_1 v_0 (1+\alpha_1)(G_2 + G_0) \end{Bmatrix}} \tag{37}$$

First, let $e^{-\beta t1Z} = 0$ and $e^{-\beta t1Z} = 1$, calculate formula (36) and gain t_{1Z}'s upper and lower bounds (that is, t_{1ZU} and t_{1ZL}). Then, use the present values of t_{1ZU} and t_{1ZL} to re-compute $e^{-\beta t1ZU}$ and $e^{-\beta t1ZL}$. Use them to recalculate formula (36) and obtain a new set of t_{1ZU} and t_{1ZL}. If ($t_{1ZU} = t_{1ZL}$) is true, we find t_{1Z}* (i.e., $t_{1ZU} = t_{1ZL} = t_{1Z}$*), otherwise, repeat the above-mentioned procedures, until ($t_{1ZU} = t_{1ZL}$) is true.

3. Numerical illustration

This section uses a simulated example to illustrate how our model and result works. First, the assumption of variables' values is displayed in Table 1.

Table 1
Assumptions of variables' values in our numerical illustration

C	K	β_2	λ	α_1	C_1	C_R	C_S	h_1	M	P_2	g
\$2	\$200	0.5	4000	0.5	\$2	\$1	\$0.1	\$0.4	\$2500	5000	0.018
π	β_1	x	θ_1	θ_2	h_3	φ	h	β	P_1	α_2	α_3
0.4	-0.70	20%	0.3	0.3	\$0.4	0.51	\$0.4	1	10000	0.1	0.1

Secondly, we verify $E[TCU(t_{1Z})]$'s convexity, that is $\delta(t_{1Z}) > t_{1Z} > 0$ must hold (see Eq. (A-7) in Appendix A). Knowing that $e^{-\beta t1Z}$ falls within [0, 1], we start with assuming $e^{-\beta t1Z} = 0$ and $e^{-\beta t1Z} = 1$. By computing Eq. (36) we find $t_{1ZL} = 0.0686$ and $t_{1ZU} = 0.3554$. Then, apply Eq. (A-7) using $e^{-\beta t1ZL}$ and $e^{-\beta t1ZU}$ we confirm that $\delta(t_{1ZL}) = 0.1821 > t_{1ZL} > 0$ and $\delta(t_{1ZU}) = 0.4929 > t_{1ZU} > 0$, respectively. Therefore, we found that for $\beta = 1$, $E[TCU(t_{1Z})]$ is convex, and the optimal t_{1Z}* value exists. Table 2 exhibits our model's applicability by showing $E[TCU(t_{1Z})]$'s convexity using a more comprehensive range of β values.

Table 2
Verifying the convexity of $E[TCU(t_{1Z})]$ using different βs

β	$\delta(t_{1ZU})$	t_{1ZU}	$\delta(t_{1ZL})$	t_{1ZL}
12	1.9037	0.3491	0.0202	0.0097
9	1.0491	0.3493	0.0265	0.0128
6	0.6454	0.3497	0.0390	0.0187
4	0.5104	0.3503	0.0571	0.0269
3	0.4727	0.3509	0.0741	0.0342
2	0.4580	0.3520	0.1054	0.0464
1	**0.4929**	**0.3554**	**0.1821**	**0.0686**
0.5	0.5934	0.3622	0.3009	0.0862
0.01	2.8999	0.7792	2.2233	0.1095

By applying sub-section 2.4's solution procedure for seeking the optimal t_{1Z}*, Table 3 depicts the iterative steps, and we find t_{1Z}* = 0.1149 and $E[TCU(t_{1Z}*)]$ = \$11,807.

Table 3
The iterative steps for seeking t_{1Z}*

Step	t_{1ZU}	$e^{-\beta t_{1ZU}}$	t_{1ZL}	$e^{-\beta t_{1ZL}}$	t_{1ZU} - t_{1ZL}	$E[TCU(t_{1ZU})]$	$E[TCU(t_{1ZL})]$
-	-	0	-	1	-	-	-
1	**0.3554**	0.7009	**0.0686**	0.9337	0.2868	$12376.84	$11915.88
2	0.1795	0.8357	0.0981	0.9065	0.0814	$11887.72	$11816.57
3	0.1354	0.8734	0.1091	0.8967	0.0263	$11817.33	$11807.62
4	0.1217	0.8854	0.1129	0.8932	0.0088	$11807.86	$11806.65
5	0.1172	0.8894	0.1142	0.8920	0.0030	$11806.68	$11806.54
6	0.1157	0.8907	0.1147	0.8916	0.0010	$11806.54	$11806.53
7	0.1152	0.8912	0.1149	0.8915	0.0003	$11806.53	$11806.52
8	0.1150	0.8913	0.1149	0.8914	0.0001	$11806.52	$11806.52
9	**0.1149**	0.8914	**0.1149**	0.8914	**0.0000**	**$11806.52**	**$11806.52**

3.1. The effect of uptime-reduction strategies

This study implements dual uptime-reduction strategies. We explicitly explore their impact on the proposed model and demonstrate the following outcomes: Fig. 7 exhibits utilization behavior concerning changes in π. As π increases, utilization knowingly declines. For outsourcing factor π = 0.4 (as our example assumes), utilization declines a 41.15% to 0.1876.

