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A model for location-assortment problem in a competitive environment

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A B S T R A C T

This paper considers simultaneously two areas of facility location and assortment planning in a competitive environment. In fact, a chain store that has competitors in the market locates a new facility. As there are different products in the market that can substitute with each other, it is intended to determine the best product assortment as well. An integer nonlinear programming problem is proposed to model the mentioned subject. For solving the model, the problem is reformulated as a mixed integer linear programming one. Therefore, a MIP solver software can be used for solving the small- and medium-size problems. For large-scale problems, a firefly algorithm is designed for obtaining a satisfactory solution. By using the proposed model, it is numerically shown that, in addition to the optimal location, it is also necessary to determine simultaneously the best product assortment for the new store. Actually, comparison results reveal that the location significantly affects the assortment scenarios for the new store. In other words, the selection of new store locations may lead to loss of large profit if the assortment planning is neglected.

1. Introduction

Today, the competition is considered as an integral part of all supply chains in a way that if any chain player becomes unaware of it, the failure probability of the supply chain will be increasing. One of the factors that has a great impact on the competitiveness of a supply chain is the decisions that are made in a chain at different strategic and tactical levels. Strategic decisions are long-term decisions that are difficult and costly to change, such as facility location, facility capacity and technology selection. On the other hand, tactical decisions are those which deal with market changes and the factors determined by other competitors like assortment, pricing and promotion decisions (Shankar et al., 2013). Both strategic and tactical competitive decisions must be considered simultaneously in supply chain network design, as the overall chain structure is shaped by these kinds of decisions and the attention to one of them can lead to ignoring other important factors in the chain. In this paper, we investigate two important and influential factors in the competition of chain stores: facility location and assortment planning; The first one is considered as a strategic decision and the second is a tactical one.

As the location of a new facility influences a company's profits greatly and cannot be changed easily due to a lot of investments, it is considered as a strategic decision (Ma et al., 2020). Many competitive location models are available in the literature, see for instance the survey papers (Plastria, 2001; Drezner, 2014; Ashtiani, 2016) and the references therein. They vary in the model components, for example: The location space may be a plane, network or discrete set. The number of new facilities can be one or more. The competition may be static or with foresight; the first one means that the competitors are already in the market and the owner of the new facility is aware of all related information and the second means that the facilities of competitors will be entered in the market when a new facility is opened. The demand points can be considered inelastic or elastic, depending on whether the goods are necessary or unnecessary. The attraction function of a customer

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towards a particular facility is also important, and it usually depends on the distance between the customer and the facility and the features of the facility that are called facility design.

On the other hand, selecting a set of products that leads to profit maximization is a major problem in planning retail operations and revenue management (Gallego and Topaloglu, 2019). One of the concepts that is widely used in assortment planning is choice models, in which customer substitution behavior and the resulting demand are specified from a given set of products (Désir et al., 2020). The need to consider customer choice behavior generates a considerable body of literature in the area of assortment optimization.

The rest of the paper is organized as follows. In Section 2, we review related studies in the literature. In Section 3, we present the problem. Section 4 introduces the exact solution method and the proposed heuristic. Numerical examples to demonstrate the efficiency of the model and the proposed methods are shown in Section 5. Finally, in section 6, the conclusion highlights the innovation of the model and concludes with the future work.

2. Literature Review

In this section, we first review the location and assortment planning literature separately, and then introduce the research gap that exists in this research area.

2.1. Competitive Location

One of the important differences among competitive facility location models is their decision variables. In many cases, the only decision variable is "location" (see, for example, Levanova & Gnusarev, 2020), but in recent years, other variables have been added to the problem. One of the most frequent variables is the design of new facilities (see, for example, Boglárka et al., 2019) and the models associated with these types of variables are called location-design problems. Some researchers have also studied the subject of both price and location in the competitive environment (see, for example, Zambrano-Rey et al., 2019). Furthermore, some papers used the other variables except those mentioned above such as the size of the facility (Zhang & Rushton, 2008) and the routing problem in the competitive environment (Hosseini-Nasab & Tavana-Chehartaghi, 2021).

Customer patronizing behavior has been also considered as one of the competitive model components (Fernández et al., 2017a). The commonly used customer choice rules in the literature can be classified into two kinds:

- The deterministic rule: A customer only patronizes the facility that attracted him/her most (Drezner, 1994a)
- The probabilistic rule: A customer splits his/her demand among all facilities in an area proportionally to the attraction of each facility to him/her (Drezner, 1994b)

Table 1 lists the most related past work to the competitive location study by categorizing according to the decision variables of the model, and customer patronizing behavior.

Table 1
Overview of the most related past studies in competitive location models

		Decision Variable		
		Location	**Location & Design**	**Location & Price**
Patronizing Behavior	**Deterministic**	Díaz et al. (2017) Fernández et al. (2017b) Ghaffarinasab (2018) Gentile et al. (2018) Iellamo et al. (2015) Khodaee et al. (2022) Rahmani & Hosseini (2021) Wang et al. (2018)	Fernández et al. (2017a) Hendrix (2016)	Arbib et al. (2020) Rohaninejad et al. (2017)
	Probabilistic	Drezner et al. (2015) Drezner et al. (2018) Gnusarev (2020) Legault & Frejinger (2022) Levanova, & Shan et al. (2019) Qi et al. (2017) Sadjadi et al. (2020)	Arrondo et al. (2015) Bagherinejad & Niknam (2018) Boglárka et al. (2019) Fernández et al. (2019) Ma et al. (2020) Redondo et al. (2015)	He et al. (2016) Kress & Pesch (2016) Li & Li (2021) Mahmoodjanloo et al. (2020) Zambrano-Rey et al. (2019)

The vast majority of competitive location models focused on only one product, and very few works have been considered multi-product cases. For example, some authors considered price variables in the competitive multi-product space, but optimal product selection has not been a decision variable (Meagher & Zauner, 2004; Kress & Pesch, 2016; Economides, 1986). To the best of our knowledge, the only papers that considered the product selection as a variable are Beresnev & Suslov (2010) and Sadjadi et al. (2020). Beresnev & Suslov (2010) proposed a model in which products selection and pricing are the decision variables and Sadjadi et al. (2020) studied a model where the location and product selection are considered as variables, and the main difference is that they did not use assortment-oriented approach in their works. In other words, the effect of substituting products with each other has been neglected.

2.2. Assortment Planning

The assortment planning literature dates back to the 1950s, when Sadowski (1959) was probably the first to address the "assortment problem". The studies related in this area are different from each other in terms of the features they have been considered, such as the consumer demand model, the products' substitution pattern, the decisions at the inventory level and consideration of assortment capacity.

Bernstein et al. (2015) categorized previous works based on the applied customer choice model - such as multinomial logit models (MNLs) and exogenous demand models - which greatly affect other problem features. The MNL model assumes that each customer visiting a store associates a utility with each product that can be decomposed into two parts: deterministic and random components. (See, for example, Besbes & Sauré, 2016). Due to some deficiencies of the MNL model, other choice models such as the nested logit model (See, for example, Gallego & Topaloglu, 2019) and a combination of the multinomial logit model (See, for example, Sen et al., 2018) have been applied to better formulate the substitution behavior. In an exogenous demand model, the demand for each product is determined in advance for all available products, and therefore, does not depend on a selected assortment (See, for example, Smith & Agrawal, 2000). Some recent work has used a ranking-based consumer choice model to depict consumer preferences, so that each customer has a ranking of his or her favorite products (See, for example, Goyal et al., 2016). See Train (2009) for an overview of these models. Researchers gradually added more variables to their models; for example, Kök & Fisher (2007) considered shelf space limitation and Yücel, et al. (2009) paid attention both to shelf space constraint and supplier selection.

Table 2 shows the most related research about the assortment planning based on the demand model and the capacity consideration.

