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Joint effects of stochastic machine failure, backorder of permissible shortage, rework, and scrap on stock replenishing decision

Yuan-Shyi Peter Chiu[a], Yu-Ru Chen[a], Victoria Chiu[b], Singa Wang Chiu[c*]

[a]*Department of Industrial Engineering & Management, Chaoyang University of Technology, Taichung 413, Taiwan*
[b]*Department of Accounting, Finance and Law, State University of New York at Oswego, Oswego, NY 13126, USA*
[c]*Department of Business Administration, Chaoyang University of Technology, Taichung 413, Taiwan*

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A B S T R A C T

With the intention of addressing product quality, machine reliability, and acceptable service level issues in real fabrication systems, this paper studies joint effects of stochastic breakdown, backorder of permissible shortage, rework, and scrap on the optimal stock replenishing policy. A decision model is developed to solve the problem, which consists of mathematical modeling, formulations, and optimization method in order to help analyze the problem, derive the system cost function, and find the optimal replenishment cycle length decision. Applicability of the research results are demonstrated by a numerical example. The proposed decision model enables production managers to not only determine the optimal stock replenishing policy, but also reveal individual impact and/or joint effects of stochastic breakdown, defective rate, backorder of allowable shortage, rework, and scrap on the replenishing decision. With such an in-depth study, diverse system characteristics become available for managerial decision-making.

1. Introduction

The present work explores joint effects of stochastic machine failure, backorder of permissible shortage, rework, and scrap on stock replenishing decision. Traditional economic production quantity (EPQ) model (Taft, 1918) employed a mathematical model to calculate total relevant production cost per unit time, balance stock holding and setup costs to determine the best lot size for a fabrication cycle. Perfect fabrication process with no permitted stock-out situation is the simple assumption of classic EPQ model. However, due to unpredictable issues in the manufacturing practices, machine failures and imperfect products are inevitable. Bielecki and Kumar (1988) analyzed and discussed values of several relevant variables in an unreliable fabrication system, which contributed to the optimality of zero-inventory policy. Groenevelt et al. (1992) presented two different inventory control policies to deal with equipment failure occurrence, namely an abort-resume (or AR) discipline and a no resumption (or NR) policy. The former conditionally resumes the interrupted batch after completion of repair of the equipment, and the

* Corresponding author
E-mail: swang@cyut.edu.tw (S.W. Chiu)

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latter unconditionally abandons the interrupted lot at the time a failure occurs. Separate analyses and results were conducted and demonstrated. Boone et al. (2000) explored a fabrication system with defective products and machine failure. Mathematical model was developed to help expose the effects of these imperfect factors in fabrication on the optimal cycle length. As a result, important guidelines for selecting suitable fabrication cycle time were presented for managerial decision makings. Chiu (2010) employed mathematical modeling, theorems of renewal reward and convexity, and a recursive algorithm to help derive the optimal run time for a fabrication system with random breakdowns under AR policy and an imperfect rework process. Results and applicability were demonstrated via a numerical example. Extra studies that investigated the effect of machine failures on manufacturing systems can also be referred elsewhere (Giri & Dohi, 2005; Chakraborty et al., 2008; Rivera-Gómez et al., 2013; Chiu et al., 2016; Zhang et al., 2016; Luong & Karim, 2017).

With the purpose of reducing quality cost in production to lower overall fabrication relevant cost, production managers often adopt rework policy to repair the imperfect products. Rosenblatt and Lee (1986) explored the effect of imperfect fabrication process on the optimal manufacturing cycle length. A constant portion of defective products is generated when an in-control status of fabrication process turns into the out-of-control status. Approximate solutions were used for finding the most economic batch size in their study. Zargar (1995) proposed two distinct rework strategies to investigate their impacts on manufacturing cycle length. Queuing models for these separate rework strategies were also constructed, and simulation techniques were employed to expose individual characteristics of these rework strategies as well as their effects on cycle length. Teunter and Flapper (2003) derived the optimal batch size for a single stage manufacturing system with defective products. Upon completion of all-unit screenings, the perfect quality items, repairable nonconforming products, and scrapped items are separated. Regular production process will be switched to rework mode when the accumulation of repairable defective products reaches a predetermined quantity '*N*', and a linear cost and time is assumed for the reworked items. Accordingly, they derived a clear equation for average profit for any given *N* as well as the optimal *N* based on this equation. Eroglu and Ozdemir (2007) examined an economic order quantity (EOQ) model by considering defective items and shortages backordered. All-unit screening separates good products, imperfect items and scraps. Effect of defective rate on optimal ordering quantity was explored together with numerical illustrations of research results. Pasandideh et al. (2013) studied a specific multi-item EPQ model featuring limited orders and rework of repairable nonconforming products, with the objectives of minimizing both the needed storage space and total inventory cost, simultaneously. Meta-heuristic algorithms (i.e., NSGA-II and MOPSO) were used to solve the proposed bi-objective nonlinear problem. Solution and performance of these algorithms (including CPU time) were confirmed and assessed via two-sample *t*-tests to demonstrate their accuracy and efficiency. Diverse aspects of fabrication systems by considering defective items and/or their quality improvement issues were also explored elsewhere (Abilash & Sivapragash, 2016; Balaji et al., 2016; Chiu et al., 2016; Jawla & Singh, 2016; Mičieta et al., 2016; Buckova et al., 2017; Chiu et al., 2017a,b; Khanna et al., 2017).

Moreover, in the situations of consolidate orders from the internal demands or intra-supply chain members, or other operating strategy viewpoints, backordering of permissible shortages can be an effective policy to lower overall operating cost and/or smooth fabrication schedule. However, to avoid excessive stock-out duration in a cycle, setting a minimum service level is required to retain customer satisfaction in terms of stock availability. Ramani and Kutty (1985) explored the effects of service level constraints on total inventory costs for a multiproduct multiple group stock system. 306 spare components of real data were collected from a transport firm for their study. Based on multi-criteria these data were first categorized into 9 subgroups and each received a specific service level constraint. Total costs for inventories are calculated based on weighted/combined service levels. Accordingly, the local and global minimal costs were found and analyzed to provide an insight into management of groups of components. Rabinowitz et al. (1995) examined a stochastic (r, Q) ordering policy inventory model with a partial backorder strategy. A controlling parameter '*b*' is assumed in their model to represent the maximal permissible level of shortage that is backordered in a given replenishment cycle time. The controlling parameter makes their proposed inventory system different from the policies of lost sales and 100% backlogging. Solution procedure was proposed to calculate the optimal values for reorder point r, fixed order quantity Q, and maximal allowable backlog level b, which minimize the expected system expenses. Chiu et al. (2007) investigated a fabrication system with allowable backlogging under service level constraint, scraps, and rework of defective products. Their objective was to determine the optimal batch size for the proposed system in terms of system cost minimization. They demonstrated that their obtained result is better than the same system when the backordering is not permitted. Furthermore, they discussed the so-called "imputed backorder cost" and its relationship with the allowable backlogging level and its impact on the proposed system cost. Jha and Shanker (2013) determined the optimal production-inventory policy for a multiple buyer fabrication system, wherein a product is manufactured by a producer to meet independent demands from multiple customers. Lead times required by different customers can be shortened at extra expedited charge and all unsatisfied demand of customers can be backordered. A service level constraint for each customer is assumed in their proposed system. With a help from an analytical model and the Lagrangian multiplier technique, they were able to simultaneously derive the optimal order quantity, lead time, and number of shipments to each customer per cycle. Additional studies (Fergany, 2016; Rakyta et al., 2016; Jindal & Solanki, 2016; Jaggi et al., 2016; Qu & Ji, 2016; Salemi, 2016; Oblak et al., 2017; van Rhyn & Hancke, 2017) have also been conducted to explore the effects of permitted backlogging and service level constraint on fabrication systems. Nevertheless, little attention has been paid to exploration of joint effects of machine failure, backorder of permissible shortage, rework and scrap on stock replenishing decision, and the present work is set to fill this gap.

2. Description and formulation of the proposed fabrication system

Consider a fabrication system has annual manufacturing rate P_1 to meet annual product demand rate λ and it is subject to stochastic machine failure with a mean of β per year where β follows Poisson distribution. In addition, its process is not perfect for a certain portion of scrap and rework-able items may randomly be produced. Also, the system permits backorder of shortage under a service level constraint $(1 - \alpha)$% where α denotes the percentage of stock-out time in a replenishment cycle length, so the stock-out situation can be retained at an acceptable level.