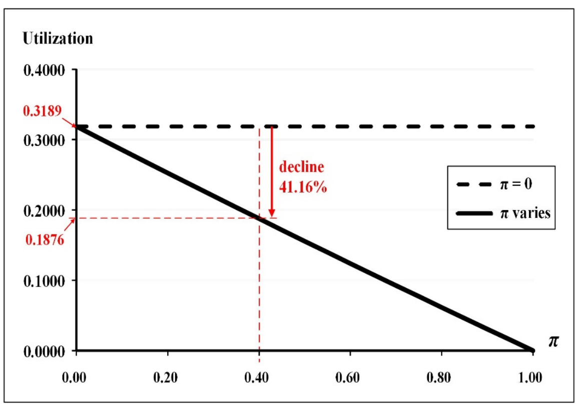

Fig. 7. Utilization behavior concerning changes in π

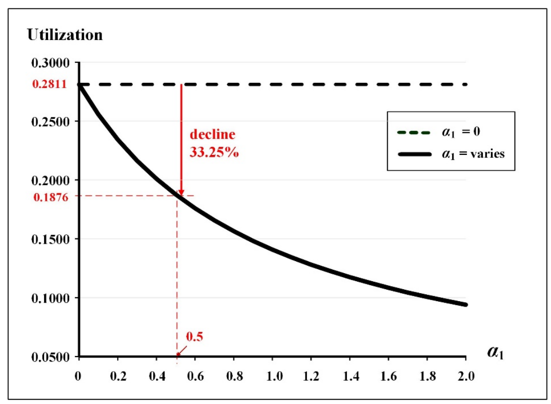

Fig. 8. The effect of variations in α_1 on utilization

Fig. 8 exhibits the effect of variations in overtime factor α_1 on utilization. As α_1 surges, the machine utilization greatly decreases. For α_1 = 0.5 (as our example assumes), utilization drops 33.25%, from 0.2811 (without overtime option) to 0.1876. Implementing dual uptime-reduction strategies (namely, the options of overtime and outsourcing) creates significant utilization decline. Fig. 9 displays the outcome of a further analysis comparing our utilization with closely-relating models. In addition, our model's utilization drops 60.7% compared to a model without using uptime-reduction strategies (Chiu et al., 2020). For a 33.25%, 41.16%, and 60.7% utilization decline, we are paying the prices of a 3.83%, 7.58%, and 14.91% increase in $E[TCU(t_{1Z}*)]$, respectively. Specifically, $E[TCU(t_{1Z}*)]$ surges to \$11,807 from \$11,371, \$10,975, and \$10,275, respectively.

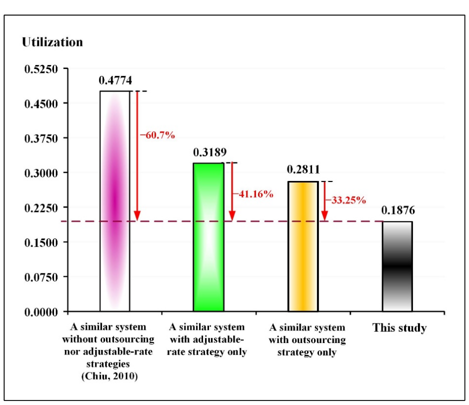

Fig. 9. Utilization comparison with other studies

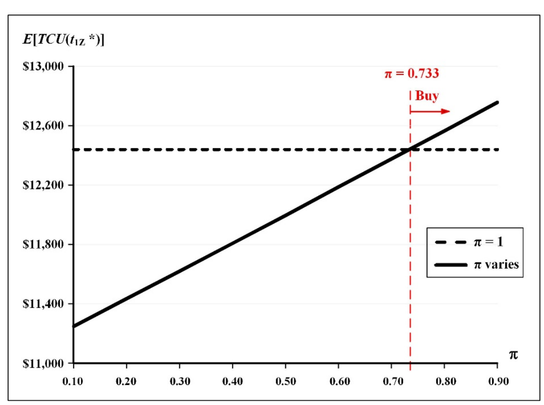

Fig. 10. The critical outsourcing factor π on 'make-or-buy' decision-making

3.2. The critical values for the make-or-buy decision-making

Fig. 10 discloses the critical outsourcing π factor (i.e., 0.733) for the make-or-buy choice. It specifies that as π rises up and over 0.733, it will be beneficial to select a '100% buy' decision. Analysis of the critical value for outsourcing cost-added β_2 factor is performed and illustrated in Figure 11. It reveals the critical outsourcing cost-added β_2 factor (i.e., 0.2476) on 'pure make' decision-making. As β_2 surges to over 24.76%, selecting a 'make' decision is more economical.