Table 2
Overview of the most related past studies in assortment planning models

		Capacity Consideration	
		Yes	**No**
Demand Model	**MNL**	Besbes & Sauré (2016) Feldman & Topaloglu (2015a)	Bernstein et al. (2015)
	Nested	Gallego & Topaloglu (2019)	
	Mix MNL	Sen et al. (2018)	Feldman & Topaloglu (2015b)
	Exogenous	Çömez-Dolgan et al. (2021) Chung et al. (2019) Hense & Hübner (2022) Kök & Fisher (2007) Smith & Agrawal (2000) Yücel et al. (2009)	**Our work**
	Choice	Désir et al. (2020) Jagabathula & Rusmevichientong (2017) Goyal et al. (2016) Transchel et al. (2022)	Aouad et al. (2018) Blanchet et al. (2016) Feldman et al. (2019) Feldman & Topaloglu (2017) Nip et al. (2022)

2.3. Contribution of the paper

As can be deduced from the literature review of both the location and assortment research, these two domains have been investigated separately in the past, whereas in practice, when a facility is opened, it is impossible to neglect choosing the products to be offered. Consumers' tastes and choices are different in various regions, and the products that are best-selling in one area of a country or a city may be under-sold in another region for various reasons, such as cultural differences, income levels, and so on. Therefore, in addition to choosing the optimal location of the new facility, the selection of the best product portfolio to be offered to the customer in accordance with the selected location must be determined. If we look at the issue from the assortment viewpoint, we will reach the same conclusion again. When we are faced with choosing a product portfolio for stores, this product portfolio should not necessarily be the same for all stores. Of course, the answer varies depending on the location of the store and the choice of customers in that area. The importance of the interdependence of these two categories becomes more apparent when this issue should be considered in a competitive environment, where the consumer can choose another facility to buy their products or services, and therefore the facility owner should increasingly pay attention to choosing the optimal place as well as the optimal product assortment.

It should also be noted that these two variables, because they affect each other, must be considered simultaneously, otherwise it may result in diminishing sales or lower profits. For example, assume that if we are faced with a pure location problem, location A is selected as the optimal solution between locations A and B, and then if we want to select its assortment, scenario X (for location A) is the optimal assortment, while if we consider these two variables together, selecting assortment Y for location B can be the optimal solution.

To the best of our knowledge, this paper is the first work to investigate the location and assortment planning of a new facility simultaneously in a competitive environment. We call this problem: competitive "location-assortment" model.

3. Proposed Model

Consider a competitive market in which stores compete with each other to sell groups of products. In this competitive market, consider a chain that has *o* stores of *m* existing ones and *m-o* belongs to the competitors. The chain wants to open a new store. There are *g* potential locations for choosing the best site for the new facility. Assume that in this competitive market, there are *n* customers that each of them has different demands for various products.

On the other hand, *p* different products are offered in the stores. For kth product, there are r_k different Stock Keeping Units (SKUs) from various brands. For the new store, in addition to choosing the optimal location, we must specify which products and SKUs should be offered.

For example, Fig 1 shows two samples of assortment scenarios.

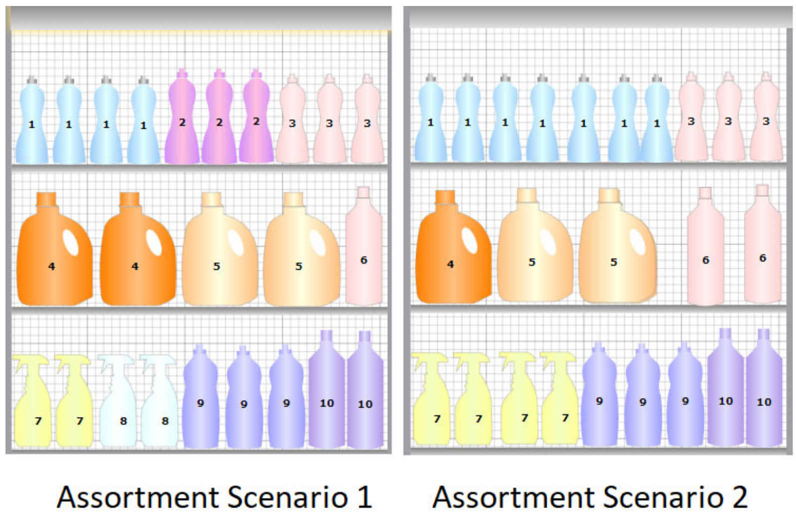

Fig. 1. Different assortment scenario

As can be seen, on the first floor (from above) there is a product in both scenarios, with the difference that in scenario 1, three different SKUs (brands) and in scenario 2, two different SKUs are located, and in this scenario, SKU 2 is removed and replaced its vacancy with SKU 1. This is also seen in other floors.

Assume that the size of the potential locations for the stores is the same. In this model, assortment-based substitution is considered for the customer demand. Assortment-based substitution is the switch to an available variant by a customer when her favorite product is not carried in the store. Therefore, if we assume that the demand of tth SKU of kth product for ith customer when the product is available in the store is D_{ikt}, the effective demand rate function under this substitution model is:

$$w_{ikts} = D_{ikt} + \sum_{t' \notin S} \alpha_{iktt'} D_{ikt'} \tag{1}$$

Where $\alpha_{iktt'}$ is the probability of substituting tth SKU of kth product for t'^{th} SKU and S is the assortment scenario of the new store.

The products are assumed to be necessary and so, the demand is inelastic. Therefore, the demands of all customers are met by the existing stores. When a new store enters the market for offering a given product, some parts of the market share of the existing stores will be cannibalized. In this competitive market, the chain wants to open a new store (among *g* potential locations). The chain seeks to find answers to the following two questions:

1. What is the optimal location of a new store?

2. In a new store, which scenario of assortment should be selected?

The following notations will be used throughout:

Indices:

i: Index of customers; $i = 1, \dots, n$
l: Index of existing facilities; $l = 1, \dots, m$
j: Index of potential locations; $j = 1, \dots, g$
k: Index of products; $k = 1, \dots, p$
t: Index of SKUs of kth product; $t = 1, \dots, r_k$
s: Index of assortment scenarios; $s = 1, \dots, \theta$

Data:

P_i: Location of the i[th] customer
Z_j: Location of the j[th] potential new store
F_l: Location of the l[th] existing facility
d_{il}: Distance between P_i and F_l
d'_{ij}: Distance between P_i and Z_j
β_{il}: Quality of F_l perceived by P_i
β'_{ij}: Quality of $Z_j$perceived by P_i
π_{si}: Weight for the assortment scenario *s* as perceived by P_i
Pr_{kt}: Profit of t[th] SKU k[th] product

Variable:

x_{js}: A binary variable that is equal to 1 if the s[th] assortment is chosen for store at j[th] potential location, 0 otherwise

The patronizing behavior of the customers is considered according to Huff rule. In this rule, the attraction that customer *i* feels for existing store *l* has a direct relationship with design (quality) of the store and a reverse one with a function of distance between the customer and the facility (Fernández et al., 2007)

$$A_{il} = {\beta_{il}}\Big/{g_i(d_{il})} \tag{2}$$

where $g_i(.)$ is a non-negative non-decreasing function. Similar to Eq. (2), the attraction that customer *i* feels for the new store is as follows:

$$A'_{ijs} = {\pi_{si}\beta'_{ij}}\Big/{g_i\left(d'_{ij}\right)} \tag{3}$$

Obviously, the more the attraction to one store, the higher the probability of attracting the customers by the mentioned store. Therefore, the market share of the chain from the demand of a particular customer *i* equals the total attraction of the facilities of the chain (existing and new) divided by the attraction of all facilities as follows:

$$M_i = \frac{\sum_{l=1}^{o} A_{il} + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A'_{ijs}x_{js}}{\sum_{l=1}^{m} A_{il} + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A'_{ijs}x_{js}} \tag{4}$$

The chain's total profit is as follows:

$$\sum_{i=1}^{n}\sum_{k=1}^{p}\sum_{t=1}^{r_k}\sum_{j=1}^{g}\sum_{s=1}^{\theta} Pr_{kt}\left(D_{ikt} + \sum_{t'\notin S}\alpha_{iktt'}D_{ikt'}\right)x_{js}\frac{\sum_{l=1}^{o} A_{il} + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A'_{ijs}x_{js}}{\sum_{l=1}^{m} A_{il} + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A'_{ijs}x_{js}} \tag{5}$$

Note that, in the Eq (5), depending on the location and the scenario of assortment, the demand of products for various customers is different, because, as mentioned earlier, due to the availability (or not availability) of products in the stores and the possibility of replacement, part of the demand for one product is transferred to another product.

