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Contracts design for serial delivery with connecting time spot: From a perspective of fourth party logistics

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A B S T R A C T

For a serial delivery system, the latter 3PL needs to be prepared at the transshipment node in advance to reduce the total delivery time. In this paper, we propose the concept of Connecting Time Spot (CTS) to help 4PL schedule the latter 3PL when to wait at the transshipment node. We study a serial delivery system with a 4PL and two 3PLs, where 4PL designs optimal contracts with two types of CTS (GCTS is derived by system parameter and DCTS is determined by 4PL's optimization) to induce 3PLs to exert the optimal effort levels. We analyze the effects of CTS on the system profit in the centralized system. For the decentralized system, we particularly investigate the optimal contracts in three penalty modes which are according to the occupancy of the warehouses. The results show that CTS can avoid 3PLs' idle resources and enhance the system profit for serial delivery both in the centralized system and the decentralized system. Compared with GCTS, DCTS has a better performance in enhancing the system profits. Also, the optimal incentive contracts achieve Pareto improvement for system profits. Interestingly, one 3PL's delivery penalty mode will not affect the other 3PL's optimal contracts.

Notation	Definition
i	Index of 3PL i=1,2
c_i	3PLi's effort cost
k_i	3PLi's cost coefficient, $k_i > 0$
p	Penalty coefficient from client, $p > 0$
t_i	3PLi's real delivery time
a_i	3PLi's general delivery time, $a_i > 0$
b_i	3PLi's output coefficient, $b_i > 0$
m_i	3PLi's random factor
w	3PL2's waiting cost coefficient, $w > 0$
e_i	3PLi's effort level
I_i	3PLi's incentive contracts
π_i	3PLi's profit
α_i	3PLi's fixed payment, $\alpha_i > 0$
β_{i1}	3PLi's tardiness penalty
β_{i2}	3PLi's earliness penalty
R_i	{3PLi's reservation utility, $R_i > 0$

Abbreviation	Definition
NCTS	Without connecting time spot
GCTS	Given connecting time spot
DCTS	Decision connecting time spot
C	The centralized system
D	The decentralized system
CN	Case without CTS in centralized system
CG	Case with GCTS in centralized system
CD	Case with DCTS in centralized system
DG	Case with GCTS in decentralized system
DD	Case with DCTS in decentralized system

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1. Introduction

Over the last decades, logistics has played an indispensable part in enterprises and businesses. With the development of economic globalization, logistics globalization gradually becomes popular (Gattorna, 1998; Clinton, 2011; Lobel & Xiao, 2017). As the shipment quantities grow rapidly, the structures of the logistics networks become increasingly complex (Vivaldini, Pires & Souza, 2008). Due to the high building and operating costs of a logistics network, some modern enterprises are willing to take logistics outsourcing services. A lot of companies need to find a third-party logistics provider (referred to as 3PL) to deliver their goods, which is called logistics outsourcing. Logistics outsourcing service enables them to pay more attention to enhancing the core-competences (Huang et al., 2013; Kumar, 2008; Hoek & Chong 2001). In such settings, the company always needs to balance the advantages of outsourcing against agency problems (Jain et al., 2013; Adcroft et al., 2007).

Due to limited information, a 4PL can hardly monitor the 3PLs logistics process all the time, resulting in the moral hazard issue between the 4PL and 3PLs. The moral hazard in outsourcing may result in an efficiency loss of the service supply chain and the loss of the company's profits. As is known to all, measuring the operational performance of a logistics outsourcing service can be challengeable (Holmstrom, 1982; Bhattacharyya & LaFontaine 1995).

As global network and information technologies blossom in the past decades, the importance of the application in logistics management is rising constantly. To integrate the 3PLs and offer further management to achieve better outsourcing performance, fourth-party logistics (referred to as 4PL) becomes increasingly popular in modern logistics supply chain management. First raised by Accenture in 1996, 4PL is defined as the integrator of the supply chain, who can provide a serial of integrated solutions by taking the best use of the capacities, resources and technologies in the supply chain (Tsai et al., 2008; Badem & Mueller, 1999). 4PL has more detailed and comprehensive information and would help those companies managing their outsourcing transportation in order to get an economical and satisfactory operation (Logan, 2000). For these reasons, 4PL is enabled to offer more professional and advanced outsourcing services than a single 3PL (Craig, 2003; Vaidyanathan, 2005). In China, Cainiao Network is a rising 4PL that is subordinate to Alibaba Group. Its smart logistics network was established by several Chinese famous logistics businesses and is aiming at enhancing logistics performance and improving customer experience by intelligent techniques.

For outsourcing logistics, many uncertain factors may influence the overall delivery quality, such as bad weather, wrong loading and traffic accidents (Kengpol & Tuammee, 2016; Shang et al., 2017; Liu et al., 2015; Liu et al., 2018; Chen & Chen, 2017). The uncertain factors can lead to low delivery quality or high cost of both 3PL and clients. Therefore, it is the major objective for 4PL to enhance the efficiency of outsourcing logistics operations and control total operation costs. However, most of the existing research focuses on serial delivery assumes that 3PLs' handover cost nothing, or take the time cost as the handling cost of the transportation location. A reasonable connection schedule for serial delivery is necessary to achieve high operational efficiency. The location of the vehicles can be monitored in real-time while technology is well-developed than ever before. Getting prepared of vehicle schedules in advance also contributes to promoting the utilization of transportation resources. In this case, 3PL will take an advantageous position in arranging logistics outsourcing tasks.

Our aim is to optimize the serial delivery connection by designing incentive contracts based on the serial delivery structure from the perspective of 4PL. A new concept called "Connecting Time Spot" (referred to as CTS) is proposed to characterize 3PLs' connection in serial delivery. The latter 3PL are told to arrive at the transshipment nodes at CTS to avoid being affected by the uncertain arrival of the former 3PL and to make an optimal arrangement of its transportation capability. By comparing the optimal solutions, this paper explores how CTS affects the decisions of 4PL and 3PLs as well as the profit of a serial delivery logistics system. We study the delivery contract design problem from the perspective of 4PL and compare the effectiveness of the optimal contracts with CTS against the outcome in the centralized system without CTS. Especially, we try to answer the following questions:

(1) How does CTS characterize the connection in a serial delivery process?
(2) How does the delivery system influence CTS? How does the CTS influence the behaviors of 4PL or 3PLs under the centralized system and the decentralized system?
(3) What kind of contract should be offered to 3PLs by 4PL? What effects will the optimal contracts with CTS do on the serial delivery?

To answer these questions, we focus on centralized decisions and decentralized decisions. First, we analyze the centralized decisions with No CTS (referred to as NCTS), Given CTS (referred to as GCTS) and Decision CTS (referred to as DCTS). Then, we derive different optimal incentive contracts from 4PL to 3PLs for the cases with the two kinds of CTS respectively in decentralized decisions. Subsequently, we analyze the game by using the Principal-Agent model. In the upper model, 4PL offers contracts to the 3PLs and maximizes her profit. In the lower model, two 3PLs determine the effort level to maximize their profits by giving contracts. The 3PLs accept the contracts as only the profits not less than their reservation utility respectively.

Our analysis shows that the 4PL and two 3PLs can achieve the optimal system profits under specific parameters ranges in the decentralized system by comparing with the centralized cases with the same CTS. The system profits are not always optimal in the decentralized system. Yet, 4PL can induce 3PLs to choose the optimal effort level by setting the parameters according to the optimal incentive contracts. In this regard, we find that the optimal system profit with DCTS is always greater than with GCTS. Although all enhanced profits are drawn by 4PL, CTS can indeed benefit from the serial delivery.

The rest of the paper is organized as follows. Some related literature reviewed is provided in Section 2. In Section 3, we describe the general models of serial delivery. In Section 4 and Section 5, we characterize the decentralized system and analyze the findings. We summarize the main contributions of this paper and future opportunities in Section 6.

2. Literature Review

Our research is related to the studies of outsourcing management and the studies of contract design for supply chain. In what follows, we will position our work and show the difference between our work and the extant literature.

2.1. Outsourcing Management for Service Supply Chain

Outsourcing has been a popular issue in supply chain management for decades. It is common sense that the supplier will have a shorter lead time with a higher cost or a longer lead time with a lower cost. The trade-off to select the optimal supplier with respect to logistics service quality is important to both companies and clients (Co et al., 2012). Bernstein and de Vericourt (2008) showed that procurement contracts with service guarantees constraints contribute to the outsourcing when facing competitive service providers. A reasonable pricing scheme in services outsourcing may increase profit, but harm social welfare (Lahiri et al., 2013). The competition of the supplier affects the optimal decision even if they requested a service at a certain price (Benjaafar et al., 2007). Jin and Ryan (2012) showed that single-sourcing is more effective than multi-sourcing when the buyer's cost increases with the number of suppliers.

There are numerous uncertain factors in the transportation process, such as severe weather, natural disasters, and heavy traffic jams. Both outsourcing logistics and self-supporting logistics can be affected by unpunctual lead time (Hertz & Alfredsson 2013). This kind of factor is called delivery time risk. Delivery time risk refers to the shipments not received as scheduled, including early delivery and tardy delivery (Fulconis 2007). Fulconis (2006) showed that delivery time and delivery quality are two of the most important things product managers will concern about delivery time. Gammelgaard et al. (2006) suggested that delivery time can be the main cause resulting in outsourcing contract termination. Selviaridis and Spring (2007) suggested that delivery time has a significant influence on the qualities of perishable products.

