The combined effect of rework, postponement, multiple shipments, and overtime producing common-component on a multiproduct vendor-client incorporated system

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CHRONICLE	ABSTRACT
	The study examines the combined effect of rework, multiple shipments, postponement, and overtime producing common-component on a multiproduct vendor-client incorporated system. Clients' product demand trend turns to diversity, quality, and rapid response in the current supply-chain environment. Under such a stiff competitive environment, today's manufacturers must effectively plan their multi-item fabrication to boost utilization and product quality, minimize total relevant costs, and meet short given order lead time. By considering the commonality of the finished goods, required quality, and completion lead time, this study presents an exact model featuring rework of defects, multiple shipments, postponement, overtime producing the mutual component, and satisfying the market needs. Through the techniques of explicitly modeling, formulating, and system cost minimization, this study simultaneously derives the optimal cycle-time and shipping frequency for the studied problem. A numerical example helps show how our model works for any given parameter values and how the variation in single and multiple factors of the problem affects the crucial system performances (e.g., total uptimes, each relevant cost, utilization, total cost, etc.) A wide variety of today's industries (e.g., automotive, household goods, etc.) and their related supply chains can utilize our decisional model to reveal in-depth managerial insights for planning their fabrication and shipments.

Nomenclature

In stage one – fabricating the mutual components

γ	=	the mutual part's completion rate compared to the finished product,
$t_{1,0}$	=	uptime with overtime option,
$t_{2,0}$	=	rework time,
$t_{3,0}$	=	delivering time,
$P_{T1,0}$	=	overtime output rate per year,
$P_{1,0}$	=	regular fabricating rate,
$\alpha_{1,0}$	=	overtime added output-rate proportion,
$P_{T2,0}$	=	reworking rate with overtime option,
$P_{2,0}$	=	regular rework rate,
$\alpha_{2,0}$	=	the relating parameter between K_{T0} and K_0,
$\alpha_{3,0}$	=	the relating parameter between C_{T0} and C_0,

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K_{T0} = setup cost with overtime option,
K_0 = regular setup cost,
C_{T0} = unit cost with overtime option,
C_0 = unit cost,
$C_{TR,0}$ = unit rework cost with overtime option,
$C_{R,0}$ = unit rework cost,
x_0 = random nonconforming proportion,
$d_{T1,0}$ = nonconforming parts' fabricating rate (i.e., $d_{T1,,0} = P_{T1,0}\,x_0$),
S_0 = setup time,
i_0 = relating parameter for unit holding cost (i.e., $h_{1,i} = i_0 C_i$),
$h_{1,0}$ = unit holding cost,
Q_0 = lot-size,
$h_{2,0}$ = unit holding cost in rework process,
λ_0 = annual requirement,
t_0^* = the sum of ($t^*{}_{1,0} + t^*{}_{2,0}$),
$H_{1,0}$ = inventory level when uptime completes,
$H_{2,0}$ = inventory level when rework completes,

In stage two – fabricating the final goods (for $i = 1, 2, \ldots, L$)
H_i = inventory level when product i's uptime completes,
D_i = fixed-quantity per shipment,
I_i = number of end products i left at the end of each $t_{n,i}$,
L = the buyer's multiproduct requirements (end products),
T_A = decision variable – rotation cycle length,
$t_{1,i}$ = final product i's fabrication uptime,
$t_{2,i}$ = rework time,
$t_{3,i}$ = delivering time,
λ_i = annual demand rate,
Q_i = lot-size,
K_i = setup cost,
$h_{1,i}$ = unit holding cost,
$h_{2,i}$ = unit holding cost in rework process,
C_i = unit cost,
$P_{1,i}$ = annual fabricating rate,
$P_{2,i}$ = annual rework rate,
x_i = random nonconforming proportion,
$d_{1,i}$ = nonconforming item i's fabricating rate,
$C_{R,i}$ = unit rework cost,
$K_{D,i}$ = fixed delivery cost,
$C_{D,i}$ = unit delivery cost,
n = another decision variable – shipment frequency per cycle,
$t_{n,i}$ = time-interval of shipments,
t_i^* = the sum of optimal $\Sigma_i(t^*{}_{1,i} + t^*{}_{2,i})$,
S_i = setup time,
$H_{1,i}$ = inventory level when uptime completes,
$H_{2,i}$ = inventory level when rework time completes,
$E[T_A]$ = the expected cycle length,
$TC(T_A, n)$ = total system cost per cycle,
$I(t)_i$ = inventory level at time t (where $i = 0, 1, 2, \ldots, L$),
$E[TC(T_A, n)]$ = the expected total system cost per cycle,
$I_d(t)_i$ = inventory level at time t (where $i = 0, 1, 2, \ldots, L$),
$I_c(t)_i$ = clients' inventory level at time t,
$E[TCU(T_A, n)]$ = the expected system cost per unit time.

1. Introduction

Clients' product demand trend turns to diversity, quality, and rapid response in the current supply-chain environment. Under such a stiff competitive environment, today's manufacturers must effectively plan their multi-item fabrication to boost utilization and product quality, minimize total relevant costs, and meet short given order lead time. In consideration of the needs of various goods with commonality and plant efficiency in terms of cost, utilization, and expediting order completion time, production planners often evaluate the alternatives of using a single piece of equipment to fabricate multiple goods in sequence, associating a postponement strategy. Wadhwa et al. (2006) examined the postponement strategies through

knowledge innovation and redesigning business processes in automotive industries. By developing managers' innovative mindset and adequately managing the existing processes knowledge, they evaluate the impact of postponement on the product flexibility, service levels, and potential benefits. In addition, they demonstrated their results using various scenarios in simulated cases. Qrunfleh and Tarafdar (2013) respectively explored the effect of strategic postponement and the role of supplier partnership on the agile and lean supply chain. The researchers also investigated the relationship between supply-chain approachability and corporation performance. They proposed a research model to study the problem and conducted a survey using data from two hundred and five managers and executives in the manufacturing and purchasing functions of business corporations in the USA. By applying the strategic-choice method and the firm resource-based supply-chain viewpoint and specific computer software, the researchers demonstrated how a proper supply-chain strategy could boost its responsiveness and performance. Finally, they found that a strategic supplier partnership can effectively enhance a lean supply chain's responsiveness; the postponement can partially increase the agile supply chain's responsiveness; both can boost the firm's performance. Mauri et al. (2021) proposed two hybrid-metaheuristics to explore a two-stage multi-item capacitated facility location system. The first stage of the studied system delivers goods from various plants to depots, and then the second stage transports goods from depots to clients. The aim is to meet demands and minimize overall operating costs under capacity constraints of plants and depots. Finally, the researchers applied the clustering search approaches and biased random-key algorithm to resolve their multiproduct system with various sets of experimental examples. Additional studies (Van Mieghem, 2004; Gualandris and Kalchschmidt, 2015; Ivanov et al., 2019) examined the impact of multi-item fabrication and different features of commonality and postponement strategies on multi-item production and supply-chain systems. In a multiproduct manufacturing plan with the postponement, an overtime arrangement helps expedite the completion uptime of making heavy quantities of mutual components. Ren et al. (2010) explored the multiple shifts vehicle routing problem with time windows and overtime. The study considered that variable demands in a finite time horizon are served by a limited set of vehicles with allowable overtime. The researcher presented a shift-dependent heuristic with overtime to construct routes for each shift and explore the tradeoff between regular operating costs (e.g., transportation, unmet demand, and diver costs) and overtime cost. The proposed heuristic can meet random demand within a tight time window and significantly save vehicle quantity and total costs. Madeira (2014) proposed a New Keynesian Phillips Curve (NKPC) model to evaluate the marginal overtime cost against the straight time. The research showed that the obtaining coefficient estimates were in line with the theory of hybrid-NKPC (which incorporates backward-looking price-setters), but not for the NKPC with only a forward-looking setting). In addition, the study exposed the characteristics of the hybrid model and the influence of employment frictions on the price changes/setting of the NK model. Keyvanshokooh et al. (2021) utilized a primal-dual approach to explore an online advance scheduling problem with overtime. Arrivals of jobs have dissimilar service times, and the services can be completed through a finite set of servers having regular or overtime capacities. Based on worst-case performance, the system selects a server, a corresponding service time, and competitive ratio (CR) for each arrival job. Then, the researchers developed an online primal-dual method to assign server-date and potential overtime plans for each arriving job. Finally, they used actual health system appointment data to demonstrate that their online scheduling algorithm significantly outperformed existing scheduling rules, such as the first-come-first-served and the nested threshold rules. Additional works (Goldenhar et al., 2003; Lambooij et al., 2007; Conway and Sturges, 2014; Chiu et al., 2020; Soriano et al., 2020) studied the effect of diverse overtime aspects on different corporation planning, manufacturing operations, and supply-chain management.

