

## Ranking DMUs using a novel combination method for integrating the results of relative closeness benevolent and relative closeness aggressive models

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### ABSTRACT

In this paper, a novel combination method is offered to integrate the results of two new relative closeness models, called relative closeness benevolent (RCB) and relative closeness aggressive (RCA) models, for ranking all DMUs. To prove the applicability of the proposed method, it is examined in three numerical examples, performance assessment problem, six nursing homes and fourteen international passenger airlines. Firstly, RCB and RCA models were formulated in order to generate the cross-efficiency intervals matrix (CEIM). After obtaining CEIM, the RC index was utilized to generate a combined cross-efficiency matrix (combined CEM). In combined CEM, target DMUs were viewed as criteria and DMUs were viewed as alternatives. After that, the weights of each criterion were generated using a new weighting method based on standard deviation technique (MSDT). Finally, all DMUs were evaluated and ranked. Comparison with existing cross-efficiency models indicates the more reliable results through the use of the proposed method.

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## 1. Introduction

The data envelopment analysis (DEA) is a linear programming model that was first described by Farrel (1957), but a mathematical model was first introduced in the Charnes et al. paper (Charnes, Cooper, & Rhodes, 1979). This non-parametric model evaluates the relative efficiency of decision making units (DMUs) with multiple inputs and outputs (Zerafat Angiz, Mustafa, & Kamali, 2013). One of the advantages of this method is that the weights for inputs and outputs are not required (Sun, Wu, & Guo, 2013). In the DEA model, the input weights and output weights are obtained by maximizing the ratio of the sum of weighted outputs to the sum of weighted inputs. It is well known that the efficiency score of each DMU cannot be greater than 1, and DMUs can be defined as being efficient DMUs if their efficiency scores are 1; otherwise the DMUs are inefficient (Wang, Chin, & Luo, 2011; Wichapa, Khokhajaikiat, & Chaiphet, 2021). Over the past four decades, DEA has been widely applied in efficiency evaluation and benchmarking in many fields (Amalnick & Saffar, 2017; Durga Prasad, Kambagowni, & Prasad, 2017; Lesik et al., 2020). However, a frequently discussed problem in DEA is that efficient DMUs cannot be discriminated between. Hence, various methods have been carried out to overcome this main drawback and to improve the discrimination power of DEA. These methods can be classified into seven groups as follows (Hosseinzadeh Lotfi et al., 2013). The first group is the cross-efficiency method. The second group is the DEA optimal weights. The third group is the super-efficiency method. The fourth group is the benchmarking idea. The fourth group is the multivariate statistical tools. The sixth group is the multi-criteria decision making (MCDM) method. The final group is various ranking methods. Although there are many methods for ranking DMUs, the cross-efficiency method is one of the most popular ranking methods in DEA. The cross-efficiency method, first introduced by Sexton et al. (1986), has long been suggested as a power tool for the ranking of DMUs based on the cross-efficiency concept. Based on this idea, a combination of self-evaluation and peer-evaluation was suggested for overcoming the weakness of DEA's discrimination power. So, the weights of all DMUs can be obtained by averaging the