An abort/resume policy is adopted when machine failure occurs. Under this policy, machine repair starts right away and the interrupted lot is resumed instantly when the repair task is accomplished. A constant machine repair time t_r is assumed, and if the practical repair time exceeds t_r, a spare machine will replace the broken one at t_r. During the manufacturing process, an x ratio of nonconforming items may be produced randomly at rate d_1, hence $d_1 = P_1x$. To guarantee the status of positive on-hand inventories, $(P_1 - d_1 - \lambda) > 0$ must satisfy. Nonconforming items are screened and the scrap (i.e., a θ portion of nonconforming items) and the rework-able (i.e., the other $1 - \theta$ portion) products are separated. At the end of uptime t_1', a rework process is used right away to repair the rework-able, at a rate of P_2 and with additional rework cost C_R per item reworked. During the reworking time t_2', there is a failure in repair portion θ_1 and they are scrapped at a unit disposal cost of C_S. Hence, the overall scrap rate in the proposed system is $\varphi = \theta + (1 - \theta)\theta_1$. Additional parameters used in this study are listed in Nomenclature in Appendix A. Due to randomness of machine failure occurring in uptime T_1, the following conditions have to be considered, respectively:

2.1 Condition 1: when t is less than t_5'

The first condition means that a stochastic machine failure takes place in the backlog being gradually satisfied time (i.e., t_5'). When machine failure occurs, an AR policy is used and fabrication of the interrupted lot is resumed right after the machine is repaired and restore. The on-hand inventory status including backordering is depicted in Fig. 1.

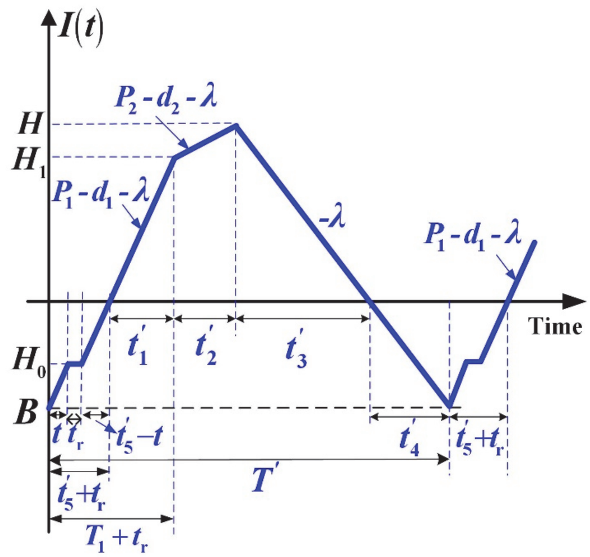

Fig. 1. The on-hand inventory status including the backlogging in the proposed fabrication system when a breakdown taking place in t_5'

It can be seen that at the time of machine breakdown taking place, the backlogging level is at H_0, and it stays at H_0 for a period of time t_r until the machine is restored. In the end of t_5', the on-hand inventory level of finished products turns into positive, and it continues to grow to level H_1, when fabrication uptime ends. Then, the reworking of rework-able nonconforming products starts in t_2', which brings the on-hand inventory level of finished products to H, when rework process ends. All finished products are depleted in the end of t_3', and inventory status turns into negative (i.e., shortage occurs) during t_4', until it reaches maximal allowable backlog level B (i.e., which is set by the service level constraint) in the end of t_4'. Then, the fabrication uptime of the next cycle starts. The following expressions can be directly observed from Fig. 1:

$$T_1 = \left(t_5^{'} + t_1^{'}\right) = \frac{Q}{P_1} \tag{1}$$

$$t_5^{'} = \frac{B}{\left(P_1 - d_1 - \lambda\right)} \tag{2}$$

$$t_1^{'} = \frac{H_1}{\left(P_1 - d_1 - \lambda\right)} \tag{3}$$

$$t_2^{'} = \frac{H - H_1}{\left(P_2 - d_2 - \lambda\right)} \tag{4}$$

$$t_3^{'} = \frac{H}{\lambda} \tag{5}$$

$$t_4^{'} = \frac{B}{\lambda} \tag{6}$$

$$T^{'} = \left(\sum_{i=1}^{5} t_i^{'}\right) + t_r \tag{7}$$

$$H_0 = B - \left(P_1 - d_1 - \lambda\right)t \tag{8}$$

$$H_1 = \left(P_1 - d_1 - \lambda\right)t_1^{'} \tag{9}$$

$$H = H_1 + \left(P_2 - d_2 - \lambda\right)t_2^{'} \tag{10}$$

The on-hand inventory levels of nonconforming products and scrap items are illustrated in Fig. 2 and Fig. 3, respectively.

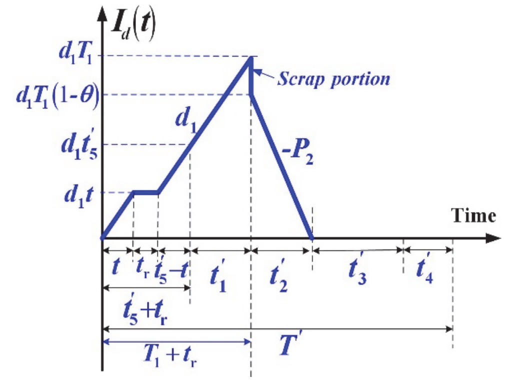

Fig. 2. The on-hand inventory level of nonconforming products in the proposed fabrication system when a breakdown taking place in t_5'

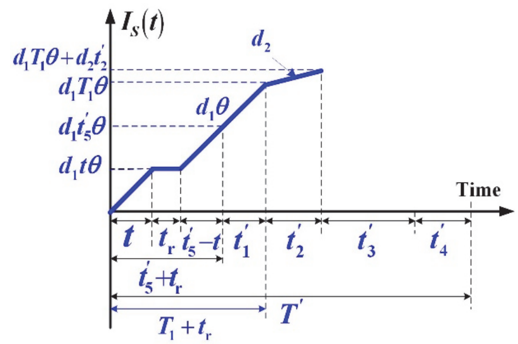

Fig. 3. The on-hand inventory level of scrap items in the proposed fabrication system when a breakdown taking place in t_5'

Note that the level of on-hand defective items (see Fig. 2) is at d_1t when a machine failure occurs, and after the machine is fixed the on-hand defective items continues to increase to d_1T_1 at the end of uptime. During the reworking time t_2', the rework-able nonconforming products $[d_1T_1(1-\theta)]$ are depleting at a rate of P_2.

$$t_2' = \frac{d_1T_1(1-\theta)}{P_2}. \tag{11}$$

Similarly, the level of on-hand scrap items stops at ($d_1t\theta$) when a breakdown occurs (Fig. 3), and after the machine is fixed the on-hand scrap items continues to increase to ($d_1T_1\theta$) in the end of uptime. Then, during reworking time t_2', the on-hand inventory of scrap items continues to grow, and at the end of rework process, the scrap items reaches the maximum level at ($d_1T_1\theta + d_2t_2'$), where

$$d_1T_1\theta + d_2t_2' = (xQ)\theta + \left[xQ(1-\theta)\right]\theta_1 = \left[\theta + (1-\theta)\theta_1\right](xQ) = \varphi(xQ). \tag{12}$$

Total relevant cost per cycle when a breakdown takes place in t_5', $TRC_1(T_1)$ comprises the fabrication setup cost, variable manufacturing and reworking costs, holding cost for reworked items, disposal cost for scrap items, backordering cost for shortage, holding cost during t_1', t_2', and t_3', fixed machine repairing cost, holding and purchasing cost for safety stock (to be used during t_r), and delivery cost for finished products. Hence, $TRC_1(T_1)$ is

$$TRC_1(T_1) = K + CQ + C_R\left[x(1-\theta)Q\right] + h_1\left(\frac{P_2 t_2'}{2}\right)(t_2') + C_S\varphi(xQ) + b\frac{B}{2}(t_4' + t_5')$$
$$+ b(H_0)t_r + h\left[\frac{H_1}{2}(t_1') + \frac{(H_1+H)}{2}(t_2') + \frac{H}{2}(t_3') + (d_1 t)t_r + \frac{d_1 T_1}{2}(T_1)\right] \tag{13}$$
$$+ M + \left[h_3(\lambda t_r)\left(t + \frac{t_r}{2}\right) + C_1(\lambda t_r)\right] + C_T\left[(1-\varphi x)Q + (\lambda t_r)\right].$$

Replacing fabrication lot-size Q with T_1P_1, Eq. (13) becomes

$$TRC_1(T_1) = K + C(T_1P_1) + C_R\left[x(1-\theta)(T_1P_1)\right] + h_1\left(\frac{P_2 t_2'}{2}\right)(t_2') + C_S\varphi\left[x(T_1P_1)\right]$$
$$+ b\frac{B}{2}(t_4' + t_5') + b(H_0)t_r + h\left[\frac{H_1}{2}(t_1') + \frac{(H_1+H)}{2}(t_2') + \frac{H}{2}(t_3') + (d_1 t)t_r + \frac{d_1 T_1}{2}(T_1)\right] \tag{14}$$
$$+ M + \left[h_3(\lambda t_r)\left(t + \frac{t_r}{2}\right) + C_1(\lambda t_r)\right] + C_T\left[(1-\varphi x)(T_1P_1) + (\lambda t_r)\right].$$

Substituting Eq. (1) to Eq. (12) in Eq. (14) and applying the expected values of x (for dealing with its randomness), yield $E[TRC_1(T_1)]$ as shown in Eq. (B-1) in Appendix B. Upon obtaining the total cost per cycle in the case that a breakdown takes place in t_5', we then examine the second situation in the following sub-section.