But the delivery can’t be monitored all the time. The unobservability gives some dishonest 3PLs a chance to cheat the client. For instance, 3PLs claim that they take a higher effort level than the actual reporting effort level in order to earn extra money according to the incentive contract. But the higher effort doesn’t perform well as the uncertain factors make the delivery time get longer than it should be. As a result, they will get a higher payment in terms of the incentive contract than telling the truth (Myerson & Roger, 1979). It is generally not easy for a company to get a whole picture of all 3PL. In other words, the companies possess limited information of all the 3PL appearing on the market. They can hardly find proper 3PLs to finish their transportation (Tsai et al., 2008). The deficiency becomes obvious when the logistics network and delivery process are complex. Incentive contracts are always competent to solve that kind of problem.

2.2. Contract Design for Logistics Supply Chain

Customer service is a central topic in service supply management. Incentive contract is always used by 4PL to take the supervision and management of 3PL (Bade & Mueller, 1999; Hertz & Alfredsson, 2003). In particular, the incentive contracts based on the theory of Principal and Agent are common for 4PL to force 3PLs to take some action that can benefit one certain supply chain. For the logistics supply chain, it remains a good way to ensure the quality of customer service. The capacity of the firm’s customer service in the delivery system can be important factors of logistics quality and operation cost. Bollapragada et al. (2004) examined the supply management optimization problem with quantity and timing uncertainty in a two-echelon service supply chain. Their study reveals the benefits of a guaranteed supply lead time. Jin and Tian (2012) formulate an optimization model to explore the logistics service in a supply chain associated with performance-based contracts. They provide theoretical insights into how logistics services drive the trade-off between reliability design and inventory level under such contracts. Logistics services even play an important role in significantly influencing customer demand. So (2000) conducted a pioneering study on the delivery service in a supply chain under competition. He finds that the monopoly case is significantly different from the competitive case. In particular, firms differentiate their delivery services under competition. Sharif et al. (2012) used a semi-fuzzy approach to examine the third-party logisticsservice provider in a supply chain. Hu and Qiang (2013) proposed a supply chain in which there are manufacturers, e-tailors, express service providers, and consumers. The quality of the logistics services is determined by the express service providers. They identify the equilibrium decisions for each party in such a supply chain. Yao and Zhang (2012) used both analytical and empirical approaches to examine logistics services in e-commerce. They derive several interesting implications for online retailers, further suggesting that such retailers either strategically set shipping and base prices, or provide menu pricing for different shipping options. Reverse selection, routing programing, operation integration, contract design and information platform design are popular issues involved in 4PL research. We focus on the incentive contract design based on the Principal-Agent model for serial delivery to optimize the efficiency of 3PLs connection and the operation cost.

2.3. Our Work

Since the literature review is based on outsourcing management of service supply chain and contract design for logistics supply chain, we find that the previous literature study on delivery systems can be classified into two groups. One group is for single 3PL or parallel 3PLs. The other group is for the routing programming for multi 3PLs. In most of the existing studies on serial delivery, it is assumed that there is no time cost of 3PLs handover in serial delivery, or just take the time cost as the handling cost of the transportation location. For routing programing, there are few researches that focus on serial delivery; they usually simply set a handling time for the connection. Generally, they assume that the next 3PL always makes an instant connection with the former one. That is to say, they take it for granted that once 3PL1 arrives at city B (the transshipment node), the 3PL2 can leave for city C (Cui et al., 2013; Huang et al., 2019). However, the time 3PL2 waiting for the 3PL1 can be considerable. The workers and the vehicles are working during the loading or unloading of the shipments. But the transportation capacities are sitting idle while waiting for the sequent 3PL2, which is needless and can be cut down by smart scheduling. A reasonable connection schedule for 3PLs connection is necessary for 4PL to optimize the outsourcing operations under serial delivery.

There is no literature aimed at the 3PL connection to programing the delivery. But the idle resources of 3PL during waiting for the subsequent 3PL can be high if the delivery is urgent or the resources is costly, for example the transportation valuables and perishable products (Iijima et al., 1996; Kant & Pal 2017; Ali et al., 2013; Rezaei-Malek et al., 2016). Even for the case not in the logistics supply chain, taking full use of the resources at hand is helpful in improving the performance of the operation system. Mak et al. (2014) showed that a compact schedule for a high-cost operating theater in the healthcare system enhances the customer satisfaction and reduces the operation cost. The main difference between this paper and previous related studies is that we characterize the connection of 3PLs in serial delivery. CTS is presented to seek for an effective appointed schedule. Then, we explore the effects of CTS on the decisions of the members both in centralized and decentralized delivery systems. Thus, this paper makes a contribution to designing contracts for serial delivery processes in the logistic supply chain by introducing a new concept of CTS.

3. Model

We consider the simplest structure of a serial delivery system for a client who wants to outsource one unit of inseparable goods to two third-party logistics (3PL) by an incentive contract designed by fourth-party logistics (4PL). It is a two-stage logistics task. The first stage is from city A to city B by 3PL1 and the other is from city B to city C by 3PL2. As requested by the client, the goods should arrive in city C as soon as possible. In other words, the 4PL receives a quantitative penalty since the goods begin to be transported from city A. At the end of the task, 3PLs will pay partial penalty to 4PL in terms of the incentive contract, and 4PL pay the entire penalty to the client in accordance with the delivery time. Especially, when 3PL2 only owns a small chunk of 3PL1s information, it will be hard for 3PL2 to forecast when 3PL1 will arrive at city B. 3PL2 can obtain inaccurate predictions of the arrival time. Meanwhile, 3PL2 can be struggling when he should wait for 3PL1 at city B. Both he and 3PL1 will be punished for delaying the total delivery time if he arrives too late. On the contrary, he will take the risk of facing a waiting cost of leaving his transportation capacities (for example, the workers and the carriers) unused if he arrives too early.

3PL1 will be punished for delaying the total delivery time if he arrives too late. On the contrary, he will take the risk of facing a waiting cost of leaving his transportation capacities (for example, the workers and the carriers) unused if he arrives too early.

We assume that 3PL has a well-developed logistics network. For one certain delivery task, the delivery time of 3PLi is given by $t_i = a_i - b_i e_i + \varepsilon_i$ for $i = 1,2$. As illustrated in Fig. 1, 3PLi's general delivery time $a_i > 0$ for $i = 1,2$ is determined by his current logistics network (marked in blue). 3PLi can make extra effort, which is called effort level $e_i \in [0,1]$ for $i = 1,2$ to shorten the delivery time by increasing the transport vehicle speed, paying overtime wage to extending working hours or other methods. 3PLi pays no extra effort to complete the logistics task, when $e_i = 0$ for $i = 1,2$. 3PLi pays the most extra effort he can, when $e_i = 1$ for $i = 1,2$. The delivery time reduction due to the effort is marked in purple, where $b_i > 0$ for $i = 1,2$ is called output coefficient of 3PLi. The output coefficient features how efficient the effort level contributes to shorten the delivery time.

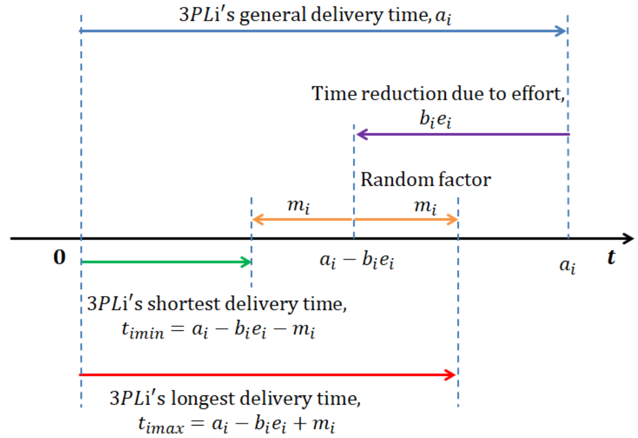


Fig. 1. 3PLi's delivery time

As mentioned, uncertain factors can affect the delivery time to longer or shorter. The influence of uncertain factors is characterized as a uniform random variable $\varepsilon_i \sim U\ [-mi,\ mi]$ for $i=1,2$ (marked in yellow). Then, we denote 3PLi's shortest delivery time $t_{i\min}$ for $i=1,2$ (marked in green) and longest delivery time $t_{i\max}$ for $i=1,2$ (marked in red). In most conditions, we are short of a real time monitoring or an overall inspection of the transportation. Moreover, even though the delivery time can be observed, it is still hard to identify how much effort the third-party logistics invest in the logistics task with these uncertain factors.

3.1. Connecting Time Spot

INSTANT CONNECTION. To be more specific, we define instant connection as an action that the latter 3PL can leave for the next shipping node immediately when the former 3PL arrived in transshipment node. The previous related studies always assume that 3PLs make an instant connection at the transshipment node. To achieve an instant connection, 3PL2 has to wait for 3PL1 at city B with whose transportation capacities in idle if without 4PL's optimization.

As Fig. 2 shows, the total delivery time of the two-stage logistics task is t_1+t_2. As the delivery times are uncertain, 3PL2's waiting time can be t_1 in the worst case when 3PL2 is totally unable to forecast 3PL1's delivery time. 3PL2's waiting cost is w and total waiting cost is wt_1. Therefore, 3PL2's total cost will be considerable if w is large enough.

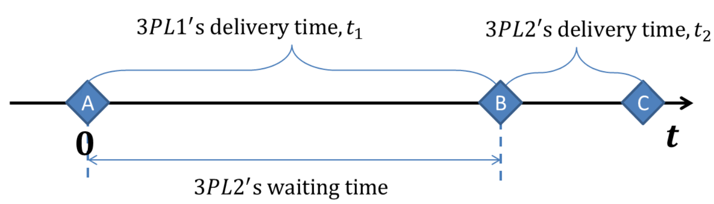


Fig. 2. Serial delivery with waiting cost and without 4PL's optimization

Actually, 3PL2 will arrive in advance, but he also won't start to wait from the beginning of the task. However, 3PL2 also won't wait in city C too late on account of the quantitative penalty paid to the client depending on the total delivery time. Under such circumstances, it is evident that the assumption of instant connection is not reasonable.