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best weights of DMUs. Finally, all DMUs can be ranked by their average cross-efficiency scores (ACE scores) in the cross-efficiency matrix. Although the cross-efficiency method has been widely studied and used all over the world, there is still one drawback. The optimal weights are not unique, so this possibly reduces the usefulness of cross-efficiency evaluation to help decision makers improve their performance (Si & Ma, 2019). To overcome the main drawback above, Sexton et al. (1986) first suggested using a secondary-goal model in the cross-efficiency method and later, Doyle and Green (1994) investigated the aggressive and benevolent formulations to deal with the non-uniqueness issue. Even though aggressive and benevolent formulations are most commonly used for ranking all DMUs, a question arises: which one is the best? Usually, the performance evaluation results obtained from aggressive and benevolent formulations may not be the same for similar problems, because each of the formulations has a different viewpoint. Certainly, each of the above viewpoints should not be ignored. Hence, it is wise to try alternative formulations and combine the results of benevolent and aggressive formulations for ranking all DMUs. Wang and Luo (2006) have first proposed the concept of efficiency assessment using virtual DMUs (ideal and anti-ideal DMUs) in the DEA literature. The ideal DMU considers the minimum input with the maximum output, while the anti-ideal DMU is the unit consuming the maximum input to produce the minimum output. Jahanshahloo et al. (2010) proposed a ranking method based on positive ideal DMUs with a common set of weights for the efficient DMUs. Wang et al. (2011) exhibited an effective model of four models based on the ideas of cross-efficiency and virtual DMUs, namely the relative closeness model (RC model), for ranking all DMUs. Sun, Wu and Gou (2013) have presented two models based on the viewpoint of virtual DMUs with common weights for performance ranking of all DMUs. Recently, Naseri and Kiaei (2016) have proposed cross-efficiency evaluation based on ideas of ideal and anti-ideal virtual DMUs' assessment. Hou, Wang, and Zhou (2018) have proposed a new formulation by the use of ideal and anti-ideal virtual DMUs' assessment for ranking all DMUs. Naseri and Kiaei (2019) proposed the new neutral models based on an ideal DMU in cross-efficiency. Inspired by the above ideas, it is wise to try new formulations and combine the ideas of benevolent, aggressive and RC models for ranking all DMUs, because neither view should be ignored. It is well known that determination of criteria weights is an important issue in the multi-criteria decision-making problem (MCDM problem). The final ranking results are highly dependent on the weights of each criterion. Several weighting methods have been presented to obtain criteria weights. These methods are divided into three categories (Keshavarz-Ghorabae, Amiri, Zavadskas, Turskis, & Antucheviciene, 2021), including subjective weighting method, objective weighting method and hybrid weighting method. In the subjective method, decision makers must determine the criteria weights. The main drawback of the subjective method is that it is a difficult task for decision-makers, and the accuracy of their preferences can be decreased by increasing the number of criteria (Alfares & Duffuaa, 2016). In the objective weighting method, the decision-makers have no role in determining criteria weights. In this method, the criteria weights are obtained using a specific computational process based on the initial information or decision matrix (Kao, 2010). There are some popular methods in this category that are often used to determine the criteria weights in the literature, such as the Entropy method (Lu & Liu, 2016), the Standard deviation method (Anitha & Das, 2019), CRITIC (Criteria Importance Through Inter-criteria Correlation) (Wichapa et al., 2021) and SECA (Simultaneous Evaluation of Criteria and Alternatives) (Keshavarz-Ghorabae, Govindan, Amiri, Zavadskas, & Antucheviciene, 2019). The last category of weighting methods is the hybrid weighting method. This method uses a combination of different views of subjective and objective weighting methods. The various methods in this category have no distinctive characteristics, and they borrow the prominent points of other weighting methods. The hybrid weighting method could give more realistic weights because this method can use the decision-makers' preferences and the information of the decision matrix (Liu, Hu, Zhang, Li, & Liu, 2020). This paper proposes a new objective weighting method based on the standard deviation technique (MSDT) for determining criteria weights. Unlike the other methods mentioned above, MSDT converts the standard deviation of each criterion in a decision matrix for estimating the criteria weights.

To this end, this research provides a novel combination method to aggregate the ideas of benevolent, aggressive and RC models for ranking all DMUs. The proposed method has been adapted from Wang's RC model (Wang et al., 2011) in the following ways: (1) we formulate two new RC models, called RCB and RCA models, based on ideas of benevolent, aggressive and Wang's RC models (Wang et al., 2011). The RCB and RCA models are utilized to generate the RC benevolent cross-efficiency matrix (RCB-CEM) and the RC aggressive cross-efficiency matrix (RCA-CEM) respectively, and then a combined CEM is generated using the RC index. (2) In the combined CEM, a target DMU and a DMU in the decision matrix are viewed as criterion and alternative respectively, and the MSDT is used to generate the criteria weights of the decision matrix (combined CEM) for evaluating the final weights of each DMU.

The remainder of this article is organized in the following manner. In the next section some mathematical models used in this article are presented. The proposed method and numerical examples are presented in Section 3 and Section 4 respectively. Finally, the conclusion is given in Section 5.

## 2. Background

### 2.1 CCR model

The mathematical model of the CCR model was first proposed by Charnes, Cooper and Rhodes (1979). The classic CCR model is used to evaluate the efficiency score of DMUs with multiple inputs and outputs. Many researchers have carried out the application of the CCR model in various fields (Al-Faraj, Alidi, & Bu-Bshait, 1993; Fancello, Carta, & Serra, 2020; Liang,