2.2 Condition 2: when t is less than T_1, but it is greater than t_5'

The second condition stands when a stochastic machine failure takes place in the positive stock growing stage (see Fig. 4). Note that the on-hand level of perfect quality products arrives at H_2 when a machine failure occurs, and after the machine is fixed, the on-hand perfect quality products continues to climb to H_1 at the end of uptime, and it reaches H at the end of rework process. Then, at the end of t_3' all perfect quality items are consumed. During t_4', the stock-out situation begins and it continues until the end of t_4' when shortages reach the maximal permissible level B (determined according to a pre-set service level constraint). In situation 2, Eq. (1) to Eq. (7) and Eq. (9) to Eq. (12) exposed in previous subsection are still in effect. The following new equation is added in analysis of situation 2:

$$H_2 = (P_1 - d_1 - \lambda)(t - t_5'). \tag{15}$$

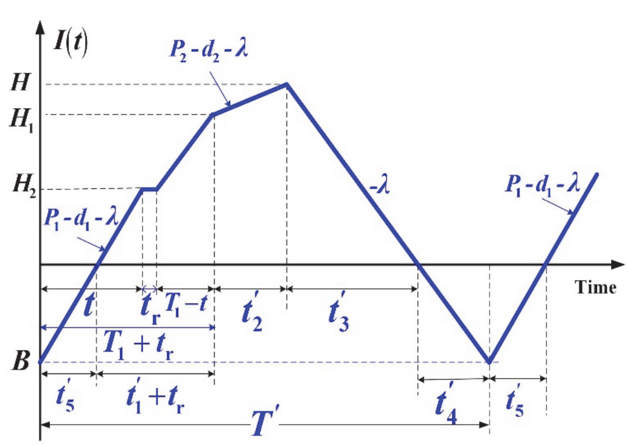

Fig. 4. The on-hand inventory status including the backlogging in the proposed fabrication system when a breakdown taking place in t_1'

The on-hand inventory levels of nonconforming products and scrap items are depicted in Fig. 5 and Fig. 6, respectively.

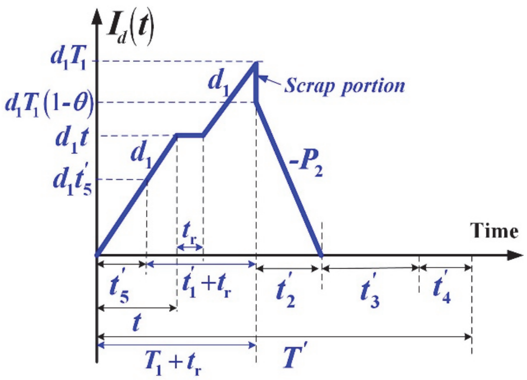

Fig. 5. The on-hand inventory level of nonconforming products in the proposed fabrication system when a breakdown taking place in t_1'

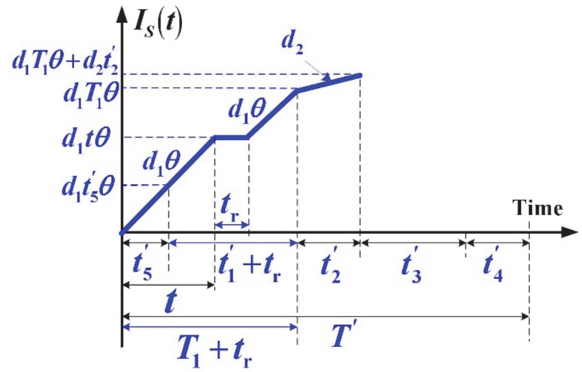

Fig. 6. The on-hand inventory level of scrap items in the proposed fabrication system when a breakdown taking place in t_1'

Note that the level of on-hand defective items (see Fig. 5) is at d_1t when a machine failure occurs, and after the machine is repaired and restored, the on-hand defective items continues to increase to d_1T_1. During the reworking time t_2', the reworkable nonconforming products $[d_1T_1(1-\theta)]$ are depleting at a rate of P_2. Similarly, the level of on-hand scrap items stops at $(d_1t\theta)$ when a breakdown occurs (Fig. 6), and after the machine is repaired and restored, the on-hand scrap items continues to increase to $(d_1T_1\theta)$ at the end of uptime. Then, in the end of rework process, the scrap items reaches maximum level at $(d_1T_1\theta + d_2t_2')$ or $\varphi(xQ)$ (see Eq. (12)). Total relevant cost per cycle in the case that a breakdown takes place in t_1', $TRC_2(T_1)$ comprises the fabrication setup cost, variable manufacturing and reworking costs, holding cost for reworked items, disposal cost for scrap items, backordering cost for shortage, holding cost during t_1', t_2', and t_3', fixed machine repairing cost, holding and purchasing cost for safety stock, and delivery cost for finished products. Therefore, $TRC_2(T_1)$ is

$$\begin{aligned}TRC_2(T_1) &= K + CQ + C_R\left[x(1-\theta)Q\right] + h_1\left(\frac{P_2t_2'}{2}\right)(t_2') + C_S\varphi(xQ) + b\frac{B}{2}(t_4'+t_5')\\&+h\left[\frac{H_2}{2}(t-t_5') + \frac{(H_2+H_1)}{2}(T_1-t) + \frac{(H_1+H)}{2}(t_2') + \frac{H}{2}(t_3') + (H_2+d_1t)t_r + \frac{d_1T_1}{2}(T_1)\right]\\&+M+\left[h_3(\lambda t_r)\left(t+\frac{t_r}{2}\right) + C_1(\lambda t_r)\right] + C_T\left[(1-\varphi x)Q + (\lambda t_r)\right]\end{aligned} \tag{16}$$

Replacing fabrication lot-size Q with T_1P_1, Eq. (16) yields

$$\begin{aligned}TRC_2(T_1) &= K + C\,(T_1P_1) + C_R\left[x\,(1-\theta)(T_1P_1)\right] + h_1\left(\frac{P_2t_2'}{2}\right)(t_2') + C_S\varphi x\,(T_1P_1) + b\,\frac{B}{2}(t_4'+t_5')\\&+h\left[\frac{H_2}{2}(t-t_5') + \frac{(H_2+H_1)}{2}(T_1-t) + \frac{(H_1+H)}{2}(t_2') + \frac{H}{2}(t_3') + (H_2+d_1t)t_r + \frac{d_1T_1}{2}(T_1)\right]\\&+M+\left[h_3(\lambda t_r)\left(t+\frac{t_r}{2}\right) + C_1(\lambda t_r)\right] + C_T\left[(1-\varphi x)(T_1P_1) + (\lambda t_r)\right].\end{aligned} \tag{17}$$

Substituting Eq. (1) to Eq. (7), Eq. (9) to Eq. (12), and Eq. (15) in Eq. (17) and applying the expected values of x (for dealing with its randomness), with extra derivations yields $E[TRC_2(T_1)]$ as displayed in

Eq. (B-2) in Appendix B. Finally, we examine the third situation in the following sub-section.