Meeting order with a fixed-quantity multiple shipment policy helps reduce clients' inventory effectively. Hence, it is a commonly used delivery plan in vendor-client incorporated systems. Khan et al. (2014) explored the combined impact of learning in fabrication and quality inspection errors on an integrated supply chain system. The researchers presented a mathematical model with learning in fabrication at the vendor-side and quality inspection errors at the client-side to decide the optimal inventory for the integrated supply chain system. They demonstrated the model's applicability using numerical examples and discussed how it could benefit managerial decisions in investment, personnel training, and process and product design. Karakaya et al. (2021) analyzed the service systems' benefits and impact from preannounce delivery time and price to customers. The researchers used numerical studies to explore customers' expectations and reactions by knowing their anticipated services' price and delivery time. Additional works (Sahebi et al., 2019;Mabrouk, 2020; Sumrit, 2020; Tran et al., 2020; Farmand et al., 2021; Sazvar et al., 2021) studied the influence of diverse products' transportation plans on various supply-chain operations. Product quality is one of the crucial factors for current vendors to stay competitive in today's markets. However, real fabrication systems inevitably generate defeats. Repairing the defectives to meet clients' anticipation becomes vendors' essential task. Raviv (2013) presented an algorithm to maximize the profit of a serial fabrication line with screening stations and rework. The results help determine the optimal design of quality control stations and fabrication rates that bring the most profits. Ponte et al. (2021) applied control engineering approaches to a hybrid fabrication and re-fabrication system with product quality classification and returned goods and evaluated the supply chain's inventory performance and Bullwhip effect in a supply chain. Different re-fabrication lead times and operations are assigned to explore the potential benefits, operational costs, and savings from the quality grading. Additional works (Eskandari and Hosseinzadeh, 2014; Sahebi et al., 2019; Assia et al., 2020; Sztorc and Savenkovs, 2020; Son and Van Hop, 2021; Abukhader and Onbaşıoğlu, 2021) explored the influence of diverse fabrication quality situations on various manufacturing systems and supply chains. For few prior studies that examined the combined effect of rework, multiple shipments, overtime producing common-component, and postponement on a multiproduct vendor-client incorporated system, we wish to bridge the gap.

2. Description, modelling, and assumption

The present work investigates the collective impact of postponement, overtime fabrication of common parts, multi-shipment, and rework on a multiproduct vendor-buyer coordinated problem. A two-stage manufacturing scheme is designed to explore the problem. All necessary common parts are made in the 1st stage, and the customer's multiproduct requirements are produced in the 2nd stage. We assume a constant mutual/common part's completion proportion γ and production rate $P_{1,0}$. An overtime plan is utilized to raise common parts' output rate by $\alpha_{1,0}$ to $P_{T1,0}$ to shorten its uptime. Eq. (1) to Eq. (3) express the relationship between overtime and regular output rates and relevant costs:

$$P_{T1,0} = P_{1,0}\left(1+\alpha_{1,0}\right), \tag{1}$$

$$K_{T0} = K_0\left(1+\alpha_{2,0}\right), \tag{2}$$

$$C_{T0} = C_0\left(1+\alpha_{3,0}\right), \tag{3}$$

where C_{T0}, K_{T0}, C_0, K_0, $\alpha_{3,0}$, and $\alpha_{2,0}$ denote the different overtime unit and setup costs, regular unit and setup costs, and connecting factors of overtime and regular rates. Constant demand rates λ_i are assumed for L multiproduct, where $i = 1, 2, \ldots,$ L. However, the manufacturing rates $P_{1,i}$ rely on the γ (e.g., when $\gamma = 0.5$, then $P_{1,0}$ and $P_{1,i}$ both double their regular output rates comparing to that in a single-stage fabricating scheme. Nonconforming proportion x_0 and x_i are randomly fabricated in both stages, and these items are repairable by the use of $P_{T2,0}$ and $P_{2,i}$, respectively. An overtime rework-rate $P_{T2,0}$ and unit cost $C_{TR,0}$ have the following relationships with their corresponding the regular parameters $P_{2,0}$ and $C_{R,0}$:

$$P_{T2,0} = P_{2,0}\left(1+\alpha_{1,0}\right), \tag{4}$$

$$C_{TR,0} = C_{R,0}\left(1+\alpha_{3,0}\right). \tag{5}$$

Fig. 1 depicts this particular postponement problem's stock level. It discloses that when uptime of stage one completes, the stock level upsurges to $H_{1,0}$. It continues to rise to $H_{2,0}$ at the end of stage one's rework time. Starting from the 2nd stage, the common parts' inventory level depletes when the end products' production begins. Meantime, each finished product's stock climbs to $H_{1,i}$ when uptime $t_{1,i}$ completes, and it rises further to $H_{2,i}$ when the rework time $t_{1,i}$ ends (see Figure 1). Moreover, we must have $(P_{T1,0} - d_{T1,0} - \lambda_0 > 0)$ and $(P_{1,i} - d_{1,i} - \lambda_i > 0)$ to ensure stock-out situations for both stages.
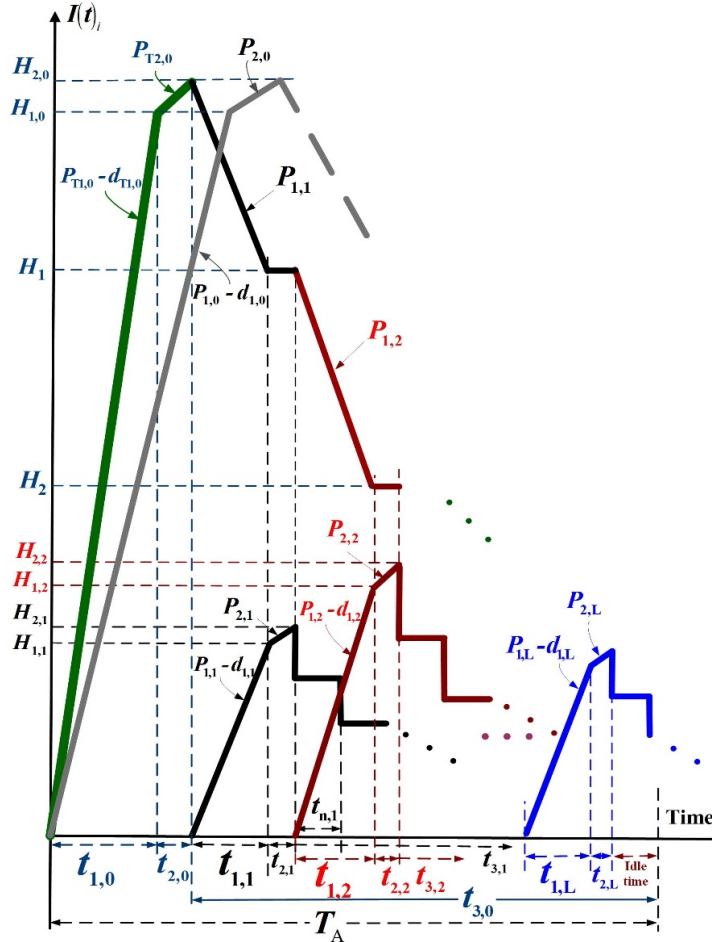