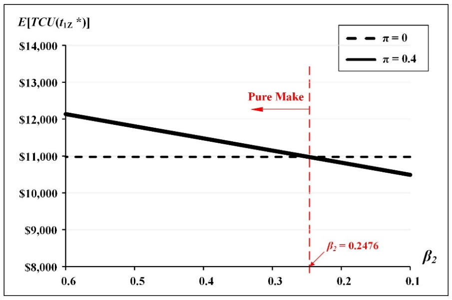

Fig. 11. The critical β_2 value for selecting a 'make' decision

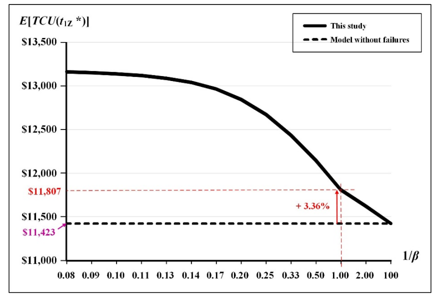

Fig. 12. Impact of $1/\beta$ on $E[TCU(t_{1Z}*)]$

3.3. The impact of random equipment failures and product quality issues

Analytical outcomes of the impact of random equipment failures on $E[TCU(t_{1Z}*)]$ is shown in Fig. 12. It shows as $1/\beta$ (i.e., the mean-time-to-failure factor) rises, $E[TCU(t_{1Z}*)]$ drops. Especially $E[TCU(t_{1Z}*)]$ decreases significantly when $1/\beta$ surges beyond 0.20. The investigation also reveals a 3.36% increase in $E[TCU(t_{1Z}*)]$ due to the random failures. Fig. 13 exposes the collective effect of π and φ on $t_{1Z}*$. It discovers that $t_{1Z}*$ substantially drops as π surges, and $t_{1Z}*$ marginally rises as φ increases. The joint effect of $1/\beta$ and φ on $E[TCU(t_{1Z}*)]$ is exposed in Fig. 14. It exhibits that $E[TCU(t_{1Z}*)]$ noticeably drops as $1/\beta$ surges, and $E[TCU(t_{1Z}*)]$ increases as φ surges. Fig. 15 exhibits joint influence of α_1 and $1/\beta$ on $t_{1Z}*$. It shows $t_{1Z}*$ noticeably declines as α_1 rises, and as $1/\beta$ increases, $t_{1Z}*$ knowingly drops, especially when $\alpha_1 \leq 1$ and $1/\beta \geq 0.2$.

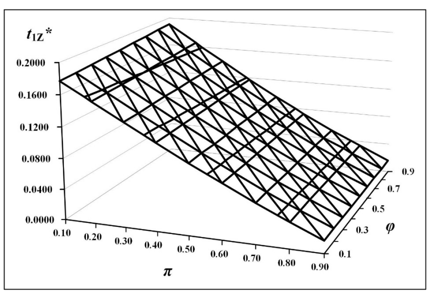

Fig. 13. The collective effect of π and φ on $t_{1Z}*$

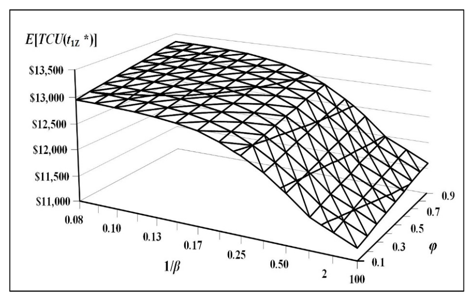

Fig. 14. $E[TCU(t_{1Z}*)]$'s performance concerning $1/\beta$ and φ

3.4. Additional analytical outcomes from system's parameters/feature

This study can explore additional analytical results from various system parameters/features. For instance, Figure 16 illustrates the collective effect of variations in unit overtime cost-added factor α_3 and outsourcing cost-added factor β_2 on $E[TCU(t_{1Z}*)]$. It discloses $E[TCU(t_{1Z}*)]$ considerably upsurges as both α_3 and β_2 rise. This example shows that β_2 has more influence on $E[TCU(t_{1Z}*)]$ than that of α_3.

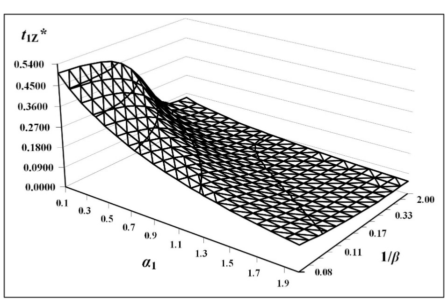

Fig. 15. The joint impact of α_1 and $1/\beta$ on $t_{1Z}*$

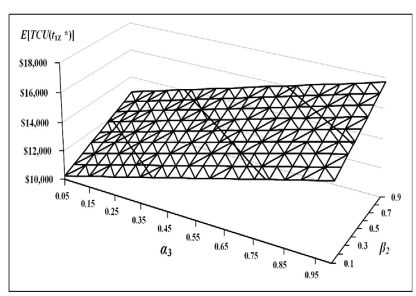

Fig. 16. The collective effect of variations in α_3 and β_2 on $E[TCU(t_{1Z}*)]$

Fig. 17 displays the joint influence of π and α_1 on utilization. It reveals that utilization considerably declines as both π and α_1 increase. This example shows that π has more effect on utilization's decline than α_1.