CONNECTING TIME SPOT. Therefore, connecting time spot (CTS) is proposed to describe the different connection type of the 3PL in serial delivery. CTS is a kind of timing that the latter 3PL in serial delivery was told to get ready to transport the goods at the transshipment node.

3.2. Three Scenarios in Serial Delivery

According to the different types of CTS, there are three scenarios for serial delivery including Scenario NCTS, Scenario GCTS and Scenario DCTS.

SCENARIO NCTS. As shown in Fig. 2, Scenario NCTS considers the waiting cost of 3PL2 and assumes that 3PL2 has no prediction of when will 3PL1arrive. So 3PL2 waits from the beginning of 3PL1 leaving for city B. In another word, 4PL doesn't set CTS, which is the worst case because 3PL2 has to wait for the longest time to avoid the effect of time uncertainty.

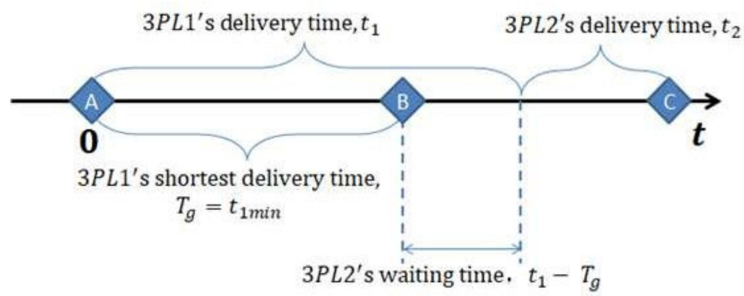


Fig. 3. Serial delivery with GCTS

SCENARIO GCTS. As illustrated in Fig. 3, Scenario GCTS considers a given connecting time spot that equals the shortest delivery time of 3PL1. It is obvious that 3PL1 will not arrive at city B before 3PL2. Although we know that 3PL2 must have to wait for a while, he still saves a waiting cost corresponding to 3PL1's shortest delivery time.

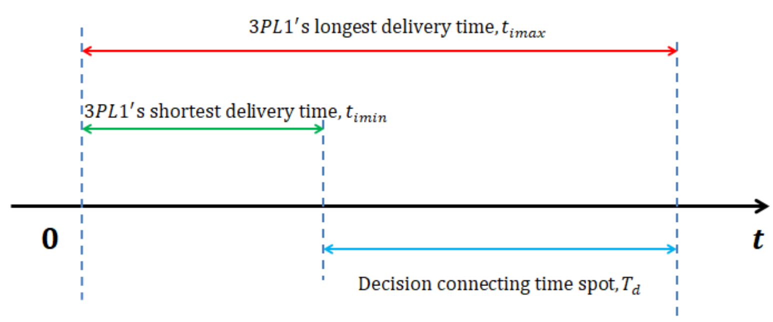


Fig. 4. Serial delivery with DCTS

SCENARIO DCTS. When it comes to Scenario DCTS, 4PL set a decision connecting time spot for 3PL2. In this case, connecting time spot is also a decision variable which is no longer directly depending on 3PL1s shortest time. On an intuitive level, the decision connecting time spot will not earlier than 3PL1s shortest delivery time. But shown in Fig. 4, it is hard to determine which 3PL will arrive at city B at first.

Fig. 5 captures 4 kinds of serial connection marked as 4 different colors. The solid points represent the 3PL with the products to be transported, while the hollow circles mean the 3PL has unloaded the products. The horizontal axis is labeled as some specific time spot. The vertical axis is divided into three parts. The highest part A is the location of City A, part B is the transshipment node City B, and the part C is the destination of the delivery task.

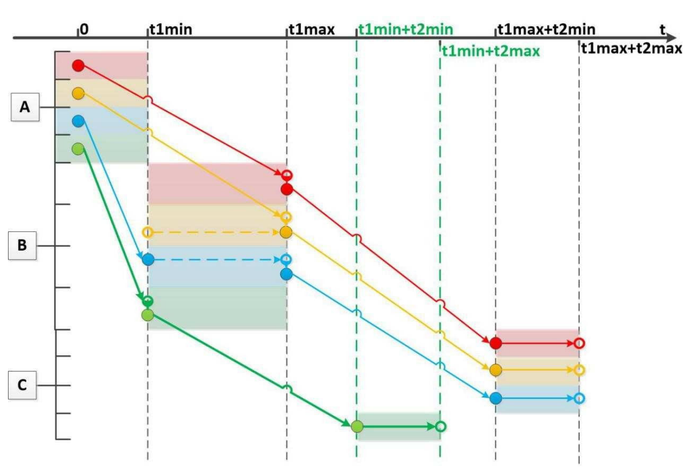


Fig. 5. Four kinds connection of serial 3PLs

The green one is an instant connection which is the best case, where 3PL1 arrives at City B in his shortest delivery time and 3PL2 arrives at the same time. Without any waiting cost, 3PLs achieve both the instant connection and earliest total delivery time.

The red delivery process is not too bad, where 3PL1 and 3PL2 arrive at City B simultaneously in 3PL1s maximal delivery time. The total delivery time is not perfect even if there is no waiting cost.

The blue one and yellow one can be the worst connection, where the 3PLs must suffer a longer total delivery time. If 3PL1 arrives at his maximal delivery time (marked as blue), 3PL2 has to wait for him to arrive early and then leave for City C.

In this condition, 3PL2 may face his largest waiting cost. If 3PL2 arrives in 3PL1s maximal delivery time (marked as yellow), the shipments have to wait for him even if 3PL1 makes a good effort to deliver them.

There is no related research mainly focused on the connection between the serial 3PLs in the management of logistics supply chain. Similar to the study in Mak et al. (2014), as transportation surges with the rapid economic growth, it is urgent to raise the efficiency of logistics transportation. To eliminate 3PL2s dilemma, we propose the concept of connecting time spots. It is for 4PL to tell 3PL2 in which time he should arrive at the transshipment node (city B).

Different from 3PLs, 4PL have all the observable information about the supply chain and the shareable information of supply chain members. The influence of 3PL1's uncertain delivery time on 3PL2's waiting cost can be greatly decreased if 4PL sets up a connecting time spot for 3PL2, at which 3PL2 should arrive in city C. In other words, 3PL2 is offered an opportunity to make an effective scheduling of his transportation capacities with the connecting time spot.

4. The Centralized System

In the centralized system (denoted by subscript C), 4PL and two 3PLs both work on maximizing the company's profit. We investigate the effects of different CTS on the decisions and results of the serial delivery optimal effort levels problem. Our analysis is based on the comparison of 3PLs effort levels and system profits in three scenarios: the serial delivery logistics system in the centralized system with NCTS (CN), with GCTS (CG) and with DCTS (CD). For ease of exposition, we consider three scenarios in a centralized system which can be summarized as Case CN, Case CG and Case CD. The scenarios structure is shown as follows:

Centralized System (No Contract) = {NCTS − Case CN, GCTS − Case CG, DCTS − Case CD}

4.1. The Optimal Effort Level

For the centralized model, 4PL and 3PLs belong to one company who aims at achieving the maximum profit of the delivery task. From Figure 6, we see that there is no contract between 4PL and 3PLs. To maximize the total profit, two 3PLs need to set their effort levels to transport the goods. The delivery time of each stage can be observed once the goods arrive.

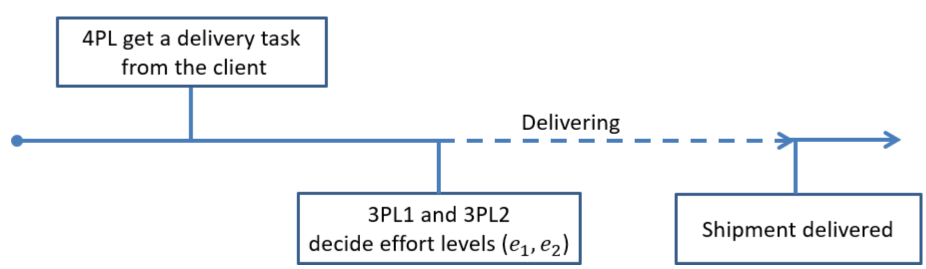


Fig. 6. Sequence of events in the centralized system

As requested, the client will give a fixed monetary payment to 4PL in advance and the goods should arrive in city C as soon as possible. In other words, the 4PL receives a quantitative penalty since the goods begin to be transported. At the end of the task, 4PL pays the penalty to the client according to the delivery time.

We summarize our model notations for the ease of reference in Nomenclature & Notation. In general, we can write that the company's optimal system profit problem in the centralized system as followed:

$$\max_{e} \Pi_c = \mathrm{E}\left[B - \sum_{i=1}^{2} c_i(e_i) - p[\max(t_1(e_1), T) + t_2(e_2)] - w\max(t_1(e_1) - T, 0) \right] \tag{1}$$

Here, the system profit contains the fixed payment from client to 4PL, 3PLs' delivery cost, penalty from the client and 3PL2's waiting cost.