Yang, Cook, & Zhu, 2006; Weber, 1996; Wei, Chen, Li, & Tsai, 2011), which proves that the CCR model is an effective method for measuring performance of all DMUs. Consider a set of  $n$  DMUs that is measured in terms of  $m$  inputs to produces  $s$  outputs. Let  $x_{ij}$  ( $i = 1, \dots, m$ ) and  $y_{rj}$  ( $r = 1, \dots, s$ ) be the values of inputs and outputs of DMU $_j$  ( $j = 1, \dots, n$ ). Let  $u_{rk}$  and  $v_{ik}$  be the weights of outputs and weights of inputs respectively. For any evaluated DMU $_k$  ( $1 \leq k \leq n$ ), the efficiency score  $E_{kk}$  can be evaluated by the CCR model as follows:

$$\begin{aligned} & \max \sum_{r=1}^s u_{rk} \cdot y_{rk} = E_{kk} \\ & \text{subject to:} \\ & \sum_{r=1}^s \mu_{rk} \cdot y_{rj} - \sum_{i=1}^m w_{ik} \cdot x_{ij} \leq 0, \quad \forall j, j = 1, 2, \dots, n \\ & \sum_{i=1}^m v_{ik} \cdot x_{ij} = 1 \\ & v_{ik} \geq 0, \quad \forall i, i = 1, 2, \dots, m \\ & u_{rk} \geq 0, \quad \forall r, r = 1, 2, \dots, s \end{aligned} \tag{1}$$

For DMU $_k$  ( $k = 1, 2, 3, \dots, n$ ), a set of optimal weights can be obtained by solving the CCR model in Equation (1). In the CCR model, DMUs are self-evaluated and termed efficient if and only if the optimal objective function is equal to 1.

### 2.2 Cross-efficiency method

The main drawback of the CCR model is that efficient DMUs cannot be fully discriminated from each other. Hence, many researchers (Sexton et al., 1986) have proposed various cross-efficiency methods to provide a full ranking for all DMUs. The cross-efficiency formulations are given below.

For each DMU $_k$  ( $k = 1, 2, \dots, n$ ), the cross-efficiency of each DMU ( $E_{kj}$ ) can be determined as follows.

$$E_{kj} = \frac{\sum_{r=1}^s u_{rk} \cdot y_{rk}}{\sum_{i=1}^m v_{ik} \cdot x_{ik}}, \quad k, j = 1, 2, 3, \dots, n \tag{2}$$

Then the average cross-efficiency score (ACE) of each DMU is defined as follows.

$$\bar{E}_j = \frac{1}{n} \sum_{k=1}^n E_{kj}, \quad k, j = 1, 2, 3, \dots, n \tag{3}$$

### 2.3 Benevolent and aggressive models

The main drawback of the cross-efficiency method is that the optimal weights of all DMUs obtained from the CCR model in Equation (1) may be not unique, which clearly cannot provide results to help decision makers improve their performance (Si & Ma, 2019; Wu, Sun, Zha, & Liang, 2011). To overcome this drawback, Doyle and Green (Doyle & Green, 1994) have proposed the well-known aggressive and benevolent models to identify the optimal weights of all DMUs. The benevolent and aggressive formulations are as follows.

$$\begin{aligned} & \max \sum_{r=1}^s u_{rk} \sum_{j=1, j \neq k}^n y_{rj} \\ & \text{subject to:} \\ & \sum_{i=1}^m v_{ik} \sum_{j=1, j \neq k}^n x_{ij} = 1, \\ & \sum_{r=1}^s u_{rk} \cdot y_{rj} - E_{kk} \cdot \sum_{i=1}^m v_{ik} \cdot x_{ij} = 0, \quad \forall j, j \neq k, j = 1, 2, 3, \dots, n, \\ & \sum_{r=1}^s \mu_{rk} \cdot Y_{rj} - \sum_{i=1}^m v_{ik} \cdot x_{ij} \leq 0, \quad \forall j, j \neq k, j = 1, 2, 3, \dots, n, \\ & u_{ik} \geq 0, \quad \forall i, i = 1, 2, 3, \dots, m, \\ & \mu_{rk} \geq 0, \quad \forall r, r = 1, 2, 3, \dots, s \quad \text{and} \end{aligned} \tag{4}$$

$$\begin{aligned} & \min \sum_{r=1}^s u_{rk} \sum_{j=1, j \neq k}^n y_{rj} \\ & \text{subject to: the same constraints as in Eq. (4)} \end{aligned} \tag{5}$$

Eq. (4) and Eq. (5) represent the benevolent and aggressive models, which aim to maximize and minimize respectively the cross efficiency of the integrated unit consisting of the other DMUs while maintaining the self-evaluation efficiency of a particular DMU under evaluation. Since both models optimize the weights