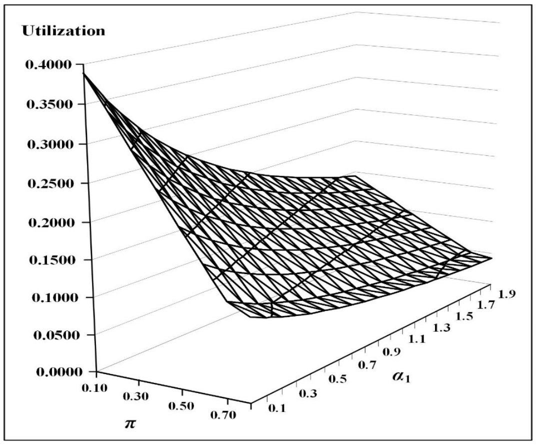

Fig. 17. The combined influence of π and α_1 on utilization

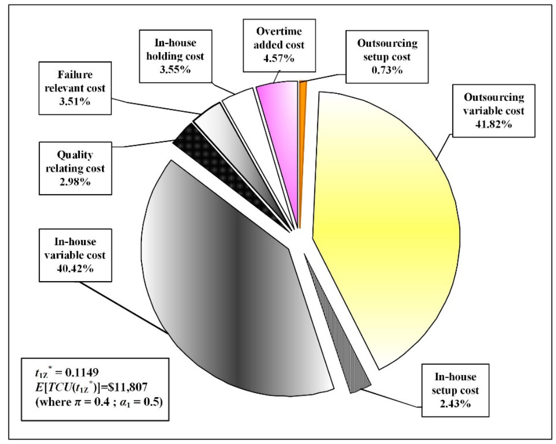

Fig. 18. $E[TCU(t_{1Z}*)]$'s cost contributors

Furthermore, this study can offer details of cost contributors of $E[TCU(t_{1Z}*)]$ as displayed in Fig. 18. It specifies that two main cost-contributors are the outsourcing and in-house variable costs, each contributes 41.82% and 40.42%. This example shows random machine failures and product-quality relevant costs each takes 3.51% and 2.98%. Moreover, Fig. 19 reveals crucial managerial information to facilitate an effective and economic uptime- or utilization-reduction strategy. This example suggests the following steps to effectively and economically reduce uptime/utilization: (i) starts with letting π = 0 and increasing α_1 (refer to the bold dash-line); (ii) once utilization decreases to 0.2638 (i.e., at π = 0 and α_1 increased to 0.815), switches π = 0.436 and resets α_1 = 0; then, keeps π = 0.436 and begins to upsurge α_1 (refer to the bold solid line).

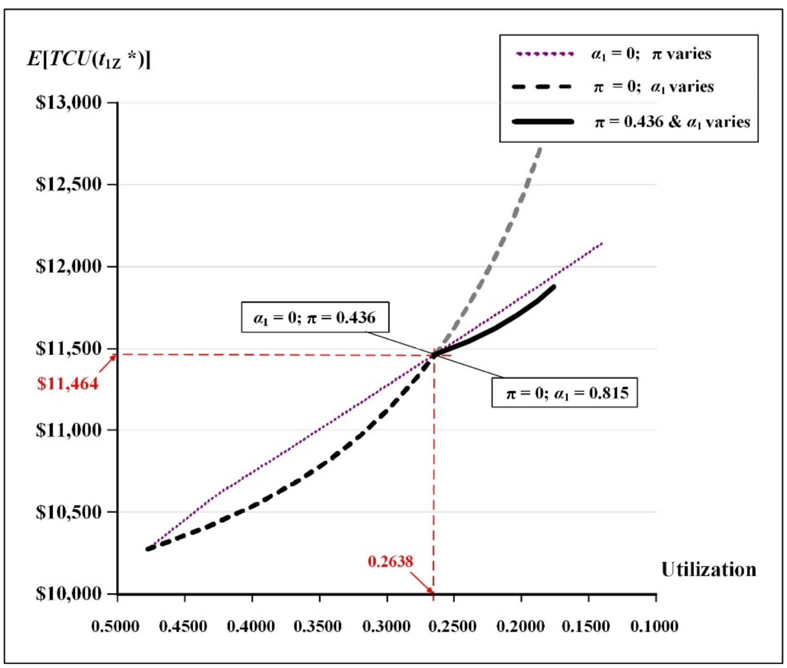

Fig. 19. The crucial managerial information to facilitate an effective and economic uptime- or utilization-reduction strategy

3.5. Discussion & limitation

We present a batch replenishing uptime model in line with the cases of no or just one equipment failure happening in a replenishing cycle. Table B-1 (in Appendix B) presents the probabilities of various Poisson-distributed failure rates. In addition, it indicates that for a "good" condition machine (or has an annual mean failure rate of less than one), our study is appropriate for exploring the specific problem, for it has over 99.39% chance of no or only one failure occurrence (see Table B-1). Moreover, for "fair" condition equipment (or has an annual mean failure rate of fewer than four breakdowns), our study is suitable for it has a 93.30% chance of no or only one failure happening (Table B-1). However, if the fabrication machine is in "worse" case (or with over four random breakdowns per year), our model's suitability will decrease to less than 85.50%. Hence, we suggest that production practitioners need to construct a multi-failure model for this specific situation.