2.3 Condition 3: when t is greater than or equal to T_1

The third condition means that no machine failure takes place during the fabrication uptime (see Fig. 7). The following expressions can be directly obtained by observing Fig. 7:

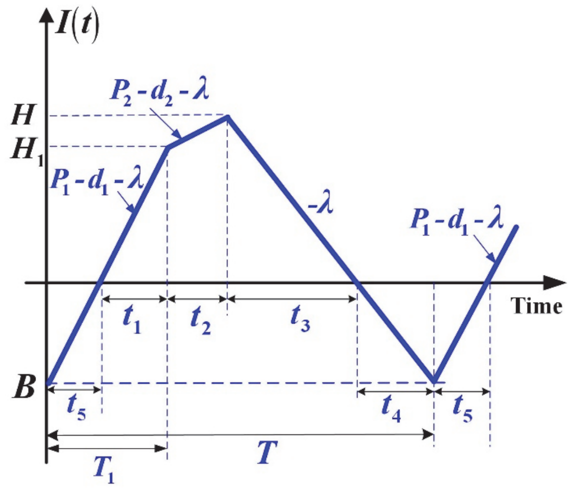

Fig. 7. The on-hand inventory status including the backlogging in the proposed fabrication system when breakdown does not take place during fabrication uptime

$$T_1 = \left(t_5 + t_1\right) = \frac{Q}{P_1}, \tag{18}$$

$$t_5 = \frac{B}{\left(P_1 - d_1 - \lambda\right)}, \tag{19}$$

$$t_1 = \frac{H_1}{\left(P_1 - d_1 - \lambda\right)}, \tag{20}$$

$$t_2 = \frac{H - H_1}{\left(P_2 - d_2 - \lambda\right)}, \tag{21}$$

$$t_3 = \frac{H}{\lambda}, \tag{22}$$

$$t_4 = \frac{B}{\lambda}, \tag{23}$$

$$T = \sum_{i=1}^{5} t_i, \tag{24}$$

$$H_1 = \left(P_1 - d_1 - \lambda\right) t_1, \tag{25}$$

$$H = H_1 + \left(P_2 - d_2 - \lambda\right) t_2, \tag{26}$$

$$t_2 = \frac{d_1 T_1 \left(1 - \theta\right)}{P_2}. \tag{27}$$

Total relevant cost per cycle in the case that no breakdown takes place in uptime, $TRC_3(T_1)$ comprises the fabrication setup cost, variable manufacturing and reworking costs, holding cost for reworked items, disposal cost for scrap items, backordering cost for shortage, holding cost during t_1, t_2, and t_3, holding and purchasing cost for safety stock, and the delivery cost for finished products. Hence, $TRC_3(T_1)$ is

$$TRC_3(T_1) = K + CQ + C_R\left[x(1-\theta)Q\right] + h_1\left(\frac{P_2 t_2}{2}\right)(t_2) + C_S\varphi(xQ) + b\frac{B}{2}(t_4+t_5) + h\left[\frac{H_1}{2}(t_1) + \frac{(H_1+H)}{2}(t_2) + \frac{H}{2}(t_3) + \frac{d_1 T_1}{2}(T_1)\right] + \left[h_3(\lambda t_r)T + C_1(\lambda t_r)\right] + C_T(1-\varphi x)Q. \tag{28}$$

Replacing fabrication lot-size Q with T_1P_1, Eq. (28) yields

$$TRC_3(T_1) = K + C(T_1P_1) + C_R\left[x(1-\theta)(T_1P_1)\right] + h_1\left(\frac{P_2 t_2}{2}\right)(t_2) + C_S\varphi x(T_1P_1) + b\frac{B}{2}(t_4+t_5) + h\left[\frac{H_1}{2}(t_1) + \frac{(H_1+H)}{2}(t_2) + \frac{H}{2}(t_3) + \frac{d_1 T_1}{2}(T_1)\right] + \left[h_3(\lambda t_r)T + C_1(\lambda t_r)\right] + C_T(1-\varphi x)(T_1P_1). \tag{29}$$

Substituting Eq. (18) to Eq. (27) in Eq. (29) and applying the expected values of x (for dealing with its randomness), and with extra derivations yields $E[TRC_3(T_1)]$ as given in Eq. (B-3) in Appendix B.

3. Determining the optimal fabrication uptime

Since this study considers service level constraints to prevent excessive shortage occurrences in any given replenishment cycle, the following equation (Chiu et al., 2007) is used to maintain the minimal service level at $(1-\alpha)$%:

$$B = \alpha\left(1 - E[x] - \frac{\lambda}{P_1}\right)\left(\frac{1-\varphi E[x]}{1-E[x]}\right)T_1P_1. \tag{30}$$

Moreover, since in this study a Poisson distributed machine failure rate (with β per year as the mean) is assumed, hence, the time to a breakdown occurrence obeys exponential distribution with $f(t) = \beta e^{-\beta t}$ and $F(t) = (1-e^{-\beta t})$ as its density and cumulative density functions, respectively. Therefore, the expected total system cost $E[TRCU(T_1)]$ is

$$E\left[TRCU(T_1)\right] = \frac{\left\{\int_0^{t_5'} E\left[TRC_1(T_1)\right]f(t)\,dt + \int_{t_5'}^{T_1} E\left[TRC_2(T_1)\right]f(t)\,dt + \int_{T_1}^{\infty} E\left[TRC_3(T_1)\right]f(t)\,dt\right\}}{\mathrm{E}[\boldsymbol{T}]}, \tag{31}$$

where $\mathrm{E}[\boldsymbol{T}]$ is

$$\mathrm{E}[\boldsymbol{T}] = \frac{T_1P_1}{\lambda}\left(1-\varphi E[x]\right). \tag{32}$$

Substituting $\mathrm{E}[TRC_1(T_1)]$, $\mathrm{E}[TRC_2(T_1)]$, and $\mathrm{E}[TRC_3(T_1)]$ (from Appendix B, Eqs. (B-1) to (B-3)), B, $f(t)$, and $\mathrm{E}[\boldsymbol{T}]$ in Eq. (31), and with further derivations we obtain $E[TRCU(T_1)]$ as follows:

$$E\left[TRCU(T_1)\right] = \frac{\lambda}{(1-\varphi E[x])}\left\{\frac{z_1}{T_1} + T_1\left[\begin{array}{l}\frac{v^2(h+b)}{2P_1\lambda} + \frac{v^2(h+b)}{2P_1^2(1-E[x]-\frac{\lambda}{P})} - \frac{h(1-2\varphi E[x])}{2} + \frac{hE[x]^2P_1\varphi(1-\theta)}{2P_2} \\ + \frac{E[x]^2P_1(1-\theta)}{2P_2}\left[h_1(1-\theta)-h\right] + \frac{(1-\varphi E[x])}{\lambda}\left[\frac{hP_1(1-\varphi E[x])}{2} - hv\right]\end{array}\right]\right\} + \frac{\lambda}{(1-\varphi E[x])}\left\{\begin{array}{l}C + C_RE[x](1-\theta) + C_S\varphi E[x] + \frac{vg(b-h)}{P_1} + C_T(1-\varphi E[x]) + h_3g(1-\varphi E[x]) \\ + \frac{w_1}{T_1} + w_2e^{-\beta T_1} + \frac{w_3e^{-\beta T_1}}{T_1} + \frac{w_4e^{-\beta T_1 s}}{T_1} + w_5\left(e^{-\beta T_1(1-s)} + e^{-\beta T_1 s}\right)\end{array}\right\}. \tag{33}$$

where w_1, w_2, w_3, w_4, w_5, z_1, v, and s denote the following:

$$w_1 = \left[\frac{M}{P_1} + \frac{h_3 \lambda g^2}{2P_1} + \frac{h_3 \lambda g}{\beta P_1} + \frac{C_T \lambda g}{P_1} + \frac{hE[x]g}{\beta} - \frac{bg\left(P_1 - P_1 E[x] - \lambda\right)}{\beta P_1} \right], \tag{34}$$

$$w_2 = \left[-\frac{h_3 \lambda g}{P_1} - hg + \frac{h \lambda g}{P_1} \right],\ w_3 = \left[-\frac{M}{P_1} - \frac{h_3 \lambda g^2}{2P_1} - \frac{h_3 \lambda g}{\beta P_1} - \frac{C_T \lambda g}{P_1} - \frac{hg}{\beta} + \frac{h \lambda g}{\beta P_1} \right], \tag{35}$$

$$w_4 = \frac{g\left(1 - E[x] - \dfrac{\lambda}{P_1}\right)(h + b)}{\beta},\ w_5 = \left(\frac{hvg}{P_1}\right);\ z_1 = \left[\frac{K}{P_1} + \frac{C_1 \lambda g}{P_1} \right], \tag{36}$$

$$v = \alpha \left(1 - E[x] - \frac{\lambda}{P_1}\right)\left(\frac{1 - \varphi E[x]}{1 - E[x]}\right) P_1, \tag{37}$$

$$sT_1 = \frac{B}{P_1 - P_1 E[x] - \lambda} = \frac{vT_1}{P_1 - P_1 E[x] - \lambda},\ s = \frac{v}{P_1 - P_1 E[x] - \lambda}. \tag{38}$$