Fig. 1. This study's inventory level comparing to the same problem without overtime (in grey)

Fig. 2 shows this particular postponement problem's nonconforming inventory level. It specifies that when uptimes $t_{1,0}$, and $t_{1,i}$ complete, the nonconforming inventory level surge respectively to $(d_{T1,0} t_{1,0})$ and $(d_{1,i} t_{1,i})$, where expressions for $d_{T1,0}$ and $d_{1,i\,a}$ are shown in Eq. (6) and Eq. (7). As the rework process starts, the nonconforming inventory levels start to decline at the rates of $P_{T2,0}$ and $P_{2,i}$, and they deplete to zero when the rework times $t_{2,0}$ and $t_{2,i}$ end.

$$d_{T1,0} = x_0 P_{T1,0}, \tag{6}$$
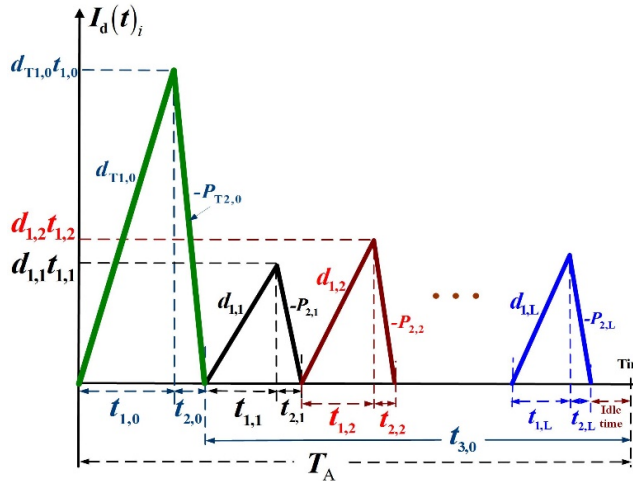
$$d_{1,i} = P_{1,i} x_i. \tag{7}$$

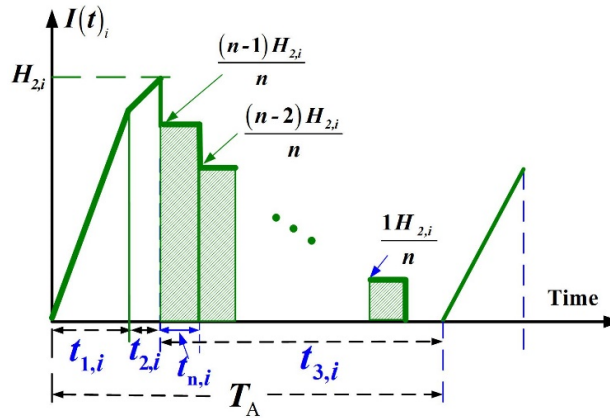**Fig. 2.** This study's nonconforming inventory level

Fig. 3. End product i's inventory level in $t_{3,i}$

2.1. Formulas of buyers' stocks

The end products' inventory levels surge to $H_{2,i}$ when their rework times complete (see Fig. 1). It follows that n equal-size multi-shipment plan initiates every $t_{n,i}$ period in $t_{3,i}$ (as shown in Fig. 3) and Eq. (8) gives the total stocks in $t_{3,i}$:

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right) H_{2,i}(t_{3,i}) = \left(\frac{n-1}{2n}\right) H_{2,i}(t_{3,i}). \tag{8}$$

The buyer side's end product i's inventory level is illustrated in Fig. 4. Eq. (9) gives the total stocks at the buyer's side.
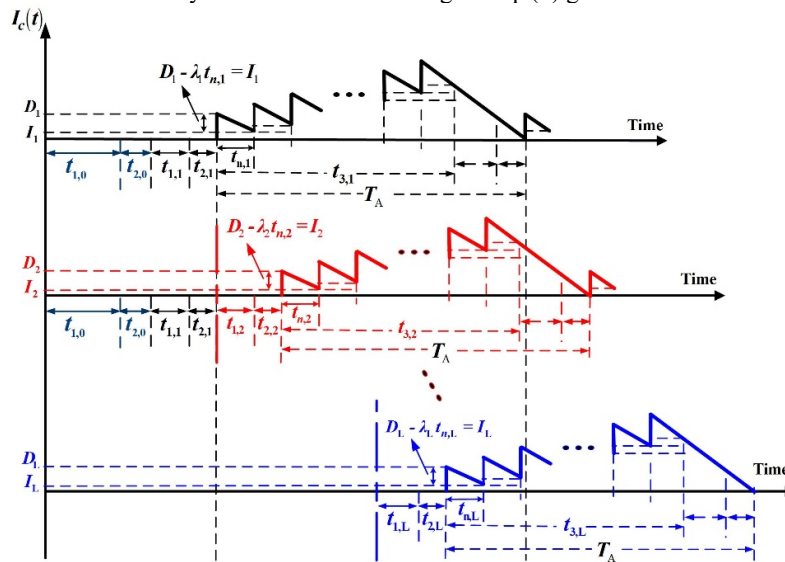

Fig. 4. Buyer side's end product i's inventory level

$$\left[\frac{n(D_i - I_i)t_{n,i}}{2} + \frac{n(n+1)}{2} I_i t_{n,i} + \frac{nI_i(t_{1,i} + t_{2,i})}{2}\right] \tag{9}$$

where

$$D_i = \frac{H_{2,i}}{n},$$ (10)

$$I_i = D_i - \lambda_i \left(t_{n,i} \right),$$ (11)

$$t_{n,i} = \frac{t_{3,i}}{n}.$$ (12)

2.2. Formulas in fabricating multiproduct

Based on the problem statement along with Fig. 1 and Fig. 2, the following formulas for $i = 1, 2, \ldots, L$ are observed:

$$Q_i = T_A \lambda_i,$$ (13)

$$T_A = \frac{Q_i}{\lambda_i} = t_{1,i} + t_{2,i} + t_{3,i},$$ (14)

$$t_{1,i} = \frac{H_{1,i}}{P_{1,i} - d_{1,i}} = \frac{Q_i}{P_{1,i}},$$ (15)

$$t_{2,i} = \frac{H_{2,i} - H_{1,i}}{P_{2,i}} = \frac{x_i Q_i}{P_{2,i}},$$ (16)

$$t_{3,i} = T_A - \left(t_{1,i} + t_{2,i} \right),$$ (17)

$$H_{1,i} = t_{1,i} \left(P_{1,i} - d_{1,i} \right),$$ (18)

$$H_{2,i} = Q_i = H_{1,i} + t_{2,i} P_{2,i}.$$ (19)

Refer to Eq. (13), we know that to accomplish making all end products the required common parts are as follows:

$$H_{2,0} = \sum_{i=1}^{L} Q_i = \sum_{i=1}^{L} \left(T_A \lambda_i \right).$$ (20)