The competitive "Location-Assortment" problem (P1) to be solved is:

$$max\ z = \sum_{i=1}^{n}\sum_{k=1}^{p}\sum_{t=1}^{r_k}\sum_{j=1}^{g}\sum_{s=1}^{\theta} Pr_{kt}w_{ikts}x_{js}\frac{\sum_{l=1}^{o} A_{il} + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A'_{ijs}x_{js}}{\sum_{l=1}^{m} A_{il} + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A'_{ijs}x_{js}} \tag{6}$$

s.t.

$$\sum_{j=1}^{g}\sum_{s=1}^{\theta} x_{js} = 1 \tag{7}$$

$$x_{js} \in \{0,1\} \qquad j = 1, \dots, g\ and\ s = 1, \dots, \theta \tag{8}$$

Where, the Eq. (6) represents the chain's profit, which must be maximized. Eq. (7) ensures that among potential locations and assortment scenarios, one of them should be chosen.

The model is an integer nonlinear programming problem with a particular structure: numerators and denominators of a ratio differ by the constants only. It has been shown in Benati & Hansen (2002) that a similar problem to P1 is NP-hard. Intuitively, the reason is easy to understand: if $g_i\left(d'_{ij}\right) \to 0$, the customers do not like to travel and tend to buy from the nearest store and therefore, the Problem P1 becomes close to the $(r|X_p) - medianoid$ problem with of 0/1 choices, whose NP-hardness was proved by Hakimi (1990).

4. Solution Methods

This section presents two methods for determining the optimal solution and then two techniques for obtaining the upper bound of the objective function in order to provide a comparison and validate two solution methods.

4.1. Integer linear formulation from fractional programming

Assume $\delta_i = \sum_{l=1}^{o} A_{il}$ and $\delta_i' = \sum_{l=1}^{o} A_{il}$ for $i = 1, \dots, n$.

The objective function can be expressed:

$$\sum_{i=1}^{n}\sum_{k=1}^{p}\sum_{t=1}^{r_k}\sum_{j=1}^{g}\sum_{s=1}^{\theta} Pr_{kt} w_{ikts} x_{js} \frac{\delta_i + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' x_{js}}{\delta_i' + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' x_{js}} \tag{9}$$

Let a variable

$$\vartheta_{ijs} = \frac{\delta_i + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' x_{js}}{\delta_i' + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' x_{js}}$$

As the denominator is positive this is equivalent to:

$$\vartheta_{ijs}\left(\delta_i' + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' x_{js}\right) = \delta_i + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' x_{js}$$

As a result

$$\vartheta_{ijs}\delta_i' + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' \vartheta_{ij} x_{js} - \delta_i - \sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' x_{js} = 0$$

Now we introduce a variable

$$\varphi_{ijs} = \vartheta_{ijs} x_{js}$$

For which the following inequalities are valid:

$$\varphi_{ijs} \le x_{js}$$

And

$$\varphi_{ijs} \ge \vartheta_{ijs} - (1 - x_{js})$$

Therefore,

$$\vartheta_{ijs}\delta_i' + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' \varphi_{ijs} - \delta_i - \sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' x_{js} = 0$$

So that

$$\vartheta_{ijs} = \frac{1}{\delta_i'}\left(\sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' x_{js} - \sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' \varphi_{ijs} + \delta_i\right)$$

Finally, the problem may be rewritten as follows (P2):

$$max\ z = \sum_{i=1}^{n}\sum_{k=1}^{p}\sum_{t=1}^{r_k}\sum_{j=1}^{g}\sum_{s=1}^{\theta} Pr_{kt} w_{ikts}\, \varphi_{ijs} \tag{10}$$

s.t.

$$\varphi_{ijs} \ge \frac{1}{\delta_i'}\left(\sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' x_{js} - \sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' \varphi_{ijs} + \delta_i\right) - (1 - x_{js}); \quad for\ every\ i, j, s \tag{11}$$

$$\varphi_{ijs} \le x_{js}; \qquad for\ every\ i, j, s \tag{12}$$

$$\sum_{j=1}^{g}\sum_{s=1}^{\theta} x_{js} = 1; \tag{13}$$

$$\varphi_{ijs} \geq 0; x_j \in \{0,1\} \qquad for\ every\ i,j,s \tag{14}$$

which can be directly handled by MIP solvers. For large-scale problems, the resulting MIP may be difficult to solve to global optimality in a reasonable computational time.

4.2. *Discrete Firefly Algorithm (DFA)*

Since the problem is NP-hard, heuristic methods must be devised to implement a procedure and to obtain an approximate solution, when the dimension of the problem makes exact methods too time-consuming.

FA is a new ecological intelligence meta-heuristic method for solving optimization problems, in which the search algorithm is inspired by firefly social behavior and the phenomenon of bioluminescent communication (Yang, 2009).

There are many applications and uses of FA in different aspects of engineering. The reader can refer to Tilahun & Ngnotchouye (2017) for further details.

To design a firefly inspired algorithm, the three rules are as follows:

1) All fireflies are homogenous, so a firefly is attracted to other fireflies despite its sex.
2) The attractiveness is proportional to the brightness that decreases as the distance between the fireflies increases. For both flashing fireflies, less light travels to the brighter one. If there is no brighter firefly than a particular one, this individual randomly moves through the search space.
3) The brightness of a firefly is determined or affected by the objective function.

For a maximization problem, brightness can simply be proportional to the value of the objective function. The main update formula for any couple of two fireflies *i* and *j* at X_i and X_j is:

$$X_i^{t+1} = X_i^t + \beta_0 e^{-\gamma r_{ij}^2}\left(X_i^t - X_j^t\right) + \alpha\varepsilon_i^t \tag{15}$$

Where α is a parameter controlling the step size, β_0 is the attractiveness at $r = 0$, the second term is due to the attraction, while the third term is randomization with the vector of random variables ε_i being drawn from a distribution. The distance between any pair of fireflies can be the Cartesian distance or the $l_2 - norm$.

In this paper, discrete firefly is used for obtaining location and assortment variables.

4.2.1. *Representation scheme*

The encoding scheme of the proposed method has been illustrated in Fig 2. This scheme denotes the location and assortment of the new store for a special firefly, which is indicated by an $g \times \theta$ matrix.

	Location (j)			
	X_{i11}^t	X_{i12}^t	…	X_{i1g}^t
	X_{i21}^t	X_{i22}^t	…	X_{i2g}^t
Assortment Scenario (s)	⋮	⋮	…	⋮
	$X_{i\theta1}^t$	$X_{i\theta2}^t$	…	$X_{i\theta g}^t$

Fig 2. Representation scheme of the solution

The location and assortment type of firefly *i* in the generation *t* can be denoted by $X_i^t = \left(X_{i11}^t, X_{i12}^t, \ldots, X_{i\theta g}^t\right)$. The value of 1 shows the location and assortment scenario of the new store.