3.1 Convexity

The first- and second-derivatives of $E[TRCU(T_1)]$ (i.e., Eq. (33)) can be obtained as follows:

$$\begin{aligned} \frac{dE\left[TRCU(T_1)\right]}{dT_1} = {} & \frac{\lambda}{(1-\varphi E[x])} \cdot \left\{ -\frac{z_1}{T_1^2} + \left[\begin{array}{l} \dfrac{v^2(h+b)}{2P_1\lambda} + \dfrac{v^2(h+b)}{2P_1^2(1-E[x]-\frac{\lambda}{P})} - \dfrac{h(1-2\varphi E[x])}{2} + \dfrac{hE[x]^2 P_1 \varphi(1-\theta)}{2P_2} \\ + \dfrac{E[x]^2 P_1(1-\theta)}{2P_2}\left[h_1(1-\theta) - h\right] + \dfrac{(1-\varphi E[x])}{\lambda}\left[\dfrac{hP_1(1-\varphi E[x])}{2} - hv\right] \end{array} \right] \right\} \\ & + \frac{\lambda}{(1-\varphi E[x])} \left\{ \begin{array}{l} -\dfrac{w_1}{T_1^2} - \beta w_2 e^{-\beta T_1} + w_3\left(-\dfrac{\beta e^{-\beta T_1}}{T_1} - \dfrac{e^{-\beta T_1}}{T_1^2}\right) + w_4\left(-\dfrac{\beta s e^{-\beta T_1 s}}{T_1} - \dfrac{e^{-\beta T_1 s}}{T_1^2}\right) \\ + w_5\left[-\beta e^{-\beta T_1}\left(e^{-\beta T_1 s}\right)^{-1} + \beta s e^{-\beta T_1}\left(e^{-\beta T_1 s}\right)^{-1} - \beta s e^{-\beta T_1 s}\right] \end{array} \right\}, \end{aligned} \tag{39}$$

and

$$\frac{d^2 E\left[TRCU(T_1)\right]}{dT_1^2} = \frac{\lambda}{(1-\varphi E[x])} \cdot \left\{ \begin{array}{l} \dfrac{2}{T_1^3}(z_1 + w_1) + \beta^2 w_2 e^{-\beta T_1} + w_3\left(\dfrac{\beta^2 e^{-\beta T_1}}{T_1} + \dfrac{2\beta e^{-\beta T_1}}{T_1^2} + \dfrac{2e^{-\beta T_1}}{T_1^3}\right) \\ + w_4\left(\dfrac{\beta^2 s^2 e^{-\beta T_1 s}}{T_1} + \dfrac{2\beta s e^{-\beta T_1 s}}{T_1^2} + \dfrac{2e^{-\beta T_1 s}}{T_1^3}\right) \\ + w_5\left[\beta^2 e^{-\beta T_1}\left(e^{-\beta T_1 s}\right)^{-1} - 2\beta^2 s e^{-\beta T_1}\left(e^{-\beta T_1 s}\right)^{-1} + \beta^2 s^2 e^{-\beta T_1}\left(e^{-\beta T_1 s}\right)^{-1} + \beta^2 s^2 e^{-\beta T_1 s}\right] \end{array} \right\}. \tag{40}$$

From Eq. (40), it is noted the first term of its RHS (right-hand side), $\lambda/(1–\varphi E[x])$ is positive, so if the second term on RHS of Eq. (40) is also positive, then $E[TRCU(T_1)]$ is convex. With extra derivations, if Eq. (41) holds, then $E[TRCU(T_1)]$ is convex:

$$z(T_1) = \frac{2z_1 + 2w_1 + 2w_3 e^{-\beta T_1} + 2w_4 e^{-\beta T_1 s}}{\left[\begin{array}{l} -T_1^2 \beta^2 w_2 e^{-\beta T_1} - T_1 \beta^2 w_3 e^{-\beta T_1} - 2\beta w_3 e^{-\beta T_1} - T_1 \beta^2 s^2 w_4 e^{-\beta T_1 s} \\ -2\beta s w_4 e^{-\beta T_1 s} - T_1^2 \beta^2 w_5 e^{-\beta T_1}\left(e^{-\beta T_1 s}\right)^{-1} + T_1^2 2\beta^2 s w_5 e^{-\beta T_1}\left(e^{-\beta T_1 s}\right)^{-1} \\ -T_1^2 \beta^2 s^2 w_5 e^{-\beta T_1}\left(e^{-\beta T_1 s}\right)^{-1} - T_1^2 \beta^2 s^2 w_5 e^{-\beta T_1 s} \end{array} \right]} > T_1 > 0 \tag{41}$$

3.2 Solution procedure for finding optimal uptime

When Eq. (41) holds, we are certain that there exists T_1* yielding minimum cost of $E[TRCU(T_1)]$. By setting the first-derivative equal to zero, we have the following:

$$\frac{dE\left[TRCU\left(T_1\right)\right]}{dT_1}=\frac{\lambda}{(1-\varphi E[x])}\left\{-\frac{z_1}{T_1^2}+\left[\begin{array}{l}\frac{v^2\left(h+b\right)}{2P_1\lambda}+\frac{v^2\left(h+b\right)}{2P_1^2(1-E[x]-\frac{\lambda}{P})}-\frac{h\left(1-2\varphi E[x]\right)}{2}+\frac{hE\left[x\right]^2P_1\varphi\left(1-\theta\right)}{2P_2}\\+\frac{E[x]^2P_1(1-\theta)}{2P_2}\left[h_1(1-\theta)-h\right]+\frac{(1-\varphi E[x])}{\lambda}\left[\frac{hP_1(1-\varphi E[x])}{2}-hv\right]\end{array}\right]\right\}$$
$$+\frac{\lambda}{(1-\varphi E[x])}\left\{\begin{array}{l}-\frac{w_1}{T_1^2}-\beta w_2e^{-\beta T_1}+w_3\left(-\frac{\beta e^{-\beta T_1}}{T_1}-\frac{e^{-\beta T_1}}{T_1^2}\right)+w_4\left(-\frac{\beta se^{-\beta T_1 s}}{T_1}-\frac{e^{-\beta T_1 s}}{T_1^2}\right)\\+w_5\left[-\beta e^{-\beta T_1}\left(e^{-\beta T_1 s}\right)^{-1}+\beta se^{-\beta T_1}\left(e^{-\beta T_1 s}\right)^{-1}-\beta se^{-\beta T_1 s}\right]\end{array}\right\}=0. \quad (42)$$

With additional derivations, Eq. (42) becomes

$$m_2T_1^2+m_1T_1+m_0=0, \quad (43)$$

where m_2, m_1, and m_0 represent the following:

$$m_2=\left\{\begin{array}{l}-h\left(1-2\varphi E[x]\right)+\frac{E[x]^2P_1\left(1-\theta\right)}{P_2}\left[h_1\left(1-\theta\right)-h\right]+\frac{hE\left[x\right]^2P_1\varphi\left(1-\theta\right)}{P_2}\\+\frac{v^2}{P_1\lambda}\left(h+b\right)+\frac{v^2}{P_1^2(1-E\left[x\right]-\frac{\lambda}{P_1})}\left(h+b\right)+\frac{h\left(1-\varphi E[x]\right)}{\lambda}\left[P_1\left(1-\varphi E\left[x\right]\right)-2v\right]\\-2\beta\left(w_2\right)e^{-\beta T_1}+2\beta w_5\left[-e^{-\beta T_1}\left(e^{-\beta T_1 s}\right)^{-1}+se^{-\beta T_1}\left(e^{-\beta T_1 s}\right)^{-1}-se^{-\beta T_1 s}\right]\end{array}\right\}, \quad (44)$$

$$m_1=2\left[-\beta w_3e^{-\beta T_1}-\beta sw_4e^{-\beta T_1 s}\right], \quad (45)$$

$$m_0=2\left[-z_1-w_1-w_3e^{-\beta T_1}-w_4e^{-\beta T_1 s}\right]. \quad (46)$$

Applying the square-root solution on Eq. (43) we find T_1* as follows:

$$T_1^*=\frac{-m_1\pm\sqrt{m_1^2-4m_2m_0}}{2m_2}. \quad (47)$$

Rearrange Eq. (47) yields

$$e^{-\beta T_1}=\frac{T_1^2m_3-\left[w_1+z_1+T_1\beta sw_4e^{-\beta T_1 s}+w_4e^{-\beta T_1 s}+T_1^2\beta sw_5e^{-\beta T_1 s}\right]}{T_1^2\left[\beta w_2e^{-\beta T_1}+\beta w_5\left[\left(e^{-\beta T_1}\right)^{-1}\right]^s-\beta sw_5\left[\left(e^{-\beta T_1}\right)^{-1}\right]^s\right]+T_1\beta w_3+w_3} \quad (48)$$

where

$$m_3=\frac{1}{2}\left\{\begin{array}{l}\left(h+b\right)\left\{\frac{v^2}{P_1\lambda}+\frac{v^2}{P_1^2(1-E\left[x\right]-\frac{\lambda}{P})}\right\}-h\left(1-2\varphi E[x]\right)+\frac{E[x]^2P_1(1-\theta)}{P_2}\left[h_1(1-\theta)-h\right]\\+\frac{hE\left[x\right]^2P_1\varphi\left(1-\theta\right)}{P_2}+\frac{hP_1(1-\varphi E[x])^2}{\lambda}-\frac{2hv(1-\varphi E[x])}{\lambda}\end{array}\right\}.$$

From Eq. (47) and Eq. (48), although T_1* cannot be obtained in a closed form, one can find it through the following searching algorithm starting from the initial bounds on $e^{-\beta T}1$.