4. Conclusions

This work studies a manufacturing system that features quality reassurances through reworking or removal of defectives, correcting probabilistic failures, and partial overtime and outsourcing options to reduce uptime. The aim is to help the management of manufacturing firms minimize operating expenditures by satisfying the client's desired quality and shorter order due dates and avoiding internal production disruption due to inevitable random defects and production equipment failures. We use the techniques of problem modeling and formulations, mathematical analyses, optimization with differential equations, and proposing a searching algorithm to carefully investigate the model, gain the function of operating expenditures, show convexity, and derive the best batch runtime decision (see Section 2). To end with our work, we offer numerical illustrations to confirm our work's applicability and disclose its capability to provide various profound crucial system information (as follows) that helps the management make strategic operating decisions (refer to Section 3):

(1) Verifying our work's applicability (Table 2) and confirming the convexity of operating expenditures function (see Table 3);
(2) The effect of uptime-reduction strategies including overtime and outsourcing on the utilization (Fig. 7 and Fig. 8);
(3) Comparison of utilization from uptime-reduction strategies with other existing studies (Fig. 9);
(4) The impact of critical outsourcing and cost-added factors π and β_2 on relevant make-or-buy decision making (Fig. 10 and Fig. 11).
(5) The influence and combined effect of factors of stochastic equipment failures, product quality, overtime, and outsourcing on system operating expenditures and the optimal runtime (Fig. 12 to Fig. 15);
(6) Additional analytical outcomes from system's parameters/features (e.g., the collective effect of variations in α_3 and β_2 on operating expenditures (Fig. 16); or the combined influence of π and α_1 on utilization (Fig. 17); or the significant contributors to the operating expenses (Fig. 18);
(7) The crucial managerial information/insight to help make an economical and effective uptime-reduction plan (Fig. 19).

Examining the influence of random annual demand on the studied problem is worth studying in the future.

Appendix – A

Details of obtaining $E[TCU(t_{1Z})]$ (Eq. (33)) and its convexity. By applying the expected values $E[x]$ to formulas (19) and (28) to deal with random nonconforming rates. Substitute formulas (19), (28), and (30) in Eq. (29), with further derivations, we gain the following $E[TCU(t_{1Z})]$:

$$E\left[TCU\left(t_{1Z}\right)\right]=\frac{\left\{\int_0^{t_{1Z}} E\left[TC\left(t_{1Z}\right)_1\right]\cdot f(t)\,dt+\int_{t_{1Z}}^{\infty} E\left[TC\left(t_{1Z}\right)_2\right]\cdot f(t)\,dt\right\}}{E[\boldsymbol{T}_Z]}$$

$$=\left[\frac{\lambda}{v_0+\dfrac{\lambda g\left(1-e^{-\beta t_{1Z}}\right)}{(t_{1Z})(1+\alpha_1)P_1}}\right]\left[\begin{array}{l}\dfrac{(1+\beta_1)K}{t_{1Z}(1+\alpha_1)P_1}+\dfrac{(1+\alpha_2)K}{t_{1Z}(1+\alpha_1)P_1}\\ +\left[(1+\beta_2)C\dfrac{\pi}{1-\pi}+(1+\alpha_3)C+(1+\alpha_3)C_RE[x](1-\theta_1)+C_SE[x]\varphi\right]\\ +(t_{1Z})\left\{\begin{array}{l}\dfrac{E[x]^2(1+\alpha_1)P_1(1-\theta_1)}{2(1+\alpha_1)P_2}\left[h_1(1-\theta_1)-h\right]+\dfrac{h}{2\lambda}\dfrac{(1+\alpha_1)P_1}{(1-\pi)^2}\cdot\\ \left\{\begin{array}{l}\dfrac{\lambda E[x](1-\pi)(1-\theta_1)}{(1+\alpha_1)P_2}\left[E[x]\varphi(1-\pi)-2\pi\right]\\ -\dfrac{\lambda(1-\pi)}{(1+\alpha_1)P_1}\left[(1+\pi)-2E[x]\varphi(1-\pi)\right]+\left[1-E[x]\varphi(1-\pi)\right]^2\end{array}\right\}\end{array}\right\}\\ +\left[M+C_1\lambda g+C_T\lambda g+h_3\left(\dfrac{\lambda g^2}{2}\right)\right]\left(\dfrac{1-e^{-\beta t_{1Z}}}{t_{1Z}(1+\alpha_1)P_1}\right)\\ +\dfrac{\left[hg\left((1+\alpha_1)P_1-\lambda\right)+h_3\lambda g\right]}{t_{1Z}(1+\alpha_1)P_1}\left(-t_{1Z}e^{-\beta t_{1Z}}-\dfrac{1}{\beta}e^{-\beta t_{1Z}}+\dfrac{1}{\beta}\right)\\ +\dfrac{h_3g}{1-\pi}\left[1-E[x]\varphi(1-\pi)\right]\left(1-e^{-\beta t_{1Z}}\right)\end{array}\right] \quad \text{(A-1)}$$

where v_0 represents:

$$v_0=\left[\frac{1}{(1-\pi)}-E[x]\varphi\right] \quad \text{(A-2)}$$

Suppose we let G_0, G_1, G_2, G_3, W_1, W_2, and W_3 stand for the following:

$$G_0 = \frac{M + C_1\lambda g + C_T\lambda g + h_3\left(\frac{\lambda g^2}{2}\right)}{(1+\alpha_1)P_1};\quad G_1 = \frac{\left[hg\left((1+\alpha_1)P_1 - \lambda\right) + h_3\lambda g\right]}{(1+\alpha_1)P_1}$$
$$G_2 = \frac{\left[hg\left((1+\alpha_1)P_1 - \lambda\right) + h_3\lambda g\right]}{(1+\alpha_1)P_1\beta};\quad G_3 = \frac{h_3 g}{1-\pi}\left[1 - E[x]\varphi(1-\pi)\right]. \tag{A-3}$$

$$\begin{aligned}
W_1 &= \frac{(1+\beta_1)K}{(1+\alpha_1)P_1} + \frac{(1+\alpha_2)K}{(1+\alpha_1)P_1}\\
W_2 &= \left[(1+\beta_2)C\frac{\pi}{1-\pi} + (1+\alpha_3)C + (1+\alpha_3)C_R E[x](1-\theta_1) + C_S E[x]\varphi\right]\\
W_3 &= \frac{E[x]^2(1+\alpha_1)P_1(1-\theta_1)}{2(1+\alpha_1)P_2}\left[h_1(1-\theta_1) - h\right]\\
&\quad + \frac{h}{2\lambda}\frac{(1+\alpha_1)P_1}{(1-\pi)^2}\begin{Bmatrix}\frac{\lambda E[x](1-\pi)(1-\theta_1)}{(1+\alpha_1)P_2}\left[E[x]\varphi(1-\pi) - 2\pi\right]\\ -\frac{\lambda(1-\pi)}{(1+\alpha_1)P_1}\left[(1+\pi) - 2E[x]\varphi(1-\pi)\right] + \left[1 - E[x]\varphi(1-\pi)\right]^2\end{Bmatrix}
\end{aligned} \tag{A-4}$$

Then, we rearrange Eq. (A-1) (i.e., $E[TCU(t_{1Z})]$) as follows:

$$E\left[TCU(t_{1Z})\right] = \frac{\lambda}{v_0 + \frac{\left(1 - e^{-\beta t_{1Z}}\right)\lambda g}{(1+\alpha_1)t_{1Z}P_1}}\begin{Bmatrix}\frac{G_0}{t_{1Z}}\left(1 - e^{-\beta t_{1Z}}\right) - G_1\left(e^{-\beta t_{1Z}}\right) + \frac{G_2}{t_{1Z}}\left(1 - e^{-\beta t_{1Z}}\right) + \\ G_3\left(1 - e^{-\beta t_{1Z}}\right) + \frac{W_1}{t_{1Z}} + W_2 + W_3(t_{1Z})\end{Bmatrix} \tag{33}$$

Apply the 1st and 2nd derivatives of $E[TCU(t_{1Z})]$, one has formulas (A-5) and (A-6) below:

$$\begin{aligned}
\frac{dE\left[TCU(t_{1Z})\right]}{d(t_{1Z})} &= \frac{(1+\alpha_1)P_1\lambda}{\left[\lambda g\left(1 - e^{-\beta t_{1Z}}\right) + (1+\alpha_1)P_1 v_0 t_{1Z}\right]^2}\cdot\\
&\begin{Bmatrix}
-\left[\lambda g\beta e^{-\beta t_{1Z}} + v_0(1+\alpha_1)P_1\right]W_1 - \left(\beta e^{-\beta t_{1Z}}t_{1Z} + e^{-\beta t_{1Z}} - 1\right)W_2\lambda g\\
+\left[-\lambda g\beta e^{-\beta t_{1Z}}{t_{1Z}}^2 - 2\lambda g e^{-\beta t_{1Z}}t_{1Z}v_0{t_{1Z}}^2(1+\alpha_1)P_1 + 2\lambda g t_{1Z}\right]W_3\\
+(1+\alpha_1)P_1(G_0 + G_2)v_0\left(\beta e^{-\beta t_{1Z}}t_{1Z} + e^{-\beta t_{1Z}} - 1\right)\\
+\left(v_0\beta e^{-\beta t_{1Z}}{t_{1Z}}^2(1+\alpha_1)P_1 - 2\lambda e^{-\beta t_{1Z}}g + \lambda g e^{-2\beta t_{1Z}} + \lambda g\right)G_3\\
-\left(-(1+\alpha_1)P_1\beta v_0 e^{-\beta t_{1Z}}{t_{1Z}}^2 - \beta\lambda e^{-\beta t_{1Z}}t_{1Z}g - \lambda e^{-2\beta t_{1Z}}g + \lambda e^{-\beta t_{1Z}}g\right)G_1
\end{Bmatrix}
\end{aligned} \tag{A-5}$$