4.2.2. *Initialization*

In this paper, the location of the new store is initialized in such a way that the total demand of each customer is calculated, and the new store is located in the closest potential location to that customer and if there are two or more customers with the same demand, one of them is randomly selected. For assortment type, the full assortment is considered as the initial solution.

4.2.3. The movement of fireflies

The movement of a firefly *i* attracted to another more attractive (brighter) firefly *j* is determined by relation (15).

4.2.4. Discretization

When firefly i moves toward firefly j, the position of firefly i changes from a binary number to a real number. Therefore, we must replace this real number by a binary one. By using the sigmoid function, the position value is limited to the interval of [0,1]. The structure of the discretization is fully described in Tilahun & Ngnotchouye (2017).

4.2.5. Pseudo code of the algorithm

The steps of the DFA can be summarized as the pseudo code shown in Fig. 3.

Run DFA
Generate initial population of fireflies X_i $(i = 1,2, \dots, n)$.
Determine objective function. Light intensity I_i at X_i
Set light absorption coefficients γ, randomization parameters α and maximum iterations (MaxItr).
while (*t* <MaxItr)
  for *i* = 1 : *n* all fireflies
    for *j* = 1 : *i*
      if $(I_j > I_i)$, Move firefly *i* towards *j* in all dimensions
        Attractiveness varies with distance *r* via $\exp[-\gamma r^2]$ for location and assortment of new store.
          $$X_i = X_i + \beta_0 e^{-\gamma r^2}\left(X_j - X_i\right) + \alpha\varepsilon_i$$
        Discrete the decision variable of *i*-th firefly.
          $$S\left(X_{ijs}^t\right) = \frac{1}{1 + \exp\left(-X_{ijs}^t\right)}$$
          Each firefly locates new store and select new assortment based on its changes of probabilities.
        Evaluate new solution (position of *i*-th firefly) and update light intensity I_i.
      end if
    end for *j*
  end for *i*
  Rank the fireflies and find the current best
End while
Show the best-known solution and its objective value

Fig. 3. Procedure of the DFA algorithm.

4.3. Upper bounds for objective function

In this study, the quality of solution obtained from the model is very important. Hence, we present below two methods that can, in a short time, provide an exact upper bound for the objective function, which allows us to estimate the accuracy of the obtained solutions by proposed methods in previous sections.

4.3.1. The continuous relaxation of P2 is a linear programming

The continuous relaxation of P2 is a linear programming problem (P3). The result of model P3 is called UB1.

$$Max\ z = \sum_{i=1}^{n}\sum_{k=1}^{p}\sum_{t=1}^{r_k}\sum_{j=1}^{g}\sum_{s=1}^{\theta} Pr_{kt} w_{ikts}\,\varphi_{ijs} \tag{16}$$

s.t.

$$\varphi_{ijs} \geq \frac{1}{\delta_i'}\left(\sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}' x_{js} - \sum_{s=1}^{\theta}\sum_{j=1}^{g} A_{ijs}'\varphi_{ijs} + \delta_i\right) - \left(1 - x_{js}\right); \quad for\ every\ i,j,s \tag{17}$$

$$\varphi_{ijs} \leq x_{js}; \qquad for\ every\ i,j,s \tag{18}$$

$$\sum_{j=1}^{g}\sum_{s=1}^{\theta} x_{js} \leq 1; \tag{19}$$

$$\varphi_{ijs} and\ x_{js} \geq 0 \qquad for\ every\ i,j,s \tag{20}$$

By using this relaxation technique, the NP-hard optimization problem (integer programming) is turned into a linear programming that is solvable in polynomial time.

4.3.2. The continuous and full assortment relaxation of the objective function of P1 is concave

Since w_{ikts} in the full assortment scenario is at its maximum value, we can prove that continuous relaxation of the following function is concave.

$$z' = \sum_{i=1}^{n}\sum_{k=1}^{p}\sum_{t=1}^{r_k} Pr_{kt} D_{ikt} \frac{\sum_{l=1}^{o} A_{il} + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A'_{ijs}x_{js}}{\sum_{l=1}^{m} A_{il} + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A'_{ijs}x_{js}} \tag{21}$$

where D_{ikt} is the full assortment scenario. In this case, the continuous relaxation of the function does not have any singularity points, which allows us to compute the second cross-derivative by standard methods. As the objective function is a sum of ratios, it is sufficient to prove that each term is concave. Therefore, consider the term *ikt:*

$$f(x_1, x_2, \dots, x_n) = Pr_{kt} D_{ikt} \frac{\sum_{l=1}^{o} A_{il} + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A'_{ijs}x_{js}}{\sum_{l=1}^{m} A_{il} + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A'_{ijs}x_{js}} \tag{22}$$

In the Hessian matrix, $H = [h_{mw}]$, where:

$$h_{mw} = \frac{\partial f(x_1, x_2, \dots, x_n)}{\partial x_m \partial x_w} = -2Pr_{kt} D_{ikt} A'_{im} A'_{iw} \frac{\sum_{l=1}^{m} A_{il} - \sum_{l=1}^{o} A_{il}}{\left(\sum_{l=1}^{m} A_{il} + \sum_{s=1}^{\theta}\sum_{j=1}^{g} A'_{ijs}x_{js}\right)^3} \tag{22}$$

Since $\sum_{l=1}^{m} A_{il} - \sum_{l=1}^{o} A_{il} \geq 0$, $h_{mw}$is always negative. Every submatrix of order two has a determinant equal to 0, whereas the diagonal elements are negative. Thus, $f(x_1, x_2, \dots, x_n)$ is concave.

The continuous relaxation of the domain is a convex set, so we can compute an upper bound by using gradient methods. The result of this method is called UB2.

5. Numerical examples

This section provides computational experiments to evaluate the performance of the proposed model. At first, a small-sized problem is solved by DICOPT as an MINLP solver available in the modeling system GAMS and the results are analyzed. Then several examples with different sizes are solved to show the efficiency and effectiveness of the proposed methods. All computational experiments are done on a Core i7 with 3.5 GHz CPU and 8 GB memory. The heuristic is coded in MATLAB R2020b.

5.1. Illustrative examples

5.1.1. Example 1

It is assumed that $m = 2, o = 1, n = 10, k = 3, t = 3, F_1 = (1,4), F_2 = (2,3), \beta_{i1} = 8, \beta_{i2} = 9$.
$g_i(d_{il}) = \varepsilon + d_{il}^2$. Table 3 depicts P_i and D_{ikt}.

Table 3
The locations and the demands of different customers

	Location	(0,0)	(2,0)	(4,1)	(1,2)	(3,2)	(2.5,2.5)	(1,3)	(1,4)	(2.5,4)	(4,4)	Profit
Product 1	**SKU 1**	5	8	10	15	13	30	50	100	100	100	6.7
	SKU 2	40	45	50	55	50	45	40	45	50	45	3.2
	SKU 3	100	100	100	90	90	50	40	10	5	5	2.0
Product 2	**SKU 1**	1	19	31	46	47	52	64	60	88	98	3.2
	SKU 2	23	30	21	51	71	13	61	74	76	40	3.8
	SKU 3	25	51	14	27	5	26	97	35	99	33	4.0
Product 3	**SKU 1**	77	106	180	37	104	35	55	39	29	30	3.0
	SKU 2	84	108	185	37	99	35	52	40	39	36	4.0
	SKU 3	83	105	180	35	114	31	53	39	35	26	5.0

Z_j are (2,4), (1,2) and (4,0) and $\beta_{ij}{}' = 8$.

For all three products, the chain's existing store sells SKUs 2 and 3 and the competitor's store offers SKUs 1 and 2. In Table 4, seven assortment scenarios and $\alpha_{iktt'}$ are depicted.