2.3. Formulas in fabricating the mutual pasts

Due the postponement strategy, the 1st manufacturing stage one must prepare adequate common parts $H_{2,i}$ for the 2nd stage's finished products' production. From Fig. 1 and Fig. 2, the following expressions are observed:

$$Q_0 = H_{2,0},$$ (21)

$$t_{1,0} = \frac{H_{1,0}}{P_{T1,0} - d_{T1,0}} = \frac{Q_0}{P_{T1,0}},$$ (22)

$$t_{2,0} = \frac{Q_0 x_0}{P_{T2,0}} = \frac{H_{2,0} - H_{1,0}}{P_{T2,0}},$$ (23)

$$H_{1,0} = \left(P_{T1,0} - d_{T1,0} \right) t_{1,0},$$ (24)

$$H_{2,0} = H_{1,0} + t_{2,0} P_{T2,0},$$ (25)

$$T_A = t_{1,0} + t_{2,0} + t_{3,0},$$ (26)

$$\lambda_0 T_A = \sum_{i=1}^{L} Q_i,$$ (27)

$$H_1 = H_{2,0} - Q_1,$$ (28)

$$H_i = H_{(i-1)} - Q_i, \text{ for } i = 2, 3, \ldots, L,$$ (29)

$$H_L = H_{(L-1)} - Q_L = 0.$$ (30)

3. Cost function and the optimal policy

The problem's cost function $TC(T_A, n)$ comprises both stages' cost about (1) setup, (2) variable, (3) rework, (4) delivering finished products, and (5) vendor's/buyer's inventory holding, as follows:

$$TC(T_A,\ n) = \begin{cases} C_{T0}Q_0 + K_{T0} + C_{TR,0}x_0Q_0 + h_{2,0}\left(t_{2,0}\right)\left(\dfrac{d_{T1,0}t_{1,0}}{2}\right) \\ +h_{1,0}\left[\dfrac{H_{1,0}t_{1,0}}{2} + \dfrac{d_{T1,0}t_{1,0}}{2}\left(t_{1,0}\right) + \dfrac{H_{2,0}+H_{1,0}}{2}\left(t_{2,0}\right) + \sum\limits_{i=1}^{L}\left[\dfrac{Q_i\left(t_{1,i}\right)}{2} + \left(t_{1,i}+t_{2,i}\right)H_i\right]\right] \end{cases}$$
$$+ \sum_{i=1}^{L}\begin{cases} C_iQ_i + K_i + C_{R,i}Q_ix_i + nK_{D,i} + C_{D,i}Q_i + h_{2,i}\left[\dfrac{d_{1,i}t_{1,i}}{2}\left(t_{2,i}\right)\right] \\ +h_{1,i}\left[\dfrac{H_{1,i}t_{1,i}}{2} + \dfrac{H_{2,i}+H_{1,i}}{2}\left(t_{2,i}\right) + H_{2,i}\left(t_{3,i}\right)\left(\dfrac{n-1}{2n}\right) + \left(t_{1,i}\right)\dfrac{d_{1,i}t_{1,i}}{2}\right] \\ +h_{3,i}\left[\dfrac{\left(D_i-I_i\right)nt_{n,i}}{2} + \dfrac{\left(n+1\right)nI_it_{n,i}}{2} + \dfrac{\left(t_{1,i}+t_{2,i}\right)nI_i}{2}\right] \end{cases} \tag{31}$$

Substituting Eq. (1) to Eq. (30) in Eq. (31) and with further derivation (refer to Appendix A), $E[TCU(T_A, n))]$ becomes as follows:

$$E\left[TCU\left(T_A,\ n\right)\right] = C_0\left(1+\alpha_{3,0}\right)\lambda_0 + \frac{K_0\left(1+\alpha_{2,0}\right)}{T_A} + \lambda_0\left(1+\alpha_{3,0}\right)C_{R,0}E\left[x_0\right]$$
$$+ h_{2,0}\lambda_0^{\ 2}T_A\left[\frac{E\left[x_0\right]^2}{2P_{2,0}\left(1+\alpha_{1,0}\right)}\right] + h_{1,0}\sum_{i=1}^{L}\left(\frac{\lambda_i^{\ 2}T_A}{2P_{1,i}}\right)$$
$$+ h_{1,0}\left\{\left(\frac{1}{2}\right)\lambda_0^{\ 2}E_{0P}T_A + \sum_{i=1}^{L}\left(-\lambda_iE_{iP}T_A\right)\left[\sum_{j=1}^{i}\left(\lambda_j\right)\right] + \left[\sum_{i=1}^{L}\left(\lambda_i\right)\right]\cdot\sum_{i=1}^{L}\left(\lambda_iT_AE_{iP}\right)\right\} \tag{32}$$
$$+ \sum_{i=1}^{L}\begin{cases} C_i\lambda_i + \dfrac{K_i}{T_A} + C_{R,i}E\left[x_i\right]\lambda_i + \dfrac{nK_{D,i}}{T_A} + C_{D,i}\lambda_i + \dfrac{h_{1,i}E_{2i}\lambda_i^{\ 2}T_A}{2} \\ +h_{2,i}\left[\dfrac{E\left[x_i\right]^2\lambda_i^{\ 2}T_A}{2P_{2,i}}\right] + \dfrac{h_{3,i}E_{iP}\lambda_i^{\ 2}T_A}{2} + \dfrac{\left(1-\lambda_iE_{iP}\right)T_A\left(h_{3,i}-h_{1,i}\right)\lambda_i}{2n} \end{cases}$$

3.1. The optimal solution

By applying the Hessian Matrix Equations (Rardin, 1998), we can verify $E[TCU(T_A, n)]$'s convexity.

$$\begin{bmatrix} T_A & n \end{bmatrix}\cdot\begin{pmatrix} \dfrac{\partial^2 E\left[TCU\left(T_A,\ n\right)\right]}{\partial T_A^{\ 2}} & \dfrac{\partial^2 E\left[TCU\left(T_A,\ n\right)\right]}{\partial T_A\partial n} \\ \dfrac{\partial^2 E\left[TCU\left(T_A,\ n\right)\right]}{\partial T_A\partial n} & \dfrac{\partial^2 E\left[TCU\left(T_A,\ n\right)\right]}{\partial n^2} \end{pmatrix}\cdot\begin{bmatrix} T_A \\ n \end{bmatrix} = \frac{2K_0\left(1+\alpha_{2,0}\right)}{T_A} + \sum_{i=1}^{L}\left(\frac{2K_i}{T_A}\right) > 0 \tag{33}$$

Eq. (33) yields positive, since K_0, T_A, $(1 + \alpha_{2,0})$, and K_i are positive. We confirm $E[TCU(T_A, n)]$ is strictly convex for all n and T_A values different from zero, and the minimum of $E[TCU(T_A, n)]$ exists. By setting the first-derivatives of $E[TCU(T_A, n)]$ concerning n and T_A equal to zero, we can derive T_A* and $n*$ simultaneously:

$$\frac{\partial E\left[TCU\left(T_A,\ n\right)\right]}{\partial n} = \sum_{i=1}^{L}\left[\frac{K_{D,i}}{T_A} - \frac{\left(1-\lambda_iE_{iP}\right)T_A\lambda_i\left(h_{3,i}-h_{1,i}\right)}{2n^2}\right] = 0 \tag{34}$$

$$\frac{\partial E\left[TCU\left(T_A,\ n\right)\right]}{\partial T_A} = \begin{cases} \dfrac{-K_0\left(1+\alpha_{2,0}\right)}{T_A^{\ 2}} + \lambda_0^{\ 2}h_{2,0}\left[\dfrac{E\left[x_0\right]^2}{2P_{2,0}\left(1+\alpha_{1,0}\right)}\right] \\ +h_{1,0}\sum\limits_{i=1}^{L}\left(\dfrac{\lambda_i^{\ 2}}{2P_{1,i}}\right) + \sum\limits_{i=1}^{L}\left\{\dfrac{-\left(K_i+nK_{D,i}\right)}{T_A^{\ 2}}\right\} \\ +h_{1,0}\left\{\left(\dfrac{1}{2}\right)E_{0P}\lambda_0^{\ 2} - \sum\limits_{i=1}^{L}\left(E_{iP}\lambda_i\right)\left[\sum\limits_{j=1}^{i}\left(\lambda_j\right)\right] + \sum\limits_{i=1}^{L}\left(\lambda_iE_{iP}\right)\cdot\left[\sum\limits_{i=1}^{L}\left(\lambda_i\right)\right]\right\} \\ +\sum\limits_{i=1}^{L}\left\{\dfrac{h_{1,i}E_{2i}\lambda_i^{\ 2}}{2} + h_{2,i}\left[\dfrac{E\left[x_i\right]^2\lambda_i^{\ 2}}{2P_{2,i}}\right] + \dfrac{h_{3,i}\lambda_i^{\ 2}E_{iP}}{2} + \dfrac{\left(h_{3,i}-h_{1,i}\right)\lambda_i\left(1-\lambda_iE_{iP}\right)}{2n}\right\} \end{cases} = 0 \tag{35}$$