DELIVERY COST. 3PLi's delivery cost is given by $c_i(e_i)=k_ie_i^2/2$ for $i=1,2$, where 1/2 is for the convenience of formulation derivation. As mentioned in Section 2, 3PLi's delivery time is given by $t_i(e_i)=a_i-b_ie_i+\varepsilon_i>0$ for $i=1,2$. It is obvious that extra effort contributes to reducing the actual delivery time and the delivery cost will increase in the extra effort level.

PENALTY. The penalty depends on the total delivery time. In Case CN and Case CG, 3PL1 won't arrive in City B than the connecting time spot where the total delivery time equals to 3PL1's delivery time and 3PL2's delivery time. But in Case CD, it is possible that the goods could wait for 3PL2 as 3PL1 arrives before the connecting time spot. Under such circumstances, the total delivery time can be the connecting time spot and 3PL2's delivery time. Without loss of generality, we formulate the total delivery time as $\max\left(t_1\left(e_1\right),T\right)+t_2\left(e_2\right)$.

WAITING COST. 3PL2s waiting cost only appears in Case CD for that both in Case CN and Case CG. Waiting cost depends on 3PLs' waiting coefficient and waiting time. Waiting costs can be huge when the waiting coefficient is large.

Obviously, the company's optimal system profit problem is an unconstrained optimization. As shown in Eq. (1), the system profit function is concave. The optimal solution can be derived by its first-order optimal condition. The optimal effort level and the optimal system profit will be given in three scenarios as follows.

4.1.1 Case CN: Centralized decision with NCTS

In Case CN, we discuss 3PLs' optimal effort level without setting CTS for 3PL2 in the centralized system. Let the connecting time spot $T=0$ in Eq. (1), the optimization problem is easy as Eq. (2). We denote e_i^{CN} as 3PLi's effort level in Case CN, for $i=1,2$.

$$\max_{e_i}\Pi^{CN}=E[B-(c_1+c_2)-p(t_1(e_1)+t_2(e_2))-wt_1] \tag{2}$$

LEMMA 1. In Case CN

(a) 3PLs optimal effort levels are

$$e_1^{CN*}=\frac{b_1(p+w)}{k_1} \tag{3}$$

$$e_2^{CN*}=\frac{b_2p}{k_2} \tag{4}$$

(b) The system profit is

$$\Pi^{CN*}=B+\frac{b_1^2(p+w)^2}{2k_1}+\frac{p^2b_2^{\ 2}}{2k_2}-p(a_1+a_2)-wa_1 \tag{5}$$

Eq. (5) is the worst profit the company can get.

4.1.2 Case CG: Centralized decision with GCTS

In Case CG, we discuss 3PLs' optimal effort level with GCTS for 3PL2 in the centralized system. Let $T=t_g=t_{1\min}$, the optimization problem of the company can be simply written as Eq, (6). It is also the objective in Case CG. We denote as 3PLi's effort level in Case CG, for $i=1,2$.

$$\max_{e_i}\Pi^{CG}=E[B-(c_1+c_2)-p(t_1(e_1)+t_2(e_2))-w(t_1-t_{1\min})] \tag{6}$$

LEMMA 2. In Case CG,

(a) 3PLis optimal effort levels are

$$e_1^{CG*}=\frac{pb_1}{k_1} \tag{7}$$

$$e_2^{CG*} = \frac{pb_2}{k_2} \tag{8}$$

b) The connecting time is

$$T^{CG*} = t_{1\min}^{*} = a_1 - \frac{pb_1^{\ 2}}{k_1} - m_1 \tag{9}$$

c) The optimal system profit is

$$\Pi^{CG*} = B + \frac{p^2 b_1^2}{2k_1} + \frac{p^2 b_2^{\ 2}}{2k_2} - p(a_1 + a_2) - wm_1 \tag{10}$$

We see that 3PL1's optimal effort level is less than the one in Case CN, and 3PL2's optimal effort is the same as it in Case CN. As $w > 0$ and $m_i > 0$, for $i = 1,2$, it is easy to get the gap of the optimal system profit between Case CN and Case CG, $\Delta\Pi_1 = \Pi^{CG*} - \Pi^{CN*} > 0$ always holds.

4.1.3 Case CD: Centralized decision with DCTS

In Case DG, we discuss 3PLs' optimal effort level with DCTS for 3PL2 in the centralized system. When connecting time spot and effort levels are decision variables, the optimization problem becomes Eq. (11). We denote e_i^{CD} is 3PLi's effort level in Case CD, for $i = 1,2$.

$$\max_{e_i, T} \Pi^{CD} = \mathrm{E}\left[B - \left(c_1 + c_2\right) - p[\max\left(t_1(e_1), T\right) + t_2(e_2)] - w\max\left(t_1(e_1) - T, 0\right)\right] \tag{11}$$

LEMMA 3. In Case CD,

(a) 3PLi's optimal effort levels are

$$e_1^{CD*} = \frac{pb_1}{k_1} \tag{12}$$

$$e_2^{CD*} = \frac{pb_2}{k_2} \tag{13}$$

(b) The decision connecting time spot is

$$T^{CD*} = a_1 + m_1 - \frac{pb_1^2}{k_1} - \frac{2m_1 p}{\left(p + w\right)} \tag{14}$$

(c) The optimal system profit is

$$\Pi^{CD*} = B - p\left(a_1 + a_2\right) + \frac{p^2 b_1^2}{2k_1} + \frac{p^2 b_2^2}{2k_2} - \frac{pwm_1}{\left(p + w\right)} \tag{15}$$

4.2. Comparison Analysis

Our analysis of the centralized system proceeds in three steps. First, we know the CTS's effects on 3PLs' effort levels in the centralized system. Second, we explore the difference between optimal GCTS and optimal DCTS. Third, we find two types of the connecting time spot have different influences on the optimal system profits in. Then we summarize the results in the following proposition.

PROPOSITION 1. In the centralized system, if $p_1 > 0,\ m_1 > 0,\ k_1 > 0,\ w > 0,$ for $i = 1,2$ then we have

$$\begin{aligned} e_1^{CN*} &> e_1^{CG*} = e_1^{CD*} \\ e_2^{CN*} &= e_2^{CG*} = e_2^{CD*} \end{aligned} \tag{16}$$

By comparing the 3PLs' optimal effort levels with different types of CTS (Eqs, (3, 4, 7, 8, 13, 14)), we obtain Proposition 1. It reveals that CTS cannot level up 3PL2's effort level and it reduces 3PL1's effort level. When it is in the centralized system for the serial delivery, 3PL2's optimal effort level is the same whether we consider the CTS or not. When in the centralized system without considering CTS, 3PL1's optimal effort level is increasing in both 3PL2's waiting cost and the penalty coefficient. But when considering CTS, 3PL1's optimal effort level is just increasing in the penalty coefficient. That is to say, CTS will reduce 3PL1's optimal effort level.Intuitively, a severe punishment can force 3PL1 to choose a higher effort level. A lower cost coefficient means it doesn't take too much to enhance the effort level. Meanwhile, 3PL1 is willing to invest more in the delivery task if his effort is easy to reduce the delivery time. But in Case CN, when 3PL1 and 3PL2 are working for the same objective to maximize the system profit of their company, 3PL1 is caught in a dilemma. He not only has to balance the delivery cost and delivery time, but also has to prevent 3PL2 from suffering a waiting cost due to his delay. Hence, 3PL1's optimal effort level is related to 3PL2's waiting cost coefficient. However, with 4PL setting a connecting time spot, 3PL2's waiting cost is simply affected by the uncertain factors existing in the first delivery stage. In other words, 3PL2 cannot save from his waiting even if 3PL1 pays extra effort to cut down delivery time. In the centralized system, two kinds of connecting time spots are given by Eq. (9) and Eq. (14). It is obvious that these CTSs are decreasing in 3PL1's penalty coefficient and output coefficient , and increasing in 3PL1's general delivery time a1 and cost coefficient .

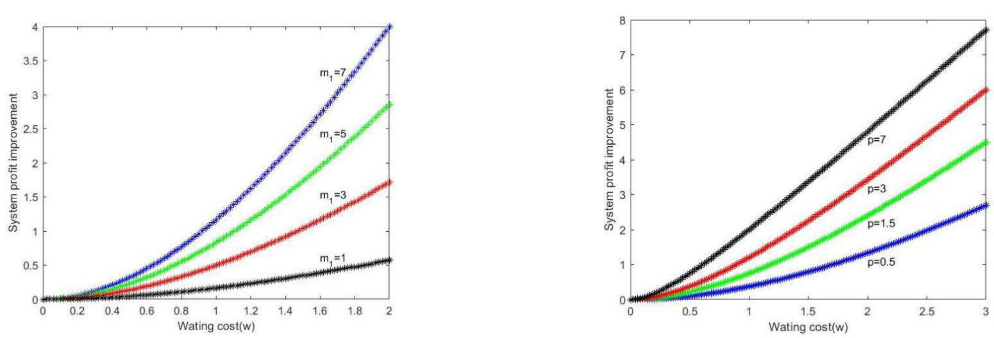
PROPOSITION 2. By given , DCTS is always late or equal to GCTS in the centralized system.

Proposition 2 implies the system's influence on CTS that both GCTS and DCTS are influenced by the uncertain factors. If the range of random factors is getting wider, 3PL1's real delivery time is more likely to be shorter.

Comparing the different parts of the CTSs, we have the conclusion in Proposition 2. That is to say, the differential of the CTS on 3PL1s random factor with decision CTS is greater than the one with given CTS. As explained in Proposition 1, quantitative proof suggests that a severe punishment, a higher output coefficient and a lower cost coefficient force 3PL1 to take a higher effort level. 3PL1's real delivery time can be shorter when 3PL1 takes a higher effort level to decrease the delivery time. On this occasion, it is no need to set an earlier CTS.