In Table 4, for example, in the scenario that only SKU 1 is present for product 1, 80% of customers that SKU 2 are their favorite and 10% of customers that want to buy SKU 3, will purchase SKU 1 instead. In dual assortments, the interpretation is slightly different. For example, in scenario 1-2 which shows the combination of SKU 1 and 2, 10% of customers whose demands are SKU 3 will buy from SKU 1 and 50% of them will meet their demands from SKU 2 and in fact 40% of them do not purchase anything.

$$\pi_{si} = 0.95, 095, 095, 1,1,1, 1.05 \; for \; s = 1,..,7$$

Table 4
The substitution probabilities for different SKUs in different assortment scenarios

Products	SKUs	Assortment scenarios					
		1	2	3	4	5	6
		1	2	3	1-2	1-3	2-3
Product 1	SKU 1	-	0.6	0.1	0.1	0.3	-
	SKU 2	0.8	-	0.7	0.5	-	0.6
	SKU 3	0.1	0.8	-	-	0.3	0.1
Product 2	SKU 1	-	0.8	0.1	0.4	0.5	-
	SKU 2	0.7	-	0.1	0.5	-	0.4
	SKU 3	0.7	0.8	-	-	0.3	0.5
Product 3	SKU 1	-	0.9	0.95	0.3	0.3	-
	SKU 2	0.9	-	0.95	0.3	-	0.3
	SKU 3	0.8	0.8	-	-	0.3	0.3

After solving the model, the chain's profit for every assortment scenario and potential location in terms of three products is shown in Table 5.

Table 5
The chain's profit for different assortment scenario

		Assortment scenarios						
		1	**2**	**3**	**4**	**5**	**6**	**7**
		1	**2**	**3**	**1-2**	**1-3**	**2-3**	**1-2-3**
Product 1	**Location 1**	2871	2367	2033	2487	2785	2370	2860
	Location 2	2699	2576	2292	2769	2686	2533	2762
	Location 3	2658	2623	2441	2279	2671	2598	2731
Product 2	**Location 1**	2739	2926	2471	2980	2974	3093	3086
	Location 2	2799	2947	2604	2991	2975	3068	3073
	Location 3	2896	2970	2788	3006	2997	3046	3060
Product 3	**Location 1**	3479	3735	4079	3618	3737	3873	3951
	Location 2	3427	3758	4209	3591	3751	3921	4010
	Location 3	3424	3773	4247	3596	3763	3940	4030

For example, regardless of products 2 and 3, if we only offer product 1 in assortment scenario 1-2 at potential location 1, the chain profit is 2487.

As can be seen from Table 5, the best assortment for product 1 is as follows:

$s = 1\ if$ the potential location 1 is selected

$s = 4\ if$ the potential location 2 is selected

$s = 7\ if$ the potential location 3 is selected

This shows that the best assortment scenario depends entirely on the location of the store. Among these three locations, the highest profit belongs to the potential location 1.

Similarly, for product 2 and 3 the results are visible in Table 5. The best location for different products (regardless of the other two products) is as follows:

Product 1: Location 1

Product 2: Location 1

Product 3: Location 3

As these products must be considered together, the best solution is to choose potential location 2 with the following scenarios for products:

Product 1: $s = 4$

Product 2: $s = 7$

Product 3: $s = 3$

This example easily shows the dependency of two variables: location and assortment. In this case, the profit of the chain is equal to 10,051.

Non-Optimal solutions comparison

In this section, we examine what is lost if we did not consider location and assortment variables simultaneously.
Table 6 provides a comparison between the optimal solution and some feasible answers.

Table 6
Comparison between different solutions

Solution	Location	Assortment for product 1	Assortment for product 2	Assortment for product 2	Objective function	% Loss
1	2	4	7	3	10,051	-
2	1	4	7	3	9,652	4.0%
3	3	4	7	3	9,586	4.6%
4	1	7	7	7	9,898	1.5%
5	2	7	7	7	9,845	2.0%
6	3	7	7	7	9,820	2.3%
7	1	6	6	6	9,335	7.1%
8	2	6	6	6	9,522	5.3%
9	3	6	6	6	9,584	4.6%
10	1	3	3	1	7,983	20.6%

In Table 6, row 1 is the optimal answer to the problem and the other 9 solutions are selected for comparison. In the solutions 2 and 3, the current optimal assortment is considered for locations 1 and 3, but the computational result shows that in these cases, we will lose 4.0% and 4.6% of profit, respectively.

The solutions 4, 5, and 6 are considered the full assortment: in these conditions, we will lose profits between 1.5% and 2.3%.

The solutions 7, 8 and 9 are the current assortment of the chain's existing store, because we want to investigate the impact of using the current assortment of the existing store in the new one. In these cases, we will lose between 4.6% and 7.1%.

Row 10 is also the worst possible answer to this example, with a profit loss of 20.6%.

As it was observed, the percentage of profit loss due to not considering two variables at the same time is significant, and this proves that these two variables are interdependent and must be considered simultaneously in order to achieve the maximum possible profit.

5.1.2. Other examples

To demonstrate the efficiency of the model, ten problems are solved, and the results have been shown in Table 7. Instances consist of different numbers of customers (n = 5, 10, 20, 50) and different numbers of facilities (m = 2, 4,10) for one product and 3 SKUs.

We randomly selected the parameters of the problems from the following intervals for each type of setting:

D_{ikt}~ U(1, 100), P_i~ U(1, 10)^2, Z_j~ U(1, 10)^2, f_l~ U(1, 10)^2 $\alpha_{iktt'}$~ U(0.1, 1), β_{il}~ U(1, 10), β'_{il}~ U(1, 10), Pr_{kt}~ U(3,6), ε=0.005.

Table 7
Results for the different problems

#	n	m	Optimal location	Optimal assortment	% Average loss of other assortment scenarios	% Average loss of other potential locations
1	5	2	2	1	8.5%	8.3%
2	5	4	1	4	7.2%	12.3%
3	10	2	5	4	11.2%	9.6%
4	10	4	5	7	5.6%	10.9%
5	20	2	2	5	10.3%	4.8%
6	20	4	4	2	4.9%	5.2%
7	20	10	3	3	8.3%	3.1%
8	50	2	1	6	7.6%	5.2%
9	50	4	7	4	10.5%	12.8%
10	50	10	1	7	12.3%	11.6%

5.1.3. Sensitivity Analysis

In this section, the changes are made in the model parameters of Example 1 and two other examples are examined and the percentage of changes in the optimal solutions is analyzed.

5.1.3.1. Probability substitution changes

Table 8 shows the rate of change of the optimal solution relative to the change of substitution probabilities. As shown in Table 8, the effect of substitution probabilities on the objective function and the optimal solution is very strong, and the estimation of this parameter must be done accurately, otherwise the wrong estimation will lead to incorrect decisions. When the probability of replacement is zero or very low, the model follows a full assortment because the products cannot be replaced with each other, but when the probability of replacement is higher, the occurrence of full assortment modes becomes lower and also the profit of the chain becomes higher. Changing the probability of replacement, in addition to affecting the store's assortment, has also affected the optimal facility location.

Table 8
Probability substitution changes effect

Example	#	Change	Opt. Location	Opt. Assortment P1	Opt. Assortment P2	Opt. Assortment P3	Objective function	% Change
1	1	$\alpha_{iktt'} = 0$	1	7	7	7	8,053	Base
	2	$\alpha_{iktt'} = 0.2$	1	7	7	7	8,605	6.9%
	3	$\alpha_{iktt'} = 0.5$	1	1	7	7	9,465	17.5%
	4	$\alpha_{iktt'} = 0.8$	2	1	7	4	10,661	32.4%
	5	$\alpha_{iktt'} = 1$	2	1	4	4	13,714	70.3%
2	1	$\alpha_{iktt'} = 0$	3	7	7	7	14,273	Base
	2	$\alpha_{iktt'} = 0.2$	3	7	4	7	15,786	10.6%
	3	$\alpha_{iktt'} = 0.5$	3	2	4	5	16,542	15.9%
	4	$\alpha_{iktt'} = 0.8$	3	2	4	5	19,725	38.2%
	5	$\alpha_{iktt'} = 1$	2	2	1	3	22,152	55.2%
3	1	$\alpha_{iktt'} = 0$	4	7	7	7	10,838	Base
	2	$\alpha_{iktt'} = 0.2$	4	2	7	7	11,402	5.2%
	3	$\alpha_{iktt'} = 0.5$	4	2	6	6	12,128	11.9%
	4	$\alpha_{iktt'} = 0.8$	1	2	6	3	13,157	21.4%
	5	$\alpha_{iktt'} = 1$	1	2	2	3	14,707	35.7%

5.1.3.2. New Store's quality changes

Table 9 shows the optimal solution changes to varying the quality of new store.