Solving the linear system of Eq. (34) and Eq. (35), the optimal production-shipment (T_A^*, n^*) policy is determined as follows:

$$T_A^* = \sqrt{\dfrac{2\left[K_0\left(1+\alpha_{2,0}\right)+\sum_{i=1}^{L}\left(nK_{D,i}+K_i\right)\right]}{\begin{aligned}&h_{2,0}E\left[x_0\right]^2\lambda_0^2\left[\dfrac{1}{P_{2,0}\left(1+\alpha_{1,0}\right)}\right]+h_{1,0}\sum_{i=1}^{L}\left(\dfrac{\lambda_i^2}{P_{1,i}}\right)\\&+h_{1,0}\left\{E_{0P}\lambda_0^2-2\sum_{i=1}^{L}\left(E_{iP}\lambda_i\right)\left[\sum_{j=1}^{i}\left(\lambda_j\right)\right]+2\sum_{i=1}^{L}\left(E_{iP}\lambda_i\right)\cdot\left[\sum_{i=1}^{L}\left(\lambda_i\right)\right]\right\}\\&+\sum_{i=1}^{L}\left\{h_{1,i}E_{2i}\lambda_i^2+\dfrac{h_{2,i}E\left[x_i\right]^2\lambda_i^2}{P_{2,i}}+h_{3,i}E_{iP}\lambda_i^2+\dfrac{\left(1-\lambda_iE_{iP}\right)\lambda_i\left(h_{3,i}-h_{1,i}\right)}{n}\right\}\end{aligned}}}$$

(36)

and

$$n^* = \sqrt{\dfrac{\left[K_0\left(1+\alpha_{2,0}\right)+\sum_{i=1}^{L}\left(K_i\right)\right]\cdot\sum_{i=1}^{L}\left[\left(1-\lambda_iE_{iP}\right)\lambda_i\left(h_{3,i}-h_{1,i}\right)\right]}{\left(\sum_{i=1}^{L}K_{D,i}\right)\left\{\begin{aligned}&h_{2,0}\lambda_0^2E\left[x_0\right]^2\left[\dfrac{1}{\left(1+\alpha_{1,0}\right)P_{2,0}}\right]+h_{1,0}\sum_{i=1}^{L}\left(\dfrac{1}{P_{1,i}}\right)\lambda_i^2\\&+h_{1,0}\left\{\lambda_0^2E_{0P}-2\sum_{i=1}^{L}\left(\lambda_iE_{iP}\right)\left[\sum_{j=1}^{i}\left(\lambda_j\right)\right]+2\left[\sum_{i=1}^{L}\left(\lambda_i\right)\right]\cdot\sum_{i=1}^{L}\left(E_{iP}\lambda_i\right)\right\}\\&+\sum_{i=1}^{L}\left\{h_{1,i}E_{2i}\lambda_i^2+\dfrac{h_{2,i}E\left[x_i\right]^2\lambda_i^2}{P_{2,i}}+h_{3,i}E_{iP}\lambda_i^2\right\}\end{aligned}\right\}}}$$

(37)

4. Numerical illustration

A simulated numerical example to utilized to exhibit the capability and applicability of our study's results. Consider a multiproduct vendor-buyer coordinated system with the postponement, overtime alternative, multi-shipment, and rework is employed to meet the needs of buyer-required five distinct items. The assumption of relevant system variables are exhibited in Tables 1 and 2. In comparison, Table B-1 (in Appendix B) displays the assumed values of corresponding parameters in a single-stage scheme.

Table 1
The assumed values of stage one's relevant variables

δ	$P_{1,0}$	λ_0	$C_{R,0}$	γ	$h_{1,0}$	$\alpha_{1,0}$	$\alpha_{3,0}$
0.5	120000	17000	$25	0.5	$8	0.5	0.25

C_0	K_0	$P_{2,0}$	x_0	$h_{2,0}$	i_0	$\alpha_{2,0}$	
$40	$8500	96000	2.5%	$8	0.2	0.1	

Table 2
The assumed values of stage two's relevant variables

Product i	$K_{D,i}$	C_i	$P_{1,i}$	x_i	$C_{D,i}$	$C_{R,i}$	λ_i	$P_{2,i}$	$h_{3,i}$	K_i	$h_{1,i}$	$h_{2,i}$
1	$1800	$40	112258	2.5%	$0.1	$25	3000	89806	$70	$8500	$16	$16
2	$1900	$50	116066	7.5%	$0.2	$30	3200	92852	$75	$9000	$18	$18
3	$2000	$60	120000	12.5%	$0.3	$35	3400	96000	$80	$9500	$20	$20
4	$2100	$70	124068	17.5%	$0.4	$40	3600	99254	$85	$10000	$22	$22
5	$2200	$80	128276	22.5%	$0.5	$45	3800	102621	$90	$10500	$24	$24

Apply equations (36), (37), and (32) to determine $T_A^* = 0.5379$, $n^* = 4$, and $E[TCU(T_A^*, n^*)] = \$2,305,879$. The behavior of $E[TCU(T_A, n)]$ regarding T_A and n is illustrated in Fig. 5. It shows the convexity of $E[TCU(T_A, n)]$. As both n and T_A deviate from their corresponding optimal points, $E[TCU(T_A, n)]$ knowingly surges.
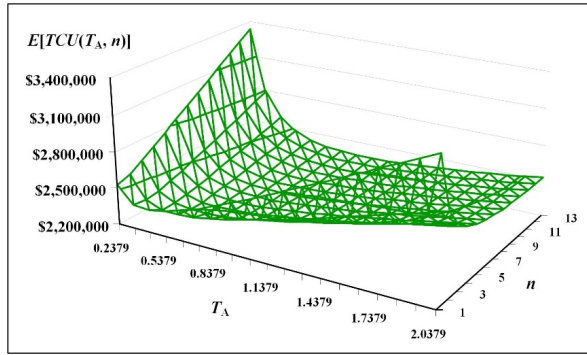
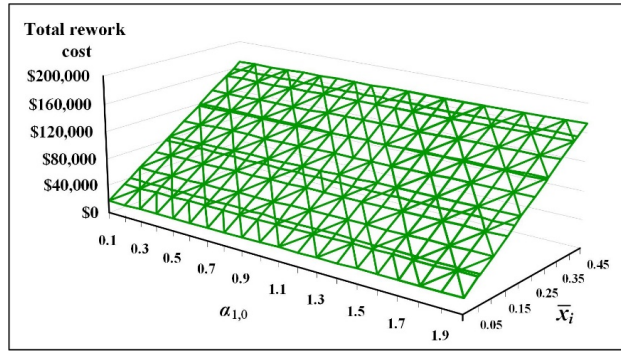**Fig. 5.** The behavior of $E[TCU(T_A, n)]$ regarding to T_A and n

Fig. 6. Combined effect of $\alpha_{1,0}$ and the mean x_i on the total rework cost