PROPOSITION 3. In the centralized system, CTS can improve the system profit. DCTS has a better performance in enhancing system profit than GCTS.

Proposition 3 reveals that CTS has positive effects on the centralized system. The profit improvement from the Case CN to Case CG simply results from the gap between 3PL1's general delivery time a1 and uncertain effect m1. It is obvious that the improvement increases in 3PL1's general delivery time and decreases with the improvement increases in 3PL1's general delivery time and decreases with the uncertain effect. Fig. 7, Fig. 8, Fig. 9 numerically demonstrate the variation trend of the profit improvement with different key parameters.

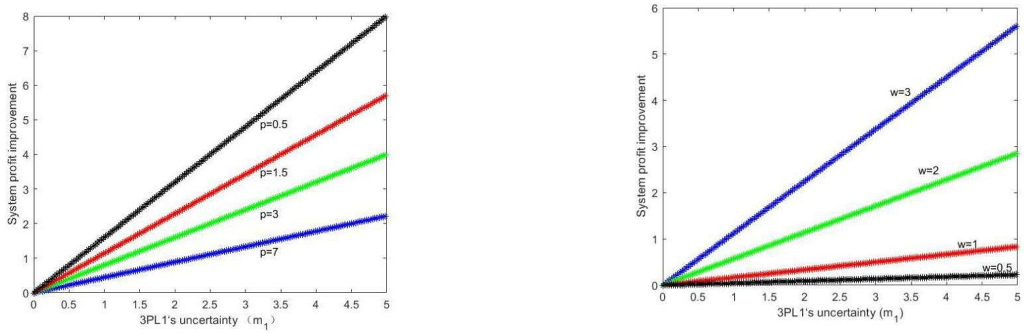


(a) With different 3PL1s uncertainty (b) With different penalty

Fig. 7. Profit improvement in waiting cost

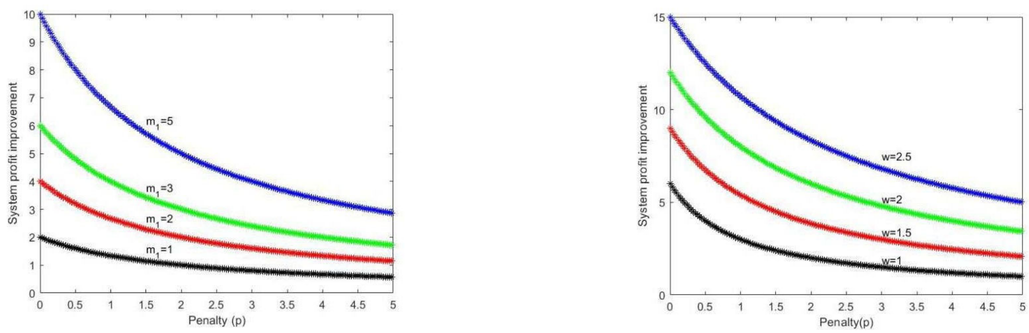
As shown in Fig. 7, the profit improvement increases in waiting cost. That is to say that the sharper the cost when 3PL2 is waiting for 3PL1, the greater improvement CTS will make. The reason for the observation is that the profit improvement is mainly saved from the proper scheduling of the operation. A larger waiting cost exactly means more cost 3PL2 can save. During the same waiting time, a sharper waiting cost can cause a bigger loss of 3PL2.

As shown in Fig. 8, as 3PL1's uncertainty increases, the profit improvement gets large. That is to say that the more uncertain of 3PL1's logistics task, the greater improvement CTS will make. From observation, the profit improvement mainly comes from the savings in the proper scheduling of the operation. If 3PL1 suffers more random obstruction in his delivery, exact delivery time will be increasingly difficult to determine. It just means more cost 3PL2 can save from being informed a proper set out time to begin his delivery. At the same time, the uptrend of profit improvement increases in 3PL1's uncertainty in Fig. 7(a) suggests the same variation.



(a) With different penalty (b) With different waiting cost

Fig. 8. Profit improvement in 3PL1's uncertainty



(a) With different 3PL1s uncertainty (b) With different waiting cost

Fig. 9. Profit improvement in penalty coefficient

As shown in Fig. 9, the profit improvements increase in the client's penalty. That is to say that the severer penalty from the client, the greater improvement CTS will make. The reason for the observation is that the profit improvement is mainly saved from the proper scheduling of the operation. If 3PL2 is afraid of being punished, he is more likely to arrive in the transshipment node. It precisely leads to a shorter waiting time of 3PLs and the cost saving from which reduces in response. At the same time, it can also be verified that the downtrend of profit improvement decreases with the client penalty in Fig. 7(b).

Proposition 3 allows us to develop some intuition that connecting time spots helps enhance the system profits than the case without considering it even in the decentralized system.

5. The Decentralized System

In the decentralized system (denoted by subscript D), we still consider a simplest serial delivery logistics system in which there are only one 4PL and two 3PLS. We investigate the effects of different CTS on 3PLs decisions and the optimal contracts for the serial delivery. In this part, we first describe the optimal contracts problem for serial delivery systems. To model the problem, we give a brief introduction to three different delivery penalty modes and incentives contracts and their general models. The conclusions of the centralized system allow us to develop some intuition that connecting time spots can help enhance the system profits than the case without considering it. Our analysis is based on the comparison of 3PLs effort levels and system profits in two scenarios: the serial delivery logistics system in the decentralized system with GCTS (DG) and with DCTS (DD). For ease of exposition, the two scenarios in the decentralized system can be summarized as Case DG and Case DD. According to the difference, there are three penalty modes: Emergency Penalty Delivery Mode (E), Tardiness Penalty Delivery Mode (T) and Tardiness and Earliness Penalty Delivery Mode (ET). Each mode has its own contract structure which is different from the other mode. In each case, we derive the corresponding optimal contracts in three penalty modes. The analytical structure is shown as follows.

$$
\begin{array}{ccc}
 & & \textit{Optimal Contracts} \\
\textit{Decentralized System} & \left\{\begin{array}{l} GCTS-Case\ DG \left\{\begin{array}{l} Mode\ E \\ Mode\ T \\ Mode\ TE \end{array}\right. \\ DCTS-Case\ DD \left\{\begin{array}{l} Mode\ E \\ Mode\ T \\ Mode\ TE \end{array}\right. \end{array}\right. &
\end{array}
$$

5.1. Penalty for Different Delivery Modes

In the decentralized system, we introduce three penalty modes. For each penalty mode, there is a specific contract structure. As shown in Fig. 10, City A is the origin, B is the transshipment node and C is the destination. According to the occupancies of the warehouses in City B and C, 3PLs will be requested by different delivery modes. Generally, the incentive contract contains a fixed payment from 4PL to 3PL and quantitative penalties. For 3PL2, his incentive contracts will include an additional compensation if 3PL1 is later than him. Parameters vary with delivery modes.

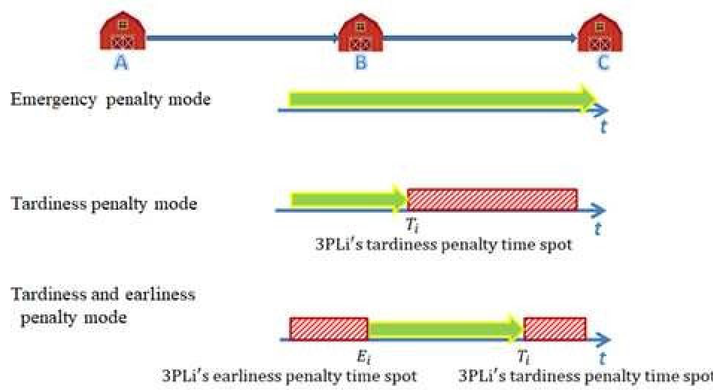


Fig. 10. Delivery modes in the decentralized system

5.1.1 Mode E: Penalty for Emergency Delivery

Emergency penalty mode is requested when the warehouse is always empty. Under this mode, 3PL will be punished quantitatively from the time he leaves for the warehouse. Algebraically, we have the incentive contracts in emergency penalty mode as follows.

$$
\begin{cases} I_1 = \alpha_1 - \beta_{11} t_1 \\ I_2 = \alpha_2 - \beta_{21} t_2 + q(t_1 - T) \end{cases} \tag{17}
$$

where β_{i1} is 3PLi's tardiness penalty coefficient that is contract parameters decided by 4PL. q is a constant compensation coefficient of 3PL2.

5.1.2 Mode T: Penalty for Tardiness Delivery

Tardiness penalty mode is request when the warehouse will be used in 3PLi's tardiness penalty time spot T_i for $i = 1,2$. Under this mode, 3PL suffers a quantitative penalty increasing in the time from him later than T_i to arrive at the warehouse. We can write the incentive contracts in tardiness penalty mode as

$$
\begin{cases} I_1 = \alpha_1 - \beta_{11} \max(t_1 - T_1, 0) \\ I_2 = \alpha_2 - \beta_{21} \max(t_2 - T_2, 0) + q(t_1 - T) \end{cases} \tag{18}
$$

5.1.3 Mode ET: Penalty for Earliness and Tardiness Delivery

Tardiness and earliness penalty mode is requested when the warehouse is available only in the period from 3PLi's earliness penalty time spot E_i to T_i . The incentive contracts in tardiness and earliness penalty mode is as followed:

$$\begin{cases} I_1 = \alpha_1 - \beta_{11}\max\left(t_1 - T_1, 0\right) - \beta_{12}\max\left(E_1 - t_1, 0\right) \\ I_2 = \alpha_2 - \beta_{21}\max\left(t_2 - T_2, 0\right) - \beta_{22}\max\left(E_2 - t_2, 0\right) + q\left(t_1 - T\right) \end{cases} \tag{19}$$

Where β_{i2} is 3PLi's earliness penalty coefficient given by 4PL.