*3.2.1 Algorithm for finding the T_1**

Since $[1 - F(T_1)] = e^{-\beta T}1$ is the complement of $F(T_1)$ and it is over the interval of [0, 1] since the range of $F(T_1)$ is [0, 1]. Set $e^{-\beta T}1 = 0$ and $e^{-\beta T}1 = 1$ initially, starting upper bound T_{1U} and lower bound T_{1L} of fabrication uptime can be calculated from Eq. (47). Next, we apply Eq. (48) with current T_{1U} and T_{1L} to update and obtain new values for $e^{-\beta T}1U$ and $e^{-\beta T}1L$, then, repeatedly calculate Eq. (47) and Eq. (48) until there is no significant difference between T_{1U} and T_{1L}. Finally, the optimal cycle length $T_1^* = T_{1U} = T_{1L}$ is found.

4. Numerical demonstration

Consider that a fabrication system for a specific product allows backordering of permissible shortage and it is subject to stochastic machine failure and random defective rate. Details of relevant parameters for this fabrication system are as follows (for definitions of parameters please refer to Nomenclature in Appendix A):

- P = 10,000 items per year;
- λ = 4,000 items per year;
- x = [0, 0.2], uniformly distributed random variable;
- P_1 = 5,000 items per year;
- β = 0.5 per year, Poisson distributed breakdown rate;
- M = \$500 per repair;
- K = \$450 per cycle;
- C = \$2 per item;
- h = \$0.8 per item;
- $(1 - \alpha)\%$ = 80%, a minimum acceptable service level.
- b = \$0.1 for each backorder item;
- C_R = \$0.5 for each reworked item;
- h_1 = \$0.8 for each reworked item;
- h_3 = \$0.6 per item;
- C_1 = \$2 per item;
- C_T = \$0.01 per unit;
- g = 0.018 years;

Applying aforementioned parameters in the proposed model, the existence of convexity of E[$TRCU(T_1)$] (i.e., Eq. (41)) is confirmed (details please refer to Appendix C). Then, a recursive solution procedure is proposed to help find the optimal fabrication cycle time T_1* for the problem. It begins by first setting two extreme boundaries for $e^{-\beta T}1$ (i.e., $e^{-\beta T}1 = 1$ and $e^{-\beta T}1 = 0$). Applying Eq. (47), one can find the initial values of the upper and lower bounds as T_{1U} = 0.5491 and T_{1L} = 0.3423. Calculating Eq. (33), E[$TRCU(T_{1U})$] = \$9,772.90 and E[$TRCU(T_{1L})$] = \$9,709.57 are obtained accordingly. Substitute current values of T_{1U} and T_{1L} in Eq. (48), one finds $e^{-\beta T}1U$ = 0.7599 and $e^{-\beta T}1L$ = 0.8427, respectively. Utilize the current values of $e^{-\beta T}1U$ and $e^{-\beta T}1L$ as the new bounds for $e^{-\beta T}1$ and recomputed Eq. (47), new bounds for T_1 can be found and they are T_{1U} = 0.4053 and T_{1L} = 0.3843. Apply them to Eq. (33), E[$TRCU(T_{1U}$ = 0.4053)] = \$9,700.32 and E[$TRCU(T_{1L}$ = 0.3843)] = \$9,699.43 can be gained accordingly. Repeating the aforementioned solution procedure (i.e., calculations of Eq. (47) and (48)) until the difference between T_{1U} and T_{1L} is insignificant (see Table 1), then T_1* is found.

Table 1
Step by step results from the proposed recursive solution procedure for determining T_1*

Step no.	β	$e^{-\beta T_{1U}}$	T_{1U}	$e^{-\beta T_{1L}}$	T_{1L}	E[$TRCU(T_{1U})$]	E[$TRCU(T_{1L})$]
1	0.5	0	0.5491	1	0.3423	\$9,772.90	\$9,709.57
2		0.7599	0.4053	0.8427	0.3843	\$9,700.32	\$9,699.43
3		0.8166	0.3910	0.8252	0.3888	\$9,699.34	\$9,699.33
4		0.8224	0.3895	0.8233	0.3893	\$9,699.33	\$9,699.33
5		0.8230	0.3894	0.8231	0.3893	\$9,699.33	\$9,699.33
6		0.8231	**0.3893**	0.8231	**0.3893**	**\$9,699.33**	**\$9,699.33**

Table 1 reconfirms that because E[$TRCU(T_1)$] is convex and the optimal T_1* falls within the interval [T_{1L}, T_{1U}], the proposed recursive solution procedure can help us find the optimal fabrication cycle time T_1* = 0.3893 and E[$TRCU(T_1$* = 0.3893)] = \$9,699.33. The effect of fabrication cycle length T_1 on E[$TRCU(T_1)$] is demonstrated in Fig. 8.

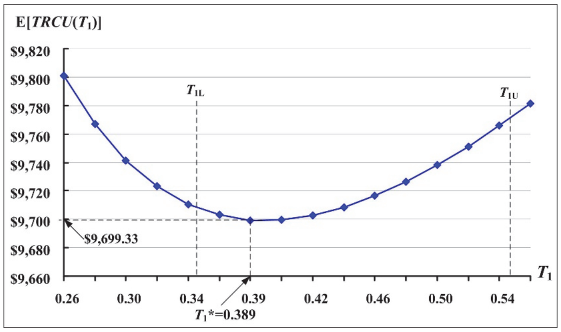

Fig. 8. The effect of fabrication cycle length T_1 on E[$TRCU(T_1)$]

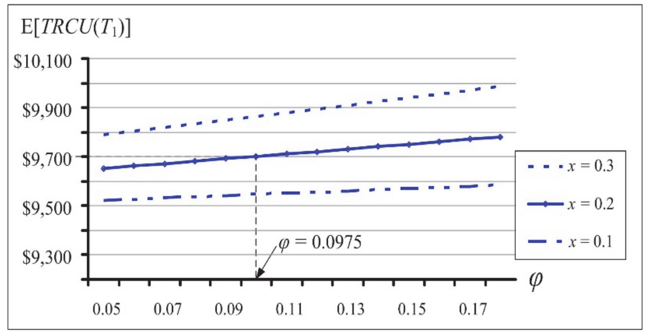

Fig. 9. Impacts from diverse defective rates and overall scrap rates on the expected total system relevant cost per unit time E[$TRCU(T_1)$]

Impacts from different defective rates and overall scrap rates on the expected total system relevant cost per unit time E[$TRCU(T_1)$] are investigated and the results are exhibited in Fig. 9. It can be seen that as either random defective rate x or overall scrap rate φ goes up, the expected system cost increases, accordingly. As the assumption in our numerical example that $x = 0.2$ and $\varphi = 0.0975$, we find the optimal E[$TRCU(T_1$*)] = \$9,699.33. Joint effects of diverse overall scrap rates and variations in time-to-failure on the expected total system cost E[$TRCU(T_1)$] are explored and the results are illustrated in Fig. 10. It is noted that as time-to-machine-failure $1/\beta$ increases (i.e., number of breakdowns per year β declines), the expected system cost decreases accordingly, and as $1/\beta$ goes up and higher than 1 year, the E[$TRCU(T_1$*)] reduces, significantly. As the assumption in our numerical example that $1/\beta = 2$ we arrive the optimal E[$TRCU(T_1$*)] = \$9,699.33. Due to the proposed system permits backordering of shortages under service level constraint $(1 - \alpha)$% (i.e., a maximal percentage α of stock-out time allowed in any given cycle length), the stock-out situation can be retained at an acceptable level. Analytical result of impact of various service levels on the expected total system cost E[$TRCU(T_1)$] are studied and the result is depicted in Fig. 11. It can be seen that as the service level $(1 - \alpha)$ is set higher, the expected system cost E[$TRCU(T_1)$] raises notably, and as the assumption in our numerical example that $(1 - \alpha)$ is set at 80%, E[$TRCU(T_1$*)] climbs to \$9,699.33.