$$\begin{aligned}
\frac{d^2E\left[TCU(t_{1Z})\right]}{d(t_{1Z})^2} &= \frac{(1+\alpha_1)P_1\lambda}{\left[\lambda g\left(1 - e^{-\beta t_{1Z}}\right) + v_0 t_{1Z}(1+\alpha_1)P_1\right]^3}\cdot\\
&\begin{Bmatrix}
\begin{pmatrix}4\beta e^{-\beta t_{1Z}}v_0\lambda g(1+\alpha_1)P_1 + \beta^2 e^{-2\beta t_{1Z}}\lambda^2 g^2 + \beta^2 e^{-\beta t_{1Z}}\lambda^2 g^2\\ +2{v_0}^2\left[(1+\alpha_1)P_1\right]^2 + \beta^2 e^{-\beta t_{1Z}}t_{1Z}v_0\lambda g(1+\alpha_1)P_1\end{pmatrix}W_1\\
+\begin{pmatrix}\beta^2 e^{-\beta t_{1Z}}v_0{t_{1Z}}^2(1+\alpha_1)P_1 + 2e^{-\beta t_{1Z}}v_0(1+\alpha_1)P_1 - 2v_0(1+\alpha_1)P_1 + 2\beta e^{-2\beta t_{1Z}}\lambda g\\ -2\beta e^{-\beta t_{1Z}}\lambda g + 2\beta e^{-\beta t_{1Z}}v_0 t_{1Z}(1+\alpha_1)P_1 + \beta^2 e^{-2\beta t_{1Z}}\lambda g t_{1Z} + \beta^2 e^{-\beta t_{1Z}}\lambda g t_{1Z}\end{pmatrix}\lambda g W_2\\
+\begin{pmatrix}\beta^2 e^{-2\beta t_{1Z}}{t_{1Z}}^2\lambda g + 2\lambda g + 2e^{-2\beta t_{1Z}}\lambda g - 4e^{-\beta t_{1Z}}\lambda g + \beta^2 e^{-\beta t_{1Z}}{t_{1Z}}^2\lambda g\\ +4\beta e^{-2\beta t_{1Z}}t_{1Z}\lambda g - 4\beta e^{-\beta t_{1Z}}t_{1Z}\lambda g + \beta^2 e^{-\beta t_{1Z}}{t_{1Z}}^3 v_0(1+\alpha_1)P_1\end{pmatrix}W_3\lambda g\\
-\begin{pmatrix}\beta^2\lambda e^{-\beta t_{1Z}}t_{1Z}g + \beta^2\lambda e^{-2\beta t_{1Z}}t_{1Z}g - 2\beta\lambda e^{-\beta t_{1Z}}g + \beta^2 e^{-\beta t_{1Z}}{t_{1Z}}^2 v_0(1+\alpha_1)P_1\\ +2\beta\lambda e^{-2\beta t_{1Z}}g - 2v_0(1+\alpha_1)P_1 + 2\beta e^{-\beta t_{1Z}}t_{1Z}v_0(1+\alpha_1)P_1 + 2e^{-\beta t_{1Z}}v_0(1+\alpha_1)P_1\end{pmatrix}(1+\alpha_1)P_1(G_0 + G_2)v_0\\
-\begin{pmatrix}4\beta\lambda g t_{1Z}v_0(1+\alpha_1)P_1 + 2\beta\lambda^2 g^2 - 2\beta\lambda^2 e^{\beta t_{1Z}}g^2 + 2\lambda g v_0(1+\alpha_1)P_1 + \beta^2\lambda^2 g^2 t_{1Z}\\ +\beta^2\lambda^2 e^{\beta t_{1Z}}g^2 t_{1Z} + \beta^2\lambda g{t_{1Z}}^2 v_0(1+\alpha_1)P_1 - 2\lambda e^{\beta t_{1Z}}g v_0(1+\alpha_1)P_1 - 2\beta\lambda e^{\beta t_{1Z}}g t_{1Z}v_0(1+\alpha_1)P_1\\ +2\beta^2\lambda e^{\beta t_{1Z}}g{t_{1Z}}^2 v_0(1+\alpha_1)P_1 + \beta^2 e^{\beta t_{1Z}}{t_{1Z}}^3{v_0}^2\left[(1+\alpha_1)P_1\right]^2\end{pmatrix}G_1 e^{-2\beta t_{1Z}}\\
-\begin{pmatrix}4\beta\lambda e^{-2\beta t_{1Z}}t_{1Z}g + 2\lambda g + 2\lambda e^{-2\beta t_{1Z}}g - 4\lambda e^{-\beta t_{1Z}}g + \beta^2\lambda e^{-2\beta t_{1Z}}{t_{1Z}}^2 g + \beta^2\lambda e^{-\beta t_{1Z}}{t_{1Z}}^2 g\\ -4\beta\lambda e^{-\beta t_{1Z}}t_{1Z}g + \beta^2 e^{-\beta t_{1Z}}{t_{1Z}}^3 v_0(1+\alpha_1)P_1\end{pmatrix}G_3 v_0(1+\alpha_1)P_1
\end{Bmatrix}
\end{aligned} \tag{A-6}$$

Since the 1st term on Eq. (A-6)'s right-hand side (RHS) is positive and if the 2nd term Eq. (A-6) is also positive, then we can confirm $E[TCU(t_{1Z})]$ is convex (i.e., if $\delta(t_{1Z}) > t_{1Z} > 0$ holds).