The incentive contracts 4PL give to 3PLs can be generally written as Eq. (19). They refer to different delivery modes with different parameter value as followed:

(1) Eq. (19) can be simplified as Eq. (18) in emergency penalty delivery mode, when $E_i = 0,\ \beta_{i2} = 0$ for $i = 1,2$.

(2) Eq. (18) can be simplified as Eq. (17) in tardiness penalty delivery mode, when and $T_i = 0$ for $i = 1,2$.

5.2. The Optimal Incentive Contracts

For the decentralized model, 4PL and two 3PLs belong to three different companies who aim at achieving the maximum profit of their own companies. From Fig. 11, we see that there is an incentive contract between 4PL and two 3PLs. To achieve their own optimal profits, two 3PLs need to set their effort levels to transport the goods. To take supervision on 3PLs, 4PL needs to set contract parameters and a proper connecting time spot. The delivery time of each stage can be observed once the goods arrive.

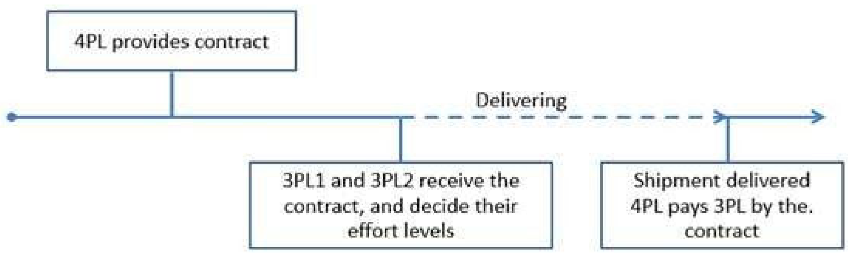


Fig. 11. Sequence of events in the decentralized system

We consider a client who wants to outsource one unit of inseparable goods to two third-party logistics (3PL) by an incentive contract provided by fourth-party logistics (4PL). It is a two-stage task that the first stage is from city A to city B by 3PL1 and the other is from city B to city C by 3PL2. The incentive contract will be given by 4PL and received by 3PL. For our basic model, we assume that the 4PL moves first and give an incentive contract$\left(\alpha,\ \beta\right)$. Then, 3PLs will decide to take the incentive contract or not. As requested, the client will give 4PL a fixed monetary payoffs B in advance and the goods should arrive in city C as soon as possible. In other words, the 4PL receives a quantitative penalty since the goods begin to be transported. At the end of the task, 3PLs will pay a partial penalty to 4PL and 4PL pays the client according to the delivery time. At the end of the task, 3PLs will pay a partial penalty to 4PL and 4PL pay all the client according to the delivery time

We characterize the 4PL's contract based on the Principal-Agent model as a profit maximization problem subject to the constraints of Individual Rationality and Incentive Compatibility. The upper model is to maximize 4PL's profit and the lower model is to maximize the profit of each 3PL. In terms of the Principal-Agent model, the Incentive Compatibility is the optimal solution of the lower model. In Case DG, the optimal incentive contract will be derived with a given connecting time spot; In Case DD, the connecting time spot will be a decision variable.

In general, we can write 4PL's optimal incentive contracts problem in the decentralized system as follows:

$$\max_{\alpha_i,\,\beta_{i1},\beta_{i2},T} \Pi_{4PL} = \mathrm{E}\left(B - p[\max\left(t_1\left(e_1\right),T\right) + t_2\left(e_2\right)] - \sum_{i=1}^{2} I_i\left(\alpha_i,\beta_{i1},\beta_{i2},T\right)\right) \tag{20}$$

$$s.t. \quad \pi_i\left(e_i\right) = I_i\left(\alpha_i,\beta_{i1},\beta_{i2},T\right) - c_i\left(e_i\right) \geq R_i \tag{21}$$

$$e_i^*\left(\alpha_i,\beta_{i1},\beta_{i2},T\right) = \arg\max\left(\pi_i\right),\ i = 1,2 \tag{22}$$

Here, 3PLs' profit functions can be written as

$$\pi_1 = \mathrm{E}\left[I_1 - c_1\right]$$
$$\pi_2 = \mathrm{E}\left[I_2 - c_2 - w\left(t_1 - t_{1\min}\right)\right]$$

where I_i is 3PLs' contract given by 4PL. c_i is 3PLs' delivery cost. R_i is 3PL's reservation utility. For 3PL1, his profit is the contract value except for the delivery cost. For 3PL2, his profit also includes the waiting cost. In equation (20), Π_{4PL} is the total profit of 4PL in the decentralized system. It is the expectation of client's fixed payment to 4PL, the penalty 4PL pays to client in terms of total delivery time and the incentive contracts 4PL gives 4PLs.

The conclusions of the centralized system allow us to develop some intuition that connecting time spot can help enhancing the system profits than the case without considering it. In Case DG, 4PL posts a given connecting time to 3PL2 and the decision variables are the contract parameters. In Case DD, the connecting time needs to be decided with the contract parameters. The fixed payment and penalty part given by the client are the same meanings as what in Eq. (1). As mentioned in Section 4.2, the incentive contracts are in corresponds to 3 kinds of delivery mode in both Case DG and Case DD.

For the constraints, Eq. (21) is the Individual Rationality. First, the left-hand side is 3PLi's profit, and the right-hand side is the 3PLi's reservation utility R_i. It represents that each 3PL will accept the incentive contract only if the profit they can earn from this task are greater than its reservation utility. In other words, they won't lose money in delivering the foods at least.

Eq. (22) is the Incentive Compatibility. It represents that 3PLs can get their optimal profits by working on the task given the optimal incentive contract. For the Principal-Agent model, the optimal solution to the lower model is the constraint of the upper model.

We summarize our model notations for the ease of reference in Nomenclature & Notation. For the lower model, 3PLi's profits functions are concave and 3PLi's optimal profits problems are unconstrained optimization. The optimal effort levels can be derived by its first order optimal condition. For the upper model, the system profit function is concave. For the Individual Rationality constraint, 4PL merely allows 3PLs to get their reservation utility. With 3PLs' optimal effort levels, the optimal incentive contract can be derived by the first order condition of Eq. (20). The optimal solutions are given by two scenarios– each of which contains three kinds of delivery modes.

5.3. Case DG: Decentralized Decision with GCTS

For Case DG, when $T = t_g = t_{1min}$, the 4PL's optimal incentive contracts problem can be written as

$$\begin{aligned} &\max_{\alpha_i,\beta_{i1},\beta_{i2}} \Pi_{4PL}^{DG} = \mathrm{E}\left(B - p[t_1(e_1) + t_2(e_2)] - \sum_{i=1}^{2} I_i(\alpha_i,\beta_{i1},\beta_{i2}) \right) \\ &s.t. \quad \pi_i(e_i) = I_i(\alpha_i,\beta_{i1},\beta_{i2}) - c_i(e_i) \geq R_i \\ &\qquad e_i^*(\alpha_i,\beta_{i1},\beta_{i2}) = \arg\max(\pi_i),\ i = 1,2 \end{aligned} \tag{23}$$

LEMMA4. In Case DG, according to the Principal-Agent model,

(a) The optimal effort levels of 3PLs are $e_1^{\mathrm{DG}*} = \dfrac{pb_1}{k_1}$ and $e_2^{\mathrm{DG}*} = \dfrac{pb_2}{k_2}$.

(b) The optimal profits of 3PLs are as follows.

$$\begin{aligned} \pi_1^{DG*} &= R_1 \\ \pi_2^{DG*} &= R_2 \end{aligned} \tag{24}$$

(c) The optimal profit of 4PL is

$$\Pi_{4PL}^{\mathrm{DG}*} = B - (R_1 + R_2)B + \frac{p^2 b_1^2}{2k_1} + \frac{p^2 {b_2}^2}{2k_2} - p(a_1 + a_2) - wm_1 \tag{25}$$

(d) The given connecting time spot is

$$T^{DG*} = t_{1\min}^* = a_1 - \frac{p{b_1}^2}{k_1} - m_1$$

(e) The optimal system profit is

$$\Pi^{\mathrm{DG}*} = B - p(a_1 + a_2) + \frac{p^2 b_1^2}{2k_1} + \frac{p^2 {b_2}^2}{2k_2} - wm_1 \tag{26}$$

As shown in Lemma4, the GCTS in the decentralized system equals the one in the centralized system. Obviously, the optimal effort levels of 3PLs and the optimal system profit with this GCTS is equal to the centralized system. For convenience, the optimal incentive contracts are given in Appendix A.

5.4 Case DD: Decentralized decision with DCTS

When T is a decision variable, the 4PL's optimal incentive contracts problem can be simply written as Eq. (20). Then we have

LEMMA5. In Case DD,

(a) The optimal effort levels of 3PLs are $e_1^{DD*} = pb_1/k_1$ and $e_2^{DD*} = pb_2/k_2$.
(b) The optimal profits of 3PLs are as follows.

$$\pi_1^{DD*} = R_1 \qquad\qquad\qquad (27)$$
$$\pi_2^{DD*} = R_2$$

(c) The optimal profit of 4PL is

$$\Pi_{4PL}^{DD*} = B - (R_1 + R_2) - p(a_1 + a_2) + \frac{p^2b_1^2}{2k_1} + \frac{p^2b_2^2}{2k_2} - \frac{pwm_1}{(p+w)} \qquad (28)$$

(d) The decision connecting time spot is

$$T^{DD*} = a_1 + m_1 - \frac{pb_1^2}{k_1} - \frac{2m_1p}{p+w} \qquad (29)$$

(e) The optimal system profit is

$$\Pi^{DD*} = B - p(a_1 + a_2) + \frac{p^2b_1^2}{2k_1} + \frac{p^2b_2^2}{2k_2} - \frac{pwm_1}{p+w} \qquad (30)$$

As shown in Lemma 5, the DCTS in the decentralized system equals to the one inthe centralized system.