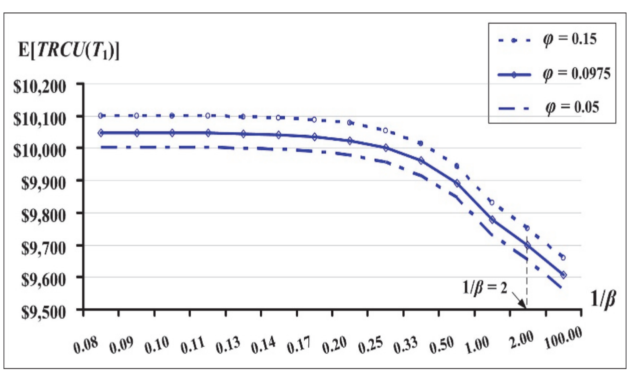

Fig. 10. Joint effects of diverse overall scrap rates and variations in time-to-failure on the expected total system cost E[$TRCU(T_1)$]

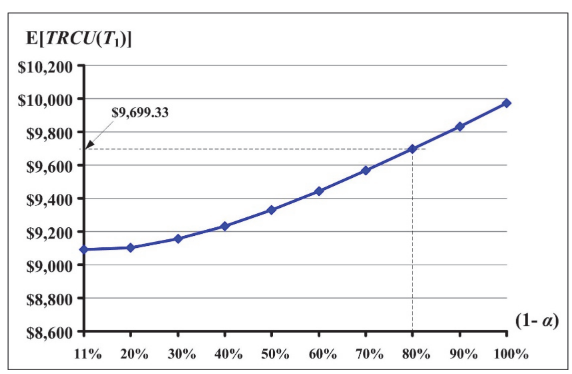

Fig. 11. Analytical result of impact of various service levels on the expected total system cost E[$TRCU(T_1)$]

Further exploratory results of the effects of different service levels on the optimal cycle length T_1*, maximal levels of stock holding H and the permissible backlog B, E[$TRCU(T_1$*)] and its increase percentage, and the cost of setting service level at $(1 - \alpha)$% are also exhibited in Table 2. It is noted that if the service level is 100%, the proposed model becomes the same as a fabrication model subjects to the stochastic breakdown, but without backlogging (Chiu, 2010).

Table 2
Exploratory results of effects of different service levels on various system parameters

$(1 - \alpha)$%	T_1*	H	B	E[$TRCU(T_1*)$]	E[$TRCU(T_1*)$] Increase %	*Extra cost due to setting service level*
100%	0.3184	1637	0	$9,974	9.72%	$883
90%	0.3508	1611	193	$9,835	8.19%	$744
80%	**0.3893**	**1574**	**425**	**$9,699**	**6.69%**	**$609**
70%	0.4354	1521	719	$9,569	5.26%	$478
60%	0.4903	1443	1079	$9,445	3.90%	$355
50%	0.5546	1326	1525	$9,332	2.65%	$241
40%	0.6261	1153	2067	$9,233	1.57%	$142
30%	0.6969	900	2684	$9,155	0.71%	$64
20%	0.7507	557	3304	$9,105	0.16%	$15
11%	0.7679	189	3760	$9,091	0%	$0

Combined impacts from variations in overall scrap rate φ and service level $(1 - \alpha)$ on the expected total system cost E[$TRCU(T_1)$] are also investigated and the results are demonstrated in Fig. 12. It is noted that as either the service level is higher or overall scrap rate goes up, the E[$TRCU(T_1)$] increases remarkably.

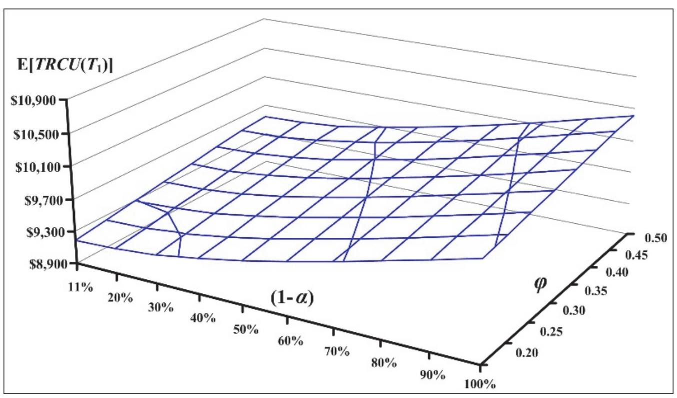

Fig. 12. Combined impacts from variations in overall scrap rate φ and service level $(1 - \alpha)$ on the expected total system cost E[$TRCU(T_1)$]

5. Concluding Remarks

A decision model is developed to investigate a realistic fabrication system incorporating diverse practical factors in production, such as random breakdown, backorder of permissible shortage, scrap, and rework of repairable items. This decision model consists of mathematical modeling, formulations, and optimization method in order to help analyze the problem, derive the system cost function, and find optimal replenishment cycle length decision, respectively. Applicability of the research results are demonstrated by a numerical example. The proposed decision model allows production managers to not only find the optimal stock replenishment policy (Fig. 8), but also thoroughly reveal individual impact and/or joint effects of stochastic breakdown, defective rate, backorder of allowable shortage, rework, and scrap on the replenishing decision (Figs. 9-12). In addition, information on detailed cost components in the specific fabrication system (Fig. 13) and contributors to any particular cost component (e.g. quality cost in production (Fig.14)) can now be available for managerial decision-making. For direction of future study, an interesting extension will be to consider a multi-shipment inventory issuing policy rather than the continuous one in the same problem.

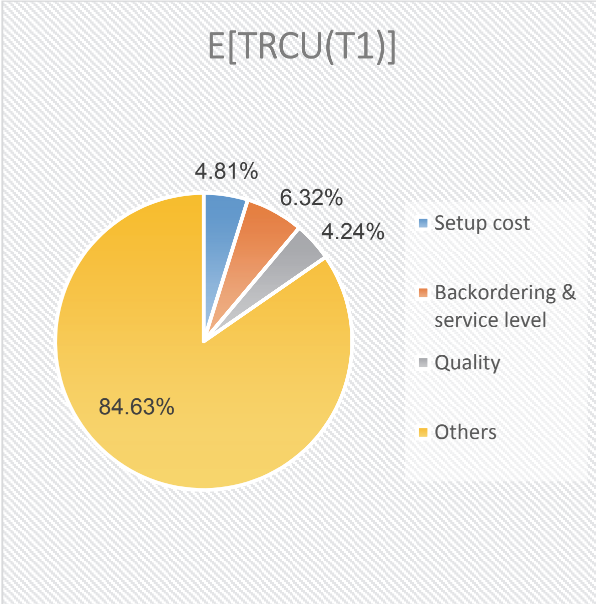

Fig. 13. Cost components of the expected total system cost E[$TRCU(T_1)$]