Table 9
New Store's quality changes effect

Example	#	Change	Opt. Location	Opt. Assortment P1	Opt. Assortment P2	Opt. Assortment P3	Objective function	% Change
1	1	$\beta'_{ij} = 8$	2	4	7	3	10,051	Base
	2	$\beta'_{ij} = 11$	2	4	7	3	10,872	8.2%
	3	$\beta'_{ij} = 12$	2	4	6	3	11,114	10.6%
	4	$\beta'_{ij} = 100$	2	4	6	3	17,444	73.6%
	5	$\beta'_{ij} = 7$	2	4	7	3	9,728	-3.2%
	6	$\beta'_{ij} = 6$	3	7	7	3	9,406	-6.4%
	7	$\beta'_{ij} = 1$	3	7	7	3	7,011	-30.2%
2	1	$\beta'_{ij} = 8$	3	4	5	7	19,526	Base
	2	$\beta'_{ij} = 11$	3	4	5	7	20,951	7.3%
	3	$\beta'_{ij} = 12$	3	4	2	6	21,752	11.4%
	4	$\beta'_{ij} = 100$	3	4	2	6	36,162	85.2%
	5	$\beta'_{ij} = 7$	3	4	5	7	18,921	-3.1%
	6	$\beta'_{ij} = 6$	3	4	7	7	18,511	-5.2%
	7	$\beta'_{ij} = 1$	2	4	7	7	14,391	-26.3%
3	1	$\beta'_{ij} = 8$	4	1	5	6	13,257	Base
	2	$\beta'_{ij} = 11$	4	1	5	6	14,463	9.1%
	3	$\beta'_{ij} = 12$	4	1	2	6	15,033	13.4%
	4	$\beta'_{ij} = 100$	4	1	2	6	26,289	98.3%
	5	$\beta'_{ij} = 7$	4	1	5	6	12,912	-2.6%
	6	$\beta'_{ij} = 6$	1	4	5	6	12,647	-4.6%
	7	$\beta'_{ij} = 1$	1	7	5	7	10,327	-22.1%

As shown in Table 9, the effect of the quality of the new facility on the optimal location and assortment is not tangible (hardly the optimal location and assortment has been changed) but it has a significant effect on the objective function value in which 1 unit increase or decrease of the quality can change objective function value about 2% - 3%.

5.1.3.3. Distance decay changes

In this section, we examine the impact of the shape of the distance effect decay functions $g_i(d_{il}) = d_{il}^{\omega}$ in the solution of the problems, more particularly with respect to the choice of the exponent ω. The five examined cases are ω = 1,2,4,8,16. The case ω = 2 is considered as the original problem.

As ω increases, the customer's willingness to use more distant facilities decreases and only refers to closer facilities. From Table 10 we can see that the locations and assortments are only slightly affected by the changes in the parameter ω. However, with the increase of ω, the amount of objective function is strongly affected, which is due to the decrease in demand for stores that are only visited by local customers.

Similarly, the rest of the model parameters can be tested, but the effect of some parameters in the model is quite clear. For example, if the variety of products is more attractive to customers (π), the model will move to a full assortment mode, and therefore the correct estimation of this parameter is very important. As another example, the higher the profit of a product, the more likely it is to be included in the assortment, especially when the probability of its replacement is low.

Table 10
Distance decay changes effect

Example	#	Change	Opt. Location	Opt. Assortment P1	Opt. Assortment P2	Opt. Assortment P3	Objective function	% Change
1	**1**	$\omega = 2$	2	4	7	3	10,051	Base
	2	$\omega = 1$	2	4	6	3	11,219	11.6%
	3	$\omega = 4$	1	1	6	3	8,235	-18.1%
	4	$\omega = 8$	1	1	6	3	6,525	-35.1%
	5	$\omega = 16$	1	1	6	3	5,862	-41.7%
2	**1**	$\omega = 2$	3	4	5	7	19,526	Base
	2	$\omega = 1$	3	4	5	6	22,142	13.4%
	3	$\omega = 4$	2	4	5	7	15,601	-20.1%
	4	$\omega = 8$	2	4	5	7	11,286	-42.2%
	5	$\omega = 16$	5	4	2	7	9,314	-52.3%
3	**1**	$\omega = 2$	4	1	5	6	13,257	Base
	2	$\omega = 1$	4	1	5	6	21,322	9.2%
	3	$\omega = 4$	1	4	5	6	16,363	-16.2%
	4	$\omega = 8$	1	4	5	6	13,785	-29.4%
	5	$\omega = 16$	1	4	7	6	12,126	-37.9%

The conclusion that can be drawn here is that in addition to this subject that is shown in the previous section about the importance and dependency of two location and assortment variables, the model parameters, especially those that have a great impact on the optimal solution, must be estimated correctly to make the right decisions.

5.2. The test problems

Several examples are provided in this section for testing the performance of the proposed methods: 1) MINLP solver (DICOPT) for P1, 2) MIP solver (CPLEX-12.6.1.0) for P2 and 3) DFA method. The instances vary in the number of customers (n = 50, 100, 200), the number of existing facilities (m = 10, 20), the number of potential locations (g = 15, 30, 60), the number of products (p = 5, 10) and the number of brands for every product (r_k=2, 4).

10 problems have been generated for every type of settings, in which, the parameters of the problems are randomly chosen from the following intervals:

D_{ikt}~ U(1, 100), P_i~ U(1, 10)^2, Z_j~ U(1, 10)^2, f_l~ U(1, 10)^2 $\alpha_{iktt'}$~ U(0.1, 1), β_{il}~ U(1, 10), β'_{il}~ U(1, 10), Pr_{kt}~ U(3,6), ε=0.005.

5.2.1. The Performance of CPLEX in comparison with upper bounds

Our first step involves evaluating CPLEX's performance against upper bounds UB1 and UB2, so that if it is valid, we can compare DFA's solution with CPLEX. Table 11 records the objective function values of ten problems of varying sizes.

Table 11
Results for the different problem size

#	n	m	Objective Function			$CPLEX/UB1$	$CPLEX/UB2$
			CPLEX	UB 1	UB 2		
1	50	5	246,506	254,130	283,340	0.97	0.87
2	50	10	127,103	132,399	146,095	0.96	0.87
3	50	20	143,699	202,393	159,666	0.71	0.90
4	100	5	216,627	228,028	243,401	0.95	0.89
5	100	10	149,865	180,560	197,191	0.83	0.76
6	100	20	272,602	320,708	278,165	0.85	0.98
7	200	5	644,912	749,898	826,810	0.86	0.78
8	200	10	571,149	751,512	664,127	0.76	0.86
9	200	20	293,371	333,376	386,014	0.88	0.76
10	200	30	416,800	473,636	463,111	0.88	0.90

As shown in Table 11, the CPLEX method provides good quality solutions even with large data sets.

5.2.2. Tuning of the DFA parameters

For the proposed DFA strategy, parameter tuning is necessary to achieve the best performance. Three parameters determine the behavior of the FA method: α, γ and β_0. In Table 12, we present the average results of five generated problems for the cases n=50, m=10, g=15, p=5, and r_k=2 and 20 runs of the DFA for each parameter value.