$$\delta(t_{1Z})=\frac{\left\{\begin{array}{l}\left(4\beta e^{-\beta t_{1Z}}v_0\lambda g(1+\alpha_1)P_1+\beta^2e^{-2\beta t_{1Z}}\lambda^2g^2+\beta^2e^{-\beta t_{1Z}}\lambda^2g^2+2v_0^2\left[(1+\alpha_1)P_1\right]^2\right)W_1\\ +\left(-2v_0(1+\alpha_1)P_1+2e^{-\beta t_{1Z}}v_0(1+\alpha_1)P_1+2\beta e^{-2\beta t_{1Z}}\lambda g-2\beta e^{-\beta t_{1Z}}\lambda g\right)W_2\lambda g\\ +\left(2e^{-2\beta t_{1Z}}\lambda g+2\lambda g-4e^{-\beta t_{1Z}}\lambda g\right)W_3\lambda g\\ -\left(-2\beta\lambda e^{-\beta t_{1Z}}g+2\beta\lambda e^{-2\beta t_{1Z}}g+2e^{-\beta t_{1Z}}v_0(1+\alpha_1)P_1-2v_0(1+\alpha_1)P_1\right)(1+\alpha_1)P_1(G_0+G_2)v_0\\ -\left(2\lambda g v_0(1+\alpha_1)P_1+2\beta\lambda^2g^2-2\beta\lambda^2e^{\beta t_{1Z}}g^2-2\lambda e^{\beta t_{1Z}}gv_0(1+\alpha_1)P_1\right)G_1e^{-2\beta t_{1Z}}\\ -\left(2\lambda e^{-2\beta t_{1Z}}g+2\lambda g-4\lambda e^{-\beta t_{1Z}}g\right)G_3v_0(1+\alpha_1)P_1\end{array}\right\}}{-\left\{\begin{array}{l}\left(\lambda g(1+\alpha_1)P_1\beta^2e^{-\beta t_{1Z}}v_0\right)W_1\\ +\left(2\beta e^{-\beta t_{1Z}}v_0(1+\alpha_1)P_1+\beta^2e^{-\beta t_{1Z}}v_0t_{1Z}(1+\alpha_1)P_1+\beta^2e^{-2\beta t_{1Z}}\lambda g+\beta^2e^{-\beta t_{1Z}}\lambda g\right)\lambda gW_2\\ +\left(\beta^2e^{-\beta t_{1Z}}t_{1Z}\lambda g+\beta^2e^{-2\beta t_{1Z}}t_{1Z}\lambda g+4\beta e^{-2\beta t_{1Z}}\lambda g-4\beta e^{-\beta t_{1Z}}\lambda g+\beta^2e^{-\beta t_{1Z}}t_{1Z}^2v_0(1+\alpha_1)P_1\right)\lambda gW_3\\ -\left(\begin{array}{l}\beta^2\lambda e^{-\beta t_{1Z}}g+\beta^2e^{-\beta t_{1Z}}t_{1Z}v_0(1+\alpha_1)P_1+\beta^2\lambda e^{-2\beta t_{1Z}}g\\ +2\beta e^{-\beta t_{1Z}}v_0(1+\alpha_1)P_1\end{array}\right)(1+\alpha_1)P_1(G_0+G_2)v_0\\ -\left(\begin{array}{l}-2\beta\lambda e^{\beta t_{1Z}}gv_0(1+\alpha_1)P_1+\beta^2\lambda^2g^2+4\beta\lambda gv_0(1+\alpha_1)P_1+\beta^2\lambda g t_{1Z}v_0(1+\alpha_1)P_1\\ +\beta^2\lambda^2e^{\beta t_{1Z}}g^2+2\beta^2\lambda e^{\beta t_{1Z}}gt_{1Z}v_0(1+\alpha_1)P_1+\beta^2e^{\beta t_{1Z}}t_{1Z}^2v_0^2\left[(1+\alpha_1)P_1\right]^2\end{array}\right)e^{-2\beta t_{1Z}}G_1\\ -\left(\begin{array}{l}\beta^2\lambda e^{-2\beta t_{1Z}}t_{1Z}g+4\beta\lambda e^{-2\beta t_{1Z}}g+\beta^2e^{-\beta t_{1Z}}t_{1Z}^2v_0(1+\alpha_1)P_1\\ +\beta^2\lambda e^{-\beta t_{1Z}}t_{1Z}g-4\beta\lambda e^{-\beta t_{1Z}}g\end{array}\right)(1+\alpha_1)P_1G_3v_0\end{array}\right\}}>t_{1Z}>0 \tag{A-7}$$

Appendix B

Table B-1

Probabilities of various Poisson distributed equipment failures rates

β	t_{1Z}*	$P(x=0)$	$P(x=1)$	$P(x\le 1)$	$P(x>1)$
6.0	0.2413	23.50%	34.03%	57.53%	42.47%
5.0	0.2039	36.07%	36.78%	72.85%	27.15%
4.0	0.1672	51.24%	34.26%	85.50%	14.50%
3.0	0.1400	65.70%	27.60%	93.30%	6.70%
2.0	0.1235	78.11%	19.30%	97.41%	2.59%
1.0	0.1149	89.14%	10.25%	99.39%	0.61%
0.5	0.1130	94.51%	5.34%	99.85%	0.15%
0.01	0.1125	99.89%	0.11%	100.00%	0.00%

$$\frac{\left(\beta t_{1Z}^*\right)^x e^{-\beta t_{1Z}^*}}{x!} \tag{B-1}$$

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