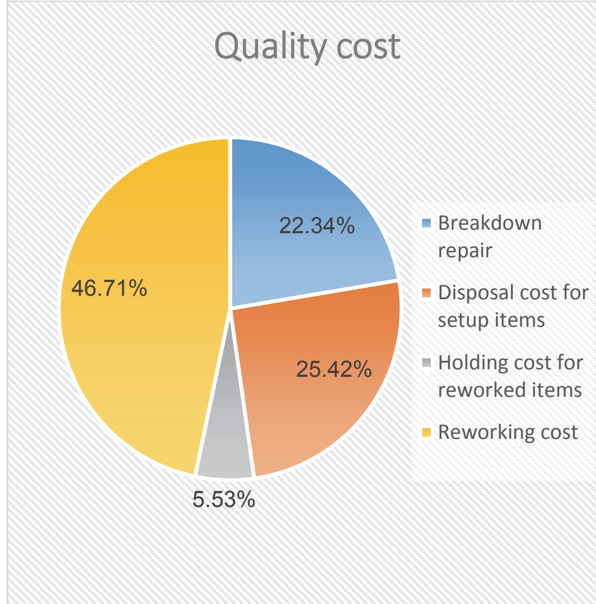

Fig. 14. Detailed contributors of the quality cost

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Appendix – A

Nomenclature

t = fabrication time before a random breakdown takes place,
β = number of breakdowns per year, a random variable that follows Poisson distribution,
M = a fixed machine repairing cost per breakdown,
T_1 = fabrication uptime, the decision variable of the proposed fabrication system,
Q = batch size per fabrication cycle,
T' = fabrication cycle time in the case that machine breakdown does take place,
t_5' = a part of fabrication uptime when negative stocks (backlog) being gradually satisfied,
t_1' = a part of fabrication uptime when positive stocks growing up,
t_2' = reworking time of nonconforming items,
t_3' = time that available finished products are depleted,
t_4' = permitted time of stock-out situation,
B = maximum level of allowable backlog/shortage,
x = defective rate during fabrication, a random variable follows uniform distribution,
θ = proportion of scrap among defective products,
θ_1 = proportion of scrap produced during rework process,
φ = overall scrap rate in the proposed system, $\varphi = \theta + (1 - \theta)\theta_1$,
H_0 = the backlog level at the time a breakdown takes place,
H_1 = the on-hand inventory level in the end of fabrication uptime,
H = the on-hand inventory level in the end of reworking time,
H_2 = the on-hand inventory level at the time a breakdown takes place,
d_2 = production rate of scrap items during the reworking time t_2',
C = unit manufacturing cost,
K = fabrication setup cost per cycle,
h = unit holding cost per unit time,
h_1 = holding cost per unit time per reworked item,
b = unit backordering cost,
h_3 = holding cost per unit time per safety item,
C_1 = cost per safety item,
C_T = delivery cost per item,
g = t_r, the fixed machine repair time,
$I(t)$ = the on-hand inventory status including backlogging at time t,
$I_d(t)$= the on-hand inventory of nonconforming products at time t,
$I_S(t)$= the on-hand inventory of scrap items at time t,
T = cycle time in the case that machine breakdown does not take place,
t_5 = a part of uptime when negative backlog being gradually satisfied in a fabrication system without breakdown occurrence,
t_1 = a part of uptime when positive stocks add up in a fabrication system without breakdown occurrence,
t_2 = reworking time in a fabrication system without breakdown occurrence,
t_3 = product depleting time in a fabrication system without breakdown occurrence,
t_4 = permitted time of stock-out situation in a fabrication system without breakdown occurrence,
$\boldsymbol{T}$ = replenishment cycle time whether a breakdown occurs or not,
$TRC_1(T_1)$ = total relevant cost per cycle in the case that a breakdown takes place in t_5',
$TRC_2(T_1)$ = total relevant cost per cycle in the case that a breakdown takes place in t_1',
$TRC_3(T_1)$ = total relevant cost per cycle in the case that no breakdown taking place,
$TRCU(T_1)$ = total system relevant cost per unit time whether a breakdown occurs or not,
$E[TRC_1(T_1)]$ = expected total relevant cost per cycle in the case that a breakdown occurs in t_5',
$E[TRC_2(T_1)]$ = expected total relevant cost per cycle in the case that a breakdown occurs in t_1',
$E[TRC_3(T_1)]$ = expected total relevant cost per cycle in the case that no breakdown occurring,
$E[TRCU(T_1)]$ = expected total system relevant cost per unit time whether a breakdown occurs or not.

Appendix – B

Substitute Eq. (1) to Eq. (12) in Eq. (14) and using the expected values $E[x]$ to cope with the randomness of nonconforming rate, and with further derivations one can obtain $E[TRC_1(T_1)]$ as shown in Eq. (B-1).

$$E[TRC_1(T_1)] = K + CT_1P_1 + C_R E[x](1-\theta)T_1P_1 + C_S E[x]\varphi T_1P_1 + M + \left[h_3(\lambda t_r)\left(t+\frac{t_r}{2}\right) + C_1(\lambda t_r)\right]$$
$$+\frac{E[x]^2 T_1^2 P_1^2(1-\theta)}{2P_2}\left[h_1(1-\theta)-h\right] + \frac{hE[x]^2 T_1^2P_1^2\varphi(1-\theta)}{2P_2} - \frac{hBT_1P_1}{\lambda}(1-\varphi E[x])$$
$$+\frac{hT_1^2P_1^2}{2\lambda}(1-\varphi E[x])^2 - \frac{hT_1^2P_1}{2}(1-2\varphi E[x]) + (h+b)\left[\frac{B^2}{2(P_1-P_1E[x]-\lambda)}+\frac{B^2}{2\lambda}\right] \tag{B-1}$$
$$+C_T\left[(1-\varphi E[x])T_1P_1 + (\lambda t_r)\right] + bt_r(B-P_1t+\lambda t) + P_1xtt_r(h+b).$$

Substitute Eq. (1) to Eq. (7), Eq. (9) to Eq. (12), and Eq. (15) in Eq. (17) and applying $E[x]$ to cope with the randomness of nonconforming rate, and with further derivations one can obtain $E[TRC_2(T_1)]$ as sho wn in Eq. (B-2).

$$E[TRC_2(T_1)] = K + CT_1P_1 + C_R E[x](1-\theta)T_1P_1 + C_S E[x]\varphi T_1P_1 + M + \left[h_3(\lambda t_r)\left(t+\frac{t_r}{2}\right) + C_1(\lambda t_r)\right]$$
$$+\frac{E[x]^2 T_1^2 P_1^2(1-\theta)}{2P_2}\left[h_1(1-\theta)-h\right] + \frac{hE[x]^2 T_1^2P_1^2\varphi(1-\theta)}{2P_2} - \frac{hBT_1P_1}{\lambda}(1-\varphi E[x])$$
$$+\frac{hT_1^2P_1^2}{2\lambda}(1-\varphi E[x])^2 - \frac{hT_1^2P_1}{2}(1-2\varphi E[x]) + (h+b)\left[\frac{B^2}{2(P_1-P_1E[x]-\lambda)}+\frac{B^2}{2\lambda}\right] \tag{B-2}$$
$$+C_T\left[(1-\varphi E[x])T_1P_1 + (\lambda t_r)\right] + htt_rP_1 - htt_r\lambda - hBt_r.$$

Substitute Eq. (18) to Eq. (27) in Eq. (29) and using $E[x]$ to cope with the randomness of nonconforming rate, and with further derivations one can obtain $E[TRC_3(T_1)]$ as shown in Eq. (B-3).

$$E[TRC_3(T_1)] = K + CT_1P_1 + C_R E[x][(1-\theta)T_1P_1] + C_S E[x](\varphi T_1P) + \left[h_3(\lambda t_r)T + C_1(\lambda t_r)\right]$$
$$-\frac{hBT_1P_1(1-\varphi E[x])}{\lambda} + \frac{E[x]^2T_1^2P_1^2(1-\theta)}{2P_2}\left[h_1(1-\theta)-h\right] + (h+b)\left[\frac{B^2}{2(P_1-P_1E[x]-\lambda)}+\frac{B^2}{2\lambda}\right] \tag{B-3}$$
$$+\frac{hE[x]^2T_1^2P_1^2\varphi(1-\theta)}{2P_2} + \frac{hT_1^2P_1^2(1-\varphi E[x])^2}{2\lambda} - \frac{hT_1^2P_1(1-2\varphi E[x])}{2} + C_T\left[(1-\varphi E[x])T_1P_1\right].$$

Appendix – C

Several different values of β have been plugged in Eq. (41) to test for convexity of the expected total system relevant cost per unit time $E[TRCU(T_1)]$, and the results are shown in Table C-1. It reveals that the proposed model can derive the optimal runtime solution for a wide-ranging breakdown rate. That is the model is suitable solving real manufacturing system that is subject to random breakdown rate per year ranging from 0 to 8. In the particular example that $\beta = 0.5$ (as highlighted in Table C-1), Eq. (41) is confirmed, since $T_{1U} = 0.5491 < z(T_{1U}) = 2.4394$ and $T_{1L} = 0.3423 < z(T_{1L}) = 2.1346$.

Table C-1
The results of convexity tests for E[$TRCU(T_1)$] by using different values of β

β	T_{1U}	$z(T_{1U})$	T_{1L}	$z(T_{1L})$
8.00	0.5277	5.7896	0.1255	0.4033
7.00	0.5279	4.1766	0.1388	0.4466
6.00	0.5282	3.1086	0.1548	0.4998
5.00	0.5285	2.3994	0.1744	0.5671
4.00	0.5291	1.9383	0.1985	0.6558
3.00	0.5301	1.6666	0.2287	0.7816
2.00	0.5320	1.5812	0.2665	0.9878
1.00	0.5378	1.8410	0.3141	1.4598
0.50	**0.5491**	**2.4394**	**0.3423**	**2.1346**
:	:	:	:	:
0.01	1.2271	6.3483	0.3730	5.4701