Table 12
The average results for different value of DFA parameters

γ	β0	Difference in Obj (%) α 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	**0.1**	3.8	2.9	2.8	3.0	3.7	4.5	4.6	5.6	6.0	6.2
	0.5	3.7	2.5	2.3	2.7	3.2	3.5	3.6	3.8	4.8	5.3
	1.0	2.7	1.7	2.0	2.5	3.1	3.3	3.4	3.5	3.8	4.9
	1.5	3.3	2.2	2.2	2.6	3.2	3.4	3.9	5.1	5.2	5.8
	2.0	3.7	2.8	1.9	2.8	3.5	3.9	4.5	5.7	5.9	6.3
0.25	**0.1**	2.6	1.9	2.0	2.1	2.6	2.9	3.5	4.1	5.2	6.1
	0.5	2.4	1.4	1.3	1.6	1.9	2.5	3.0	3.5	4.2	5.3
	1.0	2.3	1.3	1.0	1.2	1.5	1.8	2.1	3.0	3.6	4.1
	1.5	2.5	1.5	1.6	1.9	2.3	2.7	3.2	3.6	3.9	5.1
	2.0	2.6	2.1	2.0	2.6	3.0	3.2	3.4	3.9	4.9	5.8
0.5	**0.1**	2.0	1.1	0.9	1.0	1.1	1.6	1.9	2.3	3.1	3.5
	0.5	1.8	0.9	0.8	0.9	0.9	1.0	1.2	1.5	1.9	2.3
	1.0	1.4	0.8	**0.7**	0.8	0.9	1.0	1.1	1.4	1.7	2.0
	1.5	1.8	0.9	0.8	0.9	1.0	1.2	1.4	1.8	2.1	2.9
	2.0	2.1	1.0	0.9	1.1	1.5	1.6	2.1	2.5	3.5	3.8
0.75	**0.1**	2.6	1.8	2.0	2.2	2.5	2.9	3.4	3.9	4.6	5.7
	0.5	2.2	1.6	1.7	1.8	1.9	2.3	2.9	3.1	4.1	5.2
	1.0	1.8	1.2	1.3	1.5	1.8	2.0	2.3	2.6	3.7	4.4
	1.5	2.1	1.7	1.7	1.9	2.2	2.5	2.9	3.2	3.8	4.5
	2.0	2.8	2.1	1.9	2.3	2.5	2.7	3.2	3.6	4.2	5.2
0.9	**0.1**	3.9	2.9	2.9	3.2	3.6	4.5	4.9	5.2	5.6	6.3
	0.5	3.3	2.8	2.7	3.1	3.3	4.1	4.3	4.5	5.0	5.8
	1.0	2.4	1.9	2.5	2.9	3.0	3.5	3.9	4.2	4.6	5.1
	1.5	2.7	2.1	2.6	3.1	3.5	4.2	4.3	4.5	5.1	5.9
	2.0	3.5	2.6	2.8	3.3	3.8	4.8	5.2	5.5	6.0	6.6

By comparing different values, $\beta_0 = 1,\ \gamma = 0.5$ and $\alpha = 0.3$ leads to the best performance of the algorithm. The optimal solution can be found after about 500 evaluations for most cases. So, 25 fireflies and 20 generations have been selected in the computational experiment.

5.2.3. Investigating the performance of different methods

Now we can examine how different methods perform. In Table 13, the results corresponding to the 10 generated problems for the case n=50, m=10, g=15, p=5, r_k=2 and 100 runs of the DFA are presented one by one. The last two lines show the total average and total standard deviation.

We present the differences between the optimal value obtained by the optimization solvers and the best solution obtained by the DFA method in the 100 runs, in percentage. The column "Times found" refers to the number of times that DFA found the best solution. Also, the CPU time spent by the MINLP solver, MIP solver and DFA method for 10 generated problems are presented.

Table 13
The difference in objective and CPU time for the ten examples with 50 customers, 10 existing facilities, 15 potential locations, 5 products and 2 brands for each product

Problem	Difference in obj (%)	Times found	CPU seconds DICOPT	CPLEX	DFA
1	0.514	2	798.30	22.49	0.76
2	0.373	3	741.96	28.23	1.26
3	1.876	1	657.68	22.78	0.77
4	0.638	1	780.43	21.04	0.58
5	0.483	2	705.62	23.52	1.28
6	0.811	1	715.29	24.53	0.81
7	1.480	1	685.06	26.24	1.48
8	0.969	1	760.80	20.64	1.43
9	0.000	26	719.83	20.27	1.17
10	0.417	2	652.13	24.67	0.78
Average	**0.756**	**4.0**	**721.71**	**23.44**	**1.03**
Standard Deviation	**0.559**	**7.8**	**49.26**	**2.55**	**0.32**

As can be seen from Table 13, the solution of DFA is not much different from the optimization solvers in terms of the objective function, although there are cases where the difference is much larger than the average. In addition, DFA is much faster than the other two methods.

From now on, only average values are shown to check the results. Depending on the number of products, a summary table is created. Each line in it corresponds to a table like Table 10. The values in Table 14 represent the average values of the solved instances, and the standard deviation values are shown in brackets. Since the objective function difference for some problems is sometimes much larger than the average, we will also show the maximum difference in the set of 10 problems. The last line shows the average for the setting, regardless of the number of products.

Table 14
The average and standard deviation values of difference in objective and CPU time for the examples with 50 customers, 10 existing facilities, 15 potential locations, 2 brands and 5, 10 products

Number of Products	Difference in		CPU seconds		
	Obj (%)	Max (%)	DICOPT	CPLEX	DFA
5	0.76 (0.56)	1.9	721.71 (49.26)	23.44 (2.55)	1.03 (0.32)
10	0.85 (0.59)	3.2	3521.71 (215.10)	62.59 (9.67)	1.70 (0.99)
All	**0.81 (0.58)**	**3.2**	**2121.71 (132.18)**	**43.02 (6.11)**	**1.37 (0.66)**

As Table 14 shows, when the number of products increases, the solution time will increase significantly, especially in MINLP solver. The DFA is a very fast method and at the same time it produces very good quality solutions. The MIP solver is recommended till here, because although it has more CPU time than the DFA, it produces the optimal solution, and the CPU time is still short enough. This issue is examined in more detail in Table 15, where the number of potential locations has been increased.

Table 15
The average and standard deviation values of difference in objective and CPU time for the examples with 50 customers, 10 existing facilities, 10 products, 2 brands and 15, 30 and 60 potential locations

Number of Potential Locations	Difference in		CPU seconds		
	Obj (%)	Max (%)	DICOPT	CPLEX	DFA
15	0.9 (0.6)	3.2	3521.7 (215.1)	62.6 (9.7)	1.7 (1.0)
30	0.9 (0.3)	5.1	15844.5 (1025.4)	129. 3 (29.2)	4.9 (1.3)
60	0.8 (0.3)	6.3	98236.0 (10362.4)	302.3 (83.1)	13.1 (4.9)
All	**0.9 (0.4)**	**6.3**	**39200.7 (3867.6)**	**167.7 (40.7)**	**6.6 (2.4)**

As can be seen in Table 15, increasing the number of potential locations will drastically increase the MINLP solver CPU time, while MIP solver and heuristic methods are less affected. According to Table 14, the MINLP solver is no longer practical on this scale and must use the MIP solver and heuristic methods. The larger the problem size, the better the heuristic method is used due to less CPU time. But for medium-sized problems, the MIP solver is still effective.

Table 16 Shows the results of large sized instances with up to 200 customers obtained by MIP solver and heuristic method.