Obviously, the optimal effort levels of 3PLs and the optimalsystem profit with this DCTS is equal to the centralized system. For convenience, the optimal incentive contracts are given in Appendix B.

5.5. Comparison Analysis

Our analysis of the decentralized system proceeds in three steps. First, we conclude CTSs' different effects on 3PLs' effort level. Secondly, we determine how CTS benefits the serial delivery system and its members. Third, we explore how CTS changes the decisions of 4PL and 3PLS. Detailed optimal contracts are shown in Appendix B and we summarize the results in the following proposition.

PROPOSITION 4. In the decentralized system,

(a) GCTS and DCTS will reduce 3PL1's optimal effort level compared with considering no CTS.
(b) GCTS and DCTS don't change 3PL2's optimal effort level.

Comparing Lemma 1~5, Proposition 4 shows that 3PLs' optimal effort levels with GCTS equals to the optimal effort levels with DCTS regardless of the game structure. No matter which kind of CTS to be chosen, the promoting effect of 3PL2's waiting cost on 3PL1's optimal effort level will be reduced by considering CTS.

The main reason is that as CTS is used for characterizing the connection in the serial delivery, it divides the two-stage serial delivery into two parts. 3PLs are offered the opportunity to manage their delivery task without considering the objective condition resulting from each other.

PROPOSITION 5. The optimal system profits in the decentralized system can be Pareto-improved to the optimal system profits in the centralized system considering CTS.

As mentioned in Proposition 3, the system profits in the decentralized system with CTS are always greater than it without considering CTS. But the optimal system profits in the decentralized system cannot always be as much as the optimal system profits in the centralized system. Shown in Appendix B, for Case DG, if 3PLs are requested EP mode, the optimal system profits can always be derived; If 3PLs are requested TP delivery mode, the optimal system profits can be derived when 3PLs is probably delay; If 3PLs are requested ETP delivery mode, the optimal system profits cannot be derived when 3PLs is definitely not delay. Similarly, for Case DD, if 3PLs are requested EP mode, the optimal system profits can always be derived; If 3PLs are requested TP mode, the optimal system profits can be derived when 3PLs is definitely delay; If 3PLs are requested ETP delivery mode, the optimal system profits can be derived when 3PLs is probably delay.

Once the parameters are determined, 4PL will get to know if she can make extra profit with CTS and which contract should be chosen to achieve the optimal system profits. Mentioned in previous sections, the tardiness penalty date and earliness penalty date are determined by the occupancy of the housewares. But the number of days of delivery is relatively decided by the exact date when the logistics task begins. That is to say, 4PL can induce 3PLs into the corresponding conditions in different delivery mode which leads to profits improvement referring to the optimal contracts. In other words, CTS can always help 4PL get extra profits whichever delivery modes the 3PLs are asked to perform.

PROPOSITION 6. In the decentralized system, if , , , and , CTS always increases 3PLs' fixed payment.

Proposition 6 reveals that 4PL must enhance 3PL1's fixed payment. With the optimal contracts with CTS, only a higher fixed payment of 3PL can qualify the individual rationality when other parameters remain the same. The main reason for the observation is that 3PL1 suffers an extra penalty for being late to the transshipment node. According to the Principal-Agent model, 3PL1 needs a higher fixed payment to satisfy the Individual Rationality constraint. Otherwise, he won't undertake the logistics task when the profit he finally gets is lower than his reservation utility.

Proposition 6 also reveals that 4PL can achieve the optimal system profit with CTS by giving 3PL2 a less fixed payment with DCTS. The main reason for the observation is that 3PL2 gets an extra compensation when 3PL1 arrives in the transshipment node later than CTS. According to the Principal-Agent model, 3PL2 is easier to make his profit greater than the reservation utility.

By comparing the optimal contracts in Appendix B, we have the following proposition.

PROPOSITION 7. 3PL1's delivery penalty mode won't affect 3PL2's optimal contracts. 3PL2's delivery penalty mode won't affect 3PL1's optimal contracts.

Proposition 7 shows that the contracts are just determined by the delivery mode of each 3PL but the delivery mode combinations of two 3PLs. For example, if requested the TP mode, 4PL's optimal contract to 3PL1 is , no matter which mode 3PL2 will choose. It means that 4PL can manage the serial logistics task in two parts without considering the connections of the first delivery part and the second part. Therefore, the optimal contracts with CTS help 3PL2 save from waiting cost and enhance the 4PL's efficiency on operating a serial logistics. What results in the conclusion is that CTS divides the logistics task into two parts. Without considering CTS, 3PL1 collaborates with 3PL2 to deliver the goods. CTS can be a sign of the beginning of the second delivery part, which makes the second part no longer need to depend on when 3PL1 ends his delivery. From the perspective of both models and physical meanings, CTS makes the two parts of delivery tasks can be managed separately.

6. Conclusions

In this paper, we study the contracting issues of a 4PL firm outsources a two-stage delivery. The optimal contract design for such outsourcing of logistics services is complicated by two primary challenges. Firstly, the contract must solve the moral hazard problem between 4PL and 3PLs in such a setting. Secondly, a performance-based contract induces the 3PLs to take a proper effort level for achieving system optimal profits. We propose two kinds of connecting time spots to deal with the waiting cost of the serial 3PL delivery connection process. Considering three kinds of delivery modes, we derive the optimal contracts with connecting time spots for 3PL. Among the few pieces of research on serial 3PL delivery time risk management, this research is the first one that quantitatively describes the delivery connection process with connecting time spots.

Our study shows CTS can improve system profits. In the centralized system, DCTS makes a greater contribution to enhancing the system profit than GCTS. However, CTS has no effect on enhancing 3PLs' effort levels and even reduces the optimal effort levels of 3PL1 to save delivery costs. From the respective 4PL, she exploits 3PLs who can only obtain the value of their reservation utility and finally captures all the profit improvement. In the decentralized system, profit improvements cannot always be obtained. However, 4PL can induce 3PLs into the corresponding delivery mode to achieve profits improvement by the optimal contracts. Therefore, CTS can always help 4PL get extra profits whichever delivery modes the 3PLs are asked to perform in a practical sense.

Although we investigate the symmetric information setting between the 4PL and 3PLs, there are other facets of logistics outsourcing settings with CTS that deserve thorough consideration in the future. Firstly, 4PL usually doesn't have the cost information of 3PLs if they don't belong to one company in real word outsourcing. Therefore, we will focus on the settings where the 3PLs have private information to 4PL. Secondly, the CTS can be extended to Connecting Time Window (as CTW for short). In such a setting, the CTW can also be a given parameter or a decision variable. The system profit may be achieved easier or more effectively. Thirdly, there is a kind of setting that contains not only two 3PLs. The delivery may combine serial deliveries with parallel structure. We believe that each of these settings provides interesting trade-offs.

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Appendix A. The optimal contracts with GCTS