4.1. Combined effect of crucial system features

The combined impact of the overtime added-output factor $\alpha_{1,0}$ and average defective proportion x_i on the total rework cost is demonstrated in Fig.6. As the mean x_i increases, the total rework cost drastically upsurges; and as $\alpha_{1,0}$ increases, the total rework cost rises slightly. Fig. 7 exhibits the combined influence of the mean rework cost ratio $(C_{R,i} / C_i)$ and the overtime factor $\alpha_{1,0}$ on $E[TCU(T_A^*, n^*)]$. It reveals that as the mean $(C_{R,i} / C_i)$ ratio increases, $E[TCU(T_A^*, n^*)]$ rises knowingly; and as $\alpha_{1,0}$ rises, $E[TCU(T_A^*, n^*)]$ surges significantly. Hence, $\alpha_{1,0}$ has more effect on $E[TCU(T_A^*, n^*)]$ than the mean $(C_{R,i} / C_i)$ ratio.
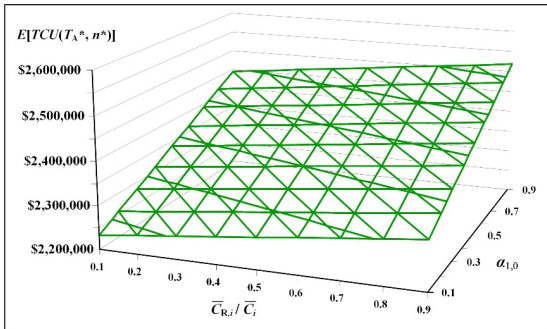
Fig. 7. Behavior of $E[TCU(T_A^*, n^*)]$ vis-à-vis the average $(C_{R,i} / C_i)$ ratio and $\alpha_{1,0}$
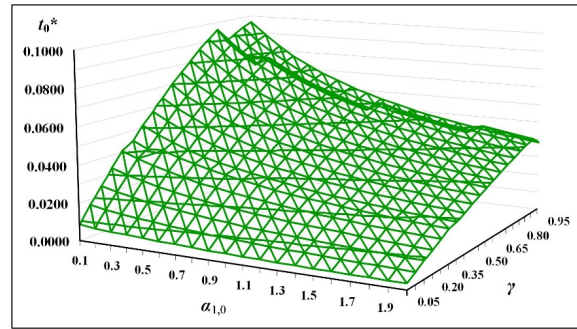

Fig. 8. Joint impact of $\alpha_{1,0}$ and γ on t_0^*

Fig. 8 illustrates the joint effect of the overtime factor $\alpha_{1,0}$ and γ on t_0^*. As $\alpha_{1,0}$ rises, t_0^* noticeably declines. In contrast, as γ increases, t_0^* upsurges radically. It also exhibits that the changing γ value between 0.8 and 0.95 has caused t_0^* to have irregular changes. The collective impact of the overtime relevant factors of unit and setup costs $\alpha_{3,0}$ and $\alpha_{2,0}$ on $E[TCU(T_A^*, n^*)]$ is demonstrated in Fig. 9. As the overtime unit cost factor $\alpha_{3,0}$ rises, $E[TCU(T_A^*, n^*)]$ severely surges, and as the overtime setup cost factor $\alpha_{2,0}$ goes up, and $E[TCU(T_A^*, n^*)]$ slightly rises. That is $\alpha_{3,0}$ has more effect on $E[TCU(T_A^*, n^*)]$ than $\alpha_{2,0}$.
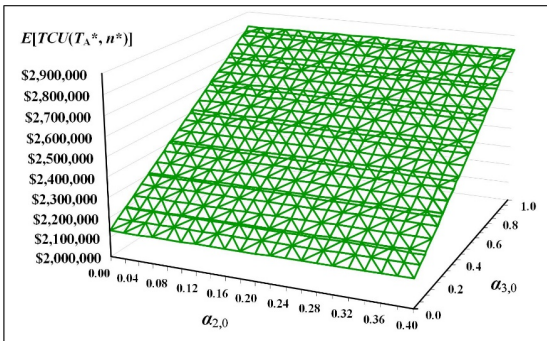

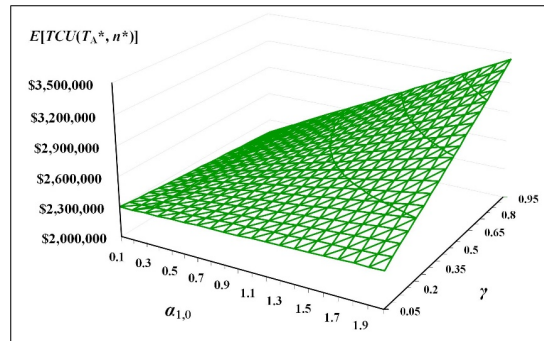**Fig. 9.** Behavior of $E[TCU(T_A^*, n^*)]$ regarding $\alpha_{2,0}$ and $\alpha_{3,0}$

Fig. 10. Combined impact of $\alpha_{1,0}$ and γ on $E[TCU(T_A^*, n^*)]$

Fig. 10 illustrates the combined effect of the overtime proportion $\alpha_{1,0}$ and γ on $E[TCU(T_A^*, n^*)]$. As both $\alpha_{1,0}$ and γ rise, $E[TCU(T_A^*, n^*)]$ increases considerably.

4.2. The influence of crucial system features

Our proposed two-stage multiproduct postponement model with multi-shipment, rework, and overtime can explore the impact of its crucial features on the problem. First of all, Fig. 11 and Fig. 12 illustrate the impact of dissimilar relationships of common component's completion rates γ and its relating values δ on $E[TCU(T_A{}^*, n^*)]$ and $T_A{}^*$, respectively. For a linear relationship between δ and γ, the analytical outcome confirms that at $\gamma = 0.5$, $E[TCU(T_A{}^*, n^*)] = \$2,305,879$ (see Fig. 11) and $T_A{}^* = 0.5379$ (refer to Fig. 12). Our model can reveal the optimal operating policy $(T_A{}^*, n^*)$ and system cost $E[TCU(T_A{}^*, n^*)]$ for any given nonlinear relationships as may exist in real application systems. For instances, when $\delta = \gamma^3$ and γ remains at 0.5, the analytical results show $E[TCU(T_A{}^*, n^*)] = \$2,237,446$ (refer to Fig. 11) and $T_A{}^* = 0.6654$ (see Fig. 12).
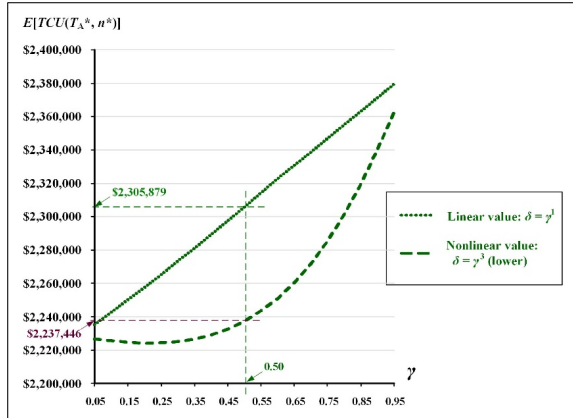

Fig. 11. Behavior of $E[TCU(T_A{}^*, n^*)]$ regarding dissimilar relationships of δ and γ
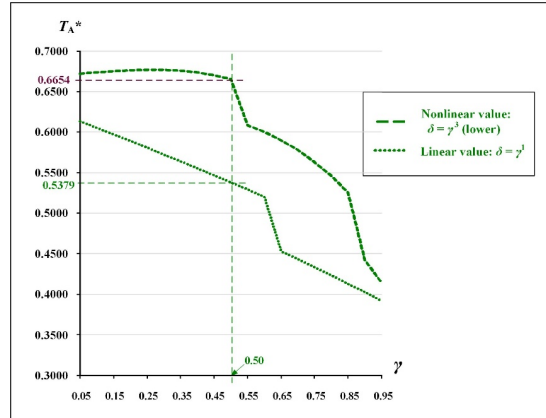
Fig. 12. The impact on $T_A{}^*$ from different relationships between δ and γ