Table 16
Results for the problems with 60 potential locations and 10 products

Number of Customers	Number of Existing Facilities	Number of Brands	Difference in		CPU Time (seconds)	
			Obj (%)	Max (%)	MIP Solver	Heuristic
50	10	2	0.8 (0.3)	6.3	302 (83)	13 (5)
		4	0.9 (0.2)	5.2	736 (275)	26 (8)
	20	2	0.9 (0.2)	7.4	642 (252)	18 (6)
		4	1.0 (0.1)	7.7	2089 (516)	39 (12)
100	10	2	0.9 (0.4)	6.9	782 (211)	25 (9)
		4	1.0 (0.3)	7.3	1624 (575)	41 (14)
	20	2	0.9 (0.1)	9.3	1281 (175)	33 (13)
		4	1.1 (0.7)	8.5	3610 (812)	57 (16)
200	10	2	0.9 (0.3)	6.9	1763 (436)	47 (16)
		4	1.2 (0.2)	9.5	4681 (939)	82 (31)
	20	2	1.0 (0.1)	9.1	3589 (982)	70 (29)
		4	1.2 (0.1)	9.7	8736 (1028)	126 (35)

According to Table 16, as the size of the problem increases, the CPU time for MIP solver increases significantly. The DFA is less sensitive to these changes. Finally, we test the solution quality of the DFA algorithm, regardless of the size of the problem. The objective of this research is to statistically compare the performance of the DFA method, using a representative set of test problems that are of diverse properties. For this purpose, like the previous tables, 10 problems with different sizes have been generated and each of them has been run 100 times by the DFA algorithm. The results are shown in Table 17.

Table 17
Results for the random problems

Problem	Number of Customers	Number of Existing Facilities	Number of Potential Locations	Number of Products	Number of Brands	Difference in obj (%)
1	50	10	60	10	2	0.741
2	50	10	60	5	2	0.555
3	200	20	30	5	2	1.023
4	100	10	15	10	2	0.825
5	50	10	15	5	4	0.629
6	200	20	30	5	4	1.037
7	200	20	60	5	2	1.088
8	100	20	15	5	4	0.866
9	50	20	15	10	2	0.638
10	100	10	15	10	4	0.863
Average	**110**	**15**	**32**	**7**	**3**	**0.827**
SD	**66**	**5**	**21**	**3**	**1**	**0.186**

The *t*-test (hypothesis testing) is used to assess the effectiveness of the DFA algorithm. Effectiveness is defined as a high probability of finding a high-quality solution. Here, the quality of a solution is measured by how close the solution is to the known global solution as shown in the last column of Table 17 “Difference in obj” which is briefly called “Diff” in the following.

The objective of the effectiveness test is to examine whether $\begin{cases} H_0: \mu_{Diff} \leq 1\% \\ H_1: \mu_{Diff} > 1\% \end{cases}$. This test checks whether the quality of the obtained solutions is more than 99 % or not. The formula for calculating the *t* value is shown in Eq. (16):

$$t = \frac{\overline{Diff} - 1}{S(Diff)/\sqrt{n}} \tag{16}$$

First, we use the Kolmogorov-Smirnov test of Normality to check whether the data has a normal distribution. The value of the K-S test statistic (D) is 0.168, the P-Value is 0.898 and therefore the data does not differ significantly from that which is normally distributed.

The value of t-test is -2.96, the P-value is 0.992 and therefore the effectiveness of the DFA algorithm for generating high-quality solutions is accepted.

5.3. Case Study

In this section, we describe an actual application of the model to find the location and assortment of a chain store’s new branch in the city of Tehran, Iran.

5.3.1. Case description

Several discount stores can be found in Tehran, Iran. Ofogh Koorosh, Haftstore, Canbo, and Winmarket are some major discount store chains in Tehran. Table 18 shows the branches of these chain stores in Tehran, as well as the number of product groups and brands available in them.

Table 18
Information about discount stores in Tehran

Discount Store	Number of stores	Number of product groups	Number of brands
Ofogh Koorosh	431	154	1224
Haftstore	103	142	975
Canbo	123	147	1015
Winmarket	67	131	726

Tehran consists of 22 districts, each of them can be considered as a customer. Ofogh Koorosh is under study and the other stores are competitors. There are 20 potential locations. Each product group contains approximately 8 brands, so there are roughly 255 assortment scenarios for each product group.

5.3.2. Model Results

Since there are a lot of variables in the problem, we use the DFA algorithm to solve the existing case. The optimal location is 17th potential site and the optimal assortment scenario of different product groups are summarized in Table 19.

Table 19
Optimal assortment for new store of Ofogh Koorosh

PG*	OSSc**	PG	OSSc	PG	OSSc	PG	OSSc	PG	OSSc
1	215	32	139	63	254	94	232	125	75
2	233	33	61	64	115	95	239	126	75
3	155	34	52	65	167	96	56	127	134
4	17	35	190	66	70	97	5	128	231
5	35	36	22	67	243	98	52	129	154
6	196	37	134	68	144	99	243	130	238
7	38	38	173	69	226	100	31	131	105
8	123	39	98	70	114	101	180	132	65
9	214	40	220	71	256	102	32	133	241
10	214	41	230	72	110	103	38	134	249
11	137	42	108	73	72	104	137	135	204
12	175	43	118	74	20	105	42	136	213
13	8	44	245	75	126	106	246	137	153
14	88	45	60	76	247	107	130	138	20
15	19	46	31	77	226	108	89	139	246
16	30	47	253	78	10	109	253	140	224
17	213	48	215	79	234	110	93	141	85

Table 19
Optimal assortment for new store of Ofogh Koorosh (Continued)

PG*	OSSc**	PG	OSSc	PG	OSSc	PG	OSSc	PG	OSSc
18	113	49	206	80	212	111	83	142	64
19	190	50	41	81	246	112	121	143	86
20	186	51	80	82	28	113	84	144	130
21	256	52	11	83	145	114	38	145	54
22	10	53	103	84	241	115	37	146	125
23	95	54	242	85	168	116	116	147	121
24	259	55	87	86	157	117	58	148	50
25	207	56	43	87	215	118	28	149	15
26	231	57	180	88	219	119	247	150	126
27	159	58	163	89	204	120	172	151	54
28	81	59	193	90	112	121	8	152	72
29	177	60	228	91	232	122	235	153	210
30	180	61	67	92	112	123	64	154	150
31	39	62	204	93	220	124	66		

* PG: Product Group
** OSSc: Optimal Assortment Scenario

6. Conclusion

In this paper, a new concept is introduced in the competitive location literature. The concept is that when locating a new store, it is also important to choose the best product portfolio offered by that store. This choice is made with an assortment-oriented approach, meaning that customers may substitute products or brands if their favorite product is not available in the store. Therefore, a model has been developed to find the best location and assortment scenario in a competitive environment. In this model, it is assumed that the competitive environment is static, which means that competitors are already present in the market and compete with each other for the same products. Customer patronizing behavior is assumed based on Huff's rule, and customers split their demands among all existing and new stores. The more attractive a store is, the more likely the customer is fascinated by it.

The model developed in this paper is an integer nonlinear programming model. We reformulate the model into a mixed integer linear problem. Therefore, a standard optimization solver can be used for obtaining the optimal solutions to small- and medium-size problems. In addition, a heuristic algorithm has been developed to solve the model for large size problems. Several examples have been solved to evaluate the efficiency of the model and the proposed methods, and the results have been analyzed, and it was found that the developed methods have good performance. The results also show the importance of considering both location and assortment variables simultaneously.

Among the possible extensions of this work, we mention the multi-facility model, in which the chain wants to locate more than one new facility. Also, it is interesting to study the proposed model for the leader-follower case. Using the model for situations where the customer chooses the closest facility is another proposal for future studies. On the other hand, there are many other choice models in the assortment literature that can be added to the model, for example MNL, Nested, Mixed MNL etc.

Also, adding other variables such as facility design, shelf space location and pricing can be considered for further research.

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