The optimal solutions of GCTS			
System profit		$\Pi_C^* = B - p(a_1 + a_2) + p^2 b_1^2 / 2k_1 + p^2 b_2^2 / 2k_2 - q_1 m_1$	
4PL's profit		$\Pi_{4PL}^* = B - (R_1 + R_2) - p(a_1 + a_2) + p^2 (\frac{b_1^2}{2k_1} + \frac{b_2^2}{2k_2}) - q_1 m_1$	
		3PL1	**3PL2**
Incentive contract		$g_1 = \alpha_1 - \beta_{11} \max(t_1 - T_1, 0) - \beta_{12} \max(S_1 - t_1, 0)$	$g_2 = \alpha_2 - \beta_{21} \max(t_2 - T_2, 0) - \beta_{22} \max(S_2 - t_2, 0) + q(t_1 - t_{1\min})$
3PL's profit fuction		$\pi_1 = \mathrm{E}[I_1 - c_1]$	$\pi_2 = \mathrm{E}[I_2 - c_2 - w(t_1 - T)]$
3PL's effort level		$e_1^* = pb_1 / k_1$	$e_2^* = pb_2 / k_2$
3PL's profit expectation		$\pi_1^* = R_1$	$\pi_2^* = R_2$
The optimal contract parameters	Emergency	$\alpha_1^* = R_1 + pa_1 - p^2 b_1^2 / 2k_1$ $\beta_{11}^* = p$	$\alpha_2^* = R_2 + pa_2 - p^2 b_2^2 / 2k_2 - (q - w) m_1$ $\beta_{21}^* = p$
	Tardiness	a) When $a_1 - pb_1^2 / k_1 - m_1 \ge T_1$, $\alpha_1^* = R_1 + p(a_1 - T_1) - p^2 b_1^2 / 2k_1$ $\beta_{11}^* = p$	a) When $a_2 - pb_2^2 / k_2 - m_2 \ge T_2$, $\alpha_2^* = R_2 + p(a_2 - T_2) - p^2 b_2^2 / 2k_2 - (q - w) m_1$ $\beta_{21}^* = p$
		b) When $a_1 - pb_1^2 / k_1 - m_1 \le T_1 \le a_1 - pb_1^2 / k_1 + m_1$, $\alpha_1^* = R_1 + p(a_1 + m_1 - T_1)/2$ $\beta_{11}^* = \frac{2m_1 p}{a_1 + m_1 - T_1 - pb_1^2 / k_1}$	b)When $a_2 - pb_2^2 / k_2 - m_2 \le T_2 \le a_2 - pb_2^2 / k_2 + m_2$, $\alpha_2^* = R_2 + p(a_2 + m_2 - T_2)/2 - (q - w) m_1$ $\beta_{21}^* = \frac{2m_2 p}{a_2 + m_2 - T_2 - pb_2^2 / k_2}$
	Earliess and Tardines	a) When $a_1 - pb_1^2 / k_1 - m_1 \ge T_1 \ge S_1$, $\alpha_1^* = R_1 + p(a_1 - T_1) - p^2 b_1^2 / 2k_1$ $\beta_{11}^* = p$ β_{12}^* take any value；	a) When $t_{2\min} = a_2 - pb_2^2 / k_2 - m_2 \ge T_2 \ge S_2$, $\alpha_2^* = R_2 + p(a_2 - T_2) - p^2 b_2^2 / 2k_2 - (q - w) m_1$ $\beta_{21}^* = p$ β_{22}^* take any value；
		b) When $S_1 \le a_1 - pb_1^2 / k_1 - m_1 \le T_1 \le a_1 - pb_1^2 / k_1 + m_1$, $\alpha_1^* = R_1 + p(a_1 + m_1 - T_1)/2$ $\beta_{11}^* = \frac{2m_1 p}{a_1 + m_1 - T_1 - pb_1^2 / k_1}$ β_{12}^* take any value，	b) When $S_2 \le a_2 - pb_2^2 / k_2 - m_2 \le T_2 \le a_2 - pb_2^2 / k_2 + m_2$, $\alpha_2^* = R_2 + p(a_2 + m_2 - T_2)/2 - (q - w) m_1$ $\beta_{21}^* = \frac{2m_2 p}{a_2 + m_2 - T_2 - pb_2^2 / k_2}$ β_{22}^* take any value；
		c) When $a_1 - pb_1^2 / k_1 - m_1 \le S_1 \le T_1 \le a_1 - pb_1^2 / k_1 + m_1$, $\alpha_1^* = R_1 + p^2 b_1^2 / 2k_1 + \beta_{11}^* \mathrm{E}_{11} + \beta_{12}^* \mathrm{E}_{12} + \beta_{12}^* (S_1 - a_1 + pb_1^2 / k_1)$ β_{11}^*, β_{12}^* satisfy $\frac{pb_1}{k_1} = \frac{\beta_{11}^* b_1 (a_1 + m_1 - T_1) + \beta_{12}^* b_1 (a_1 - m_1 - S_1)}{2k_1 m_1 + (\beta_{11}^* + \beta_{12}^*) b_1^2}$ in which， $\mathrm{E}_{11} = (a_1 - pb_1^2 / k_1 + m_1 - T_1)^2 / 4m_1$ $\mathrm{E}_{12} = (a_1 - pb_1^2 / k_1 + m_1 - S_1)^2 / 4m_1$	c) When $a_2 - pb_2^2 / k_2 - m_2 \le S_2 \le T_2 \le a_2 - pb_2^2 / k_2 + m_2$, $\alpha_2^* = R_2 + p^2 b_2^2 / 2k_2 - (q - w) m_1 + \beta_{21}^* \mathrm{E}_{21} + \beta_{22}^* \mathrm{E}_{22}$ $+ \beta_{22}^* (S_2 - a_2 + pb_2^2 / k_2)$ β_{21}^* , β_{22}^* satisfy $\frac{pb_2}{k_2} = \frac{\beta_{21}^* b_2 (a_2 + m_2 - T_2) + \beta_{22}^* b_2 (a_2 - m_2 - S_2)}{2k_2 m_2 + (\beta_{21}^* + \beta_{22}^*) b_2^2}$ in which， $\mathrm{E}_{21} = (a_2 - pb_2^2 / k_2 + m_2 - T_2)^2 / 4m_2$ $\mathrm{E}_{22} = (a_2 - pb_2^2 / k_2 + m_2 - S_2)^2 / 4m_2$

Appendix B. The optimal contracts with DCTS

The optimal solutions of DCTS			
System profit		$T^* = a_1 + m_1 - pb_1^2/k_1 - 2m_1 p/(p+w)$	
4PL's profit		$\Pi_{CD}^* = B - p(a_1 + a_2) + \dfrac{p^2 b_1^2}{2k_1} + \dfrac{p^2 b_2^2}{2k_2} - \dfrac{p^2 m_1 (q-w)}{(p+w)^2}$	
		$\Pi_{4PL}^* = B - (R_1 + R_2) - p(a_1 + a_2) + \dfrac{p^2 b_1^2}{2k_1} + \dfrac{p^2 b_2^2}{2k_2} - \dfrac{p^2 m_1 (q-w)}{(p+w)^2}$	
Incentive contract		3PL1	3PL2
3PL's profit fuction		$g_1 = \alpha_1 - \beta_{11}\max(t_1 - T_1, 0) - \beta_{12}\max(S_1 - t_1, 0)$	$g_2 = \alpha_2 - \beta_{21}\max(t_2 - T_2, 0) - \beta_{22}\max(S_2 - t_2, 0) + q(t_1 - T)$
3PL's effort level		$\pi_1 = \mathrm{E}[I_1 - c_1]$	$\pi_2 = \mathrm{E}[I_2 - c_2 - w(t_1 - T)]$
3PL's profit expectation		$e_1^* = pb_1/k_1$	$e_2^* = pb_2/k_2$
3PL's profit expectation		$\pi_1^* = R_1$	$\pi_2^* = R_2$
The optimal contract parameters	Emergency	$\alpha_1^* = R_1 + pa_1 - p^2 b_1^2/2k_1$ $\beta_{11}^* = p$	$\alpha_2^* = R_2 + pa_2 - p^2 b_2^2/2k_2 - p^2 m_1 (q-w)/(p+w)^2$ $\beta_{21}^* = p$
	Tardiness	a) When $a_1 - pb_1^2/k_1 - m_1 \ge T_1$, $\alpha_1^* = R_1 + pa_1 - p^2 b_1^2/2k_1 + p^2 m_1/(p+w)$ $\beta_{11}^* = p + w$	a) When $a_2 - pb_2^2/k_2 - m_2 \ge T_2$, $\alpha_2^* = R_2 + p(a_2 - T_2) - p^2 b_2^2/2k_2 - p^2 m_1 (q-w)/(p+w)^2$ $\beta_{21}^* = p$ b) When $a_2 - pb_2^2/k_2 - m_2 \le T_2 \le a_2 - pb_2^2/k_2 + m_2$, $\alpha_2^* = R_2 + p(a_2 + m_2 - T_2)/2 - p^2 m_1 (q-w)/(p+w)^2$ $\beta_{21}^* = \dfrac{2m_2 p}{a_2 + m_2 - T_2 - pb_2^2/k_2}$
	Earliess and Tardines	When $a_1 + m_1 - pb_1^2/k_1 - 2m_1 p/(p+w) = T \le T_1 \le a_1 - pb_1^2/k_1 + m_1$ $\beta_{11}^*, \beta_{12}^*$ satisfy $2m_1 p + \dfrac{2\beta_{12}^* m_1 w}{(p+w)} = \beta_{11}^*\left(a_1 + m_1 - T_1 - \dfrac{pb_1^2}{k_1}\right)$ $\alpha_1^* = R_1 + \dfrac{p^2 b_1^2}{2k_1} + \dfrac{\beta_{12}^* w^2 m_1}{(p+w)^2} + \dfrac{\beta_{11}^*(a_1 + m_1 - T_1 - pb_1^2/k_1)^2}{4m_1}$	a) When $t_{2\min} = a_2 - pb_2^2/k_2 - m_2 \ge T_2 \ge S_2$, $\alpha_2^* = R_2 + p(a_2 - T_2) - p^2 b_2^2/2k_2 - p^2 m_1 (q-w)/(p+w)^2$ $\beta_{21}^* = p$ β_{22}^* take any value; b) When $S_2 \le a_2 - pb_2^2/k_2 - m_2 \le T_2 \le a_2 - pb_2^2/k_2 + m_2$, $\alpha_2^* = R_2 + p(a_2 + m_2 - T_2)/2 - p^2 m_1 (q-w)/(p+w)^2$ $\beta_{21}^* = \dfrac{2m_2 p}{a_2 + m_2 - T_2 - pb_2^2/k_2}$ β_{22}^*take any value; c) When $a_2 - pb_2^2/k_2 - m_2 \le S_2 \le T_2 \le a_2 - pb_2^2/k_2 + m_2$, $\alpha_2^* = R_2 + p^2 b_2^2/2k_2 - p^2 m_1 (q-w)/(p+w)^2 + \beta_{21}^* \mathrm{E}_{21} + \beta_{22}^* \mathrm{E}_{22}$ $+ \beta_{22}^*(S_2 - a_2 + pb_2^2/k_2)$ β_{21}^* , β_{22}^* satisfy $\dfrac{pb_2}{k_2} = \dfrac{\beta_{21}^* b_2 (a_2 + m_2 - T_2) + \beta_{22}^* b_2 (a_2 - m_2 - S_2)}{2k_2 m_2 + (\beta_{21}^* + \beta_{22}^*) b_2^2}$ in which, $\mathrm{E}_{21} = (a_2 - pb_2^2/k_2 + m_2 - T_2)^2/4m_2$ $\mathrm{E}_{22} = (a_2 - pb_2^2/k_2 + m_2 - S_2)^2/4m_2$