Fig. 13 reveals the overtime $(P_{T1,0} / P_{1,0})$ ratio effect on $(t_0{}^* + t_i{}^*)$. It exposes $(t^*{}_{1,0} + t^*{}_{2,0})$ drops to 0.1336 at $(P_{T1,0} / P_{1,0}) = 1.50$. That is for a 50% added output quantity due to the overtime alternative.
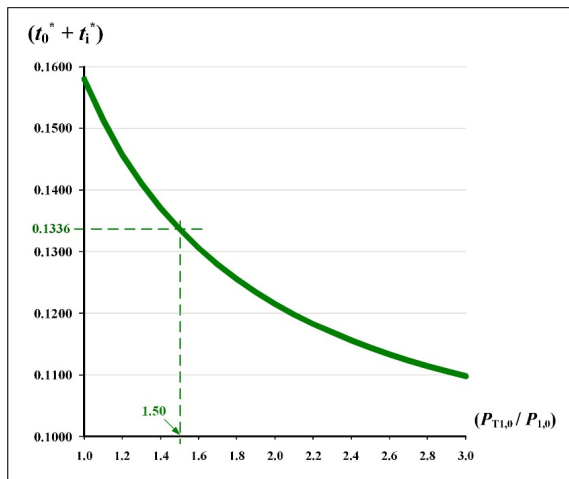

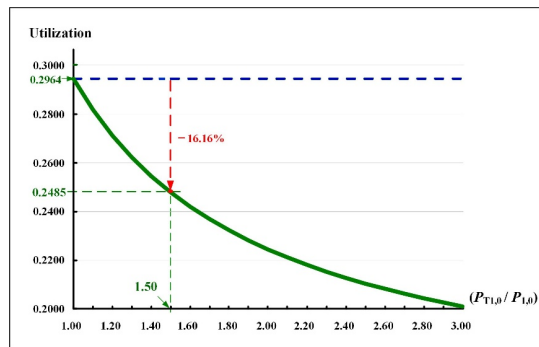**Fig. 13.** The effect of changes in $(P_{T1,0} / P_{1,0})$ on $(t^*{}_{1,0} + t^*{}_{2,0})$.

Fig. 14. Impact of $(P_{T1,0} / P_{1,0})$ on utilization

Further analysis (as illustrated in Fig. 14) shows the overtime ratio's impact on utilization. For $(P_{T1,0} / P_{1,0}) = 1.5$, the system's utilization drops to 0.2485 from 0.2964 (the no overtime alternative case), or a 16.16% decline in utilization.

Fig. 15 demonstrates the overtime ratio influence on major cost contributors in $E[TCU(T_A{}^*, n^*)]$. It reconfirms at $(P_{T1,0} /P_{1,0}) = 1.5$, $E[TCU(T_A{}^*, n^*)] = \$2,305,879$; an increase of 8.02% from \$2,134,736 (without overtime implementation). As $(P_{T1,0} /P_{1,0})$ increases, the most impact component is the overtime cost for producing mutual parts.
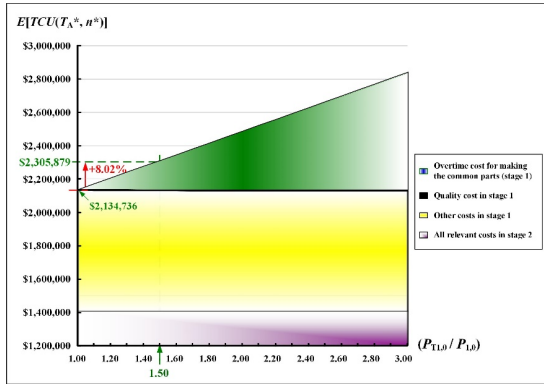
Fig. 15. The impact of $(P_{T1,0}/P_{1,0})$ on key cost contributors in $E[TCU(T_A*, n*)]$
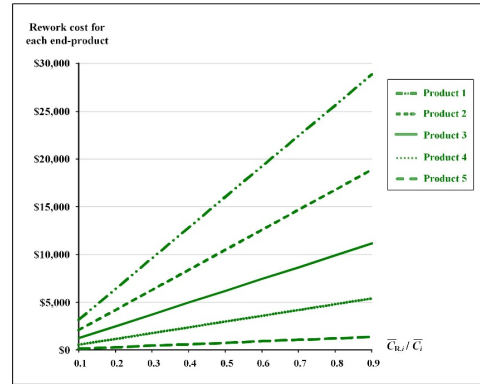

Fig. 16. The average $(C_{R,i}/C_i)$ effect on the rework cost for each end product

Our model can explicitly study additional system characteristics. Fig. 16 exposes the mean rework cost ratio $(C_{R,i}/C_i)$ effect on each end product's rework cost. As the mean $(C_{R,i}/C_i)$ ratio rises, each end product's rework cost significantly surges. Fig. 17 discloses the impact of the number of shipments n on key cost contributors in $E[TCU(T_A*, n*)]$. It reconfirms at $n* = 4$, $E[TCU(T_A*, n*)] = \$2,305,879$. As n increases, (i) buy's holding cost surges due to the number of stocks per shipment D_i decreases (see Eq. (10)); (ii) the total delivery cost surges, because the number of fixed delivery cost rises; and (iii) in contrast, the vendor's holding cost rises (see Eq. (8)).
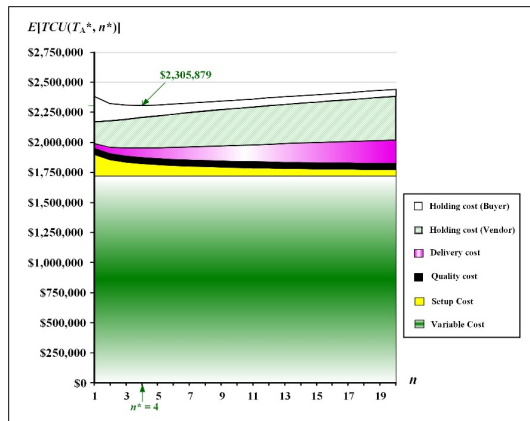

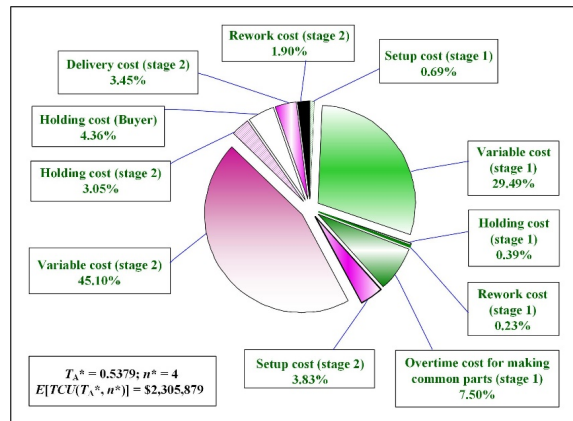**Fig. 17.** The delivery frequency n on key cost contributors in $E[TCU(T_A*, n*)]$

Fig. 18. Breakup of $E[TCU(T_A*, n*)]$

Fig. 18 explicitly illustrates the breakup of detailed cost contributors to $E[TCU(T_A*, n*)]$. It shows two main cost contributors: the variable expenses in stages 2 and 1; each contributes 45.10% and 29.49%. The overtime relating expense of 7.50% is the third large expense. It follows that the buyer's holding cost 4.36%, the end products' setup and delivery cost 3.83% and 3.45%. Then, the end products' holding and rework cost 3.05% and 1.90%. Finally, we compare our utilization with an existing model without overtime option (Chiu et al., 2016), as exhibited in Fig. 19. A further investigation, indicating that by reducing a 16.16% in utilization (declining from 0.2964 to 0.2485). However, the price paying is a 8.02% increase in $E[TCU(T_A*, n*)]$ (surging from \$2,134,736 to \$2,305,879, refer to Fig. 15).
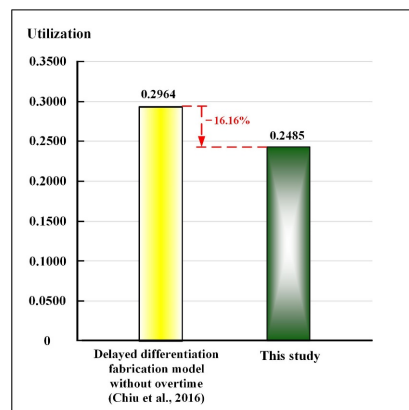

Fig. 19. Comparison of this study's utilization with an existing work without overtime

5. Conclusions

In the current stiff competitive supply-chain environment, customers' product demand trend turns to diversity, quality, and rapid response. By considering the existence of products' commonality, required quality, and short lead time, this study proposes a model featuring postponement, overtime fabricating the mutual parts, rework of defects, and multiple deliveries to assist today's producers in their multiproduct fabrication-distribution planning. Techniques of explicitly modeling, formulating, and system cost minimization were used to simultaneously derive the optimal cycle-time and shipping frequency for the studied problem. The obtaining results can be applied to a variety of today's industries (e.g., automotive, household goods, etc.) in revealing in-depth managerial insights for planning fabrication-shipment policy in their supply chains. As a demonstrating example, this model discloses the following important managerial information:

(1) The optimal fabrication-transportation policy (see Fig. 5).
(2) Joint impact of various system factors (such as overtime add-up rate, defect portion, mean rework-cost ratio, mutual part's completion rate, and overtime set-up cost) on the rework cost, total system cost, and mutual parts' uptime plus rework time. Refer to subsection 4.1., Fig. 6 to Fig. 10.
(3) The impact of dissimilar relationships of mutual component's completion rate and its corresponding values on total system cost and cycle-time (see Fig. 11 to Fig. 12).
(4) The effect of overtime-related ratios on the sum of optimal uptime and rework time, utilization, and each cost contributor (Fig. 13 to Fig. 15).
(5) The influence of variations in average rework-cost ratio and delivery frequency on the rework cost of each end product, and total system cost and separate cost contributor (see Figs. 16-17).
(6) The breakup of detailed cost contributors and a comparison of this model's utilization against a closely related previous model (refer to Fig. 18 to Fig. 19).

In summary, the significant contributions of this study are: (1) it presents a multiproduct postponement model to analyze a real fabrication-distribution problem featuring overtime and rework explicitly; and (2) it discloses diverse, in-depth system information to facilitate managerial decision makings. In addition, considering variable end products' demand rates for the studied problem is worth investigating in the future study.

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Appendix - A

The detailed derivation of Eq. (32) is as follows:

By substituting Eqs. (1) to (30) in Eq. (31), and utilizing the expected values $E[x_0]$ and $E[x_i]$ to cope with the random nonconforming proportions to compute $E[TC(T_A, n)] / E[T_A]$, we obtain $E[TCU(T_A, n)]$ as follows:

$$
\begin{aligned}
E\left[TCU\left(T_A, n\right)\right] &= \frac{E\left[TC\left(T_A, n\right)\right]}{E\left[T_A\right]} = \lambda_0\left(1+\alpha_{3,0}\right)C_0 + \frac{K_0\left(1+\alpha_{2,0}\right)}{T_A} + \lambda_0\left(1+\alpha_{3,0}\right)C_{R,0}E\left[x_0\right] \\
&+ h_{2,0}\lambda_0{}^2 T_A\left[\frac{E\left[x_0\right]^2}{2P_{2,0}\left(1+\alpha_{1,0}\right)}\right] + h_{1,0}\sum_{i=1}^{L}\left(\frac{\lambda_i^2 T_A}{2P_{1,i}}\right) \\
&+ h_{1,0}\left\{\begin{array}{l}
\left(\frac{1}{2}\right)T_A\lambda_0{}^2\left\{\left[\frac{1}{P_{1,0}\left(1+\alpha_{1,0}\right)}\right]+\left[\frac{\left[2-E\left[x_0\right]\right]E\left[x_0\right]}{P_{2,0}\left(1+\alpha_{1,0}\right)}\right]\right\} \\
+\sum_{i=1}^{L}\left\{-T_A\lambda_i\left[\frac{1}{P_{1,i}}+\frac{E\left[x_i\right]}{P_{2,i}}\right]\right\}\left[\sum_{j=1}^{i}\left(\lambda_j\right)\right]+\left[\sum_{i=1}^{L}\left(\lambda_i\right)\right]\cdot\sum_{i=1}^{L}\left[\lambda_i T_A\right]\left[\frac{1}{P_{1,i}}+\frac{E\left[x_i\right]}{P_{2,i}}\right]
\end{array}\right\} \\
&+ \sum_{i=1}^{L}\left\{\begin{array}{l}
\frac{K_i}{T_A}+C_i\lambda_i+C_{R,i}\lambda_i E\left[x_i\right]+\frac{nK_{D,i}}{T_A}+C_{D,i}\lambda_i+\frac{h_{1,i}\lambda_i^2 T_A}{2}\left[\frac{1}{\lambda_i}+\frac{\left(1-E\left[x_i\right]\right)E\left[x_i\right]}{P_{2,i}}\right] \\
+h_{2,i}\left[\frac{E\left[x_i\right]^2 T_A\lambda_i^2}{2P_{2,i}}\right]+\frac{h_{3,i}T_A\lambda_i^2}{2}\left[\frac{1}{P_{1,i}}+\frac{E\left[x_i\right]}{P_{2,i}}\right]+\frac{\left(h_{3,i}-h_{1,i}\right)T_A\lambda_i^2}{2n}\left[\frac{1}{\lambda_i}-\frac{1}{P_{1,i}}-\frac{E\left[x_i\right]}{P_{2,i}}\right]
\end{array}\right\}
\end{aligned}
\tag{A-1}
$$

Let E_{0P}, E_{2i}, and E_{iP} represents:

$$
\begin{aligned}
E_{0P} &= \left[\frac{1}{P_{1,0}\left(1+\alpha_{1,0}\right)}\right]+\left[\frac{\left[2-E\left[x_0\right]\right]E\left[x_0\right]}{\left(1+\alpha_{1,0}\right)P_{2,0}}\right]; \\
E_{2i} &= \left[\frac{1}{\lambda_i}+\frac{\left(1-E\left[x_i\right]\right)E\left[x_i\right]}{P_{2,i}}\right]; \quad E_{iP}=\left[\frac{1}{P_{1,i}}+\frac{E\left[x_i\right]}{P_{2,i}}\right] \quad for \ \ i=1, 2, \ldots, L.
\end{aligned}
\tag{A-2}
$$

Substitute Eq. (A-2) in Eq. (A-1), we obtain Eq. (32).

Appendix - B

Table B-1
Assumed values of equivalent parameters in a single-stage fabrication

Product i	C_i	λ_i	$h_{1,i}$	$P_{1,i}$	$h_{2,i}$	x_i	$P_{2,i}$	$C_{R,i}$	K_i	$K_{D,i}$	$C_{D,i}$	$h_{3,i}$
1	$80	3000	$16	58000	$16	5%	46400	$50	$17000	$1800	$0.1	$70
2	$90	3200	$18	59000	$18	10%	47200	$55	$17500	$1900	$0.2	$75
3	$100	3400	$20	60000	$20	15%	48000	$60	$18000	$2000	$0.3	$80
4	$110	3600	$22	61000	$22	20%	48800	$65	$18500	$2100	$0.4	$85
5	$120	3800	$24	62000	$24	25%	49600	$70	$19000	$2200	$0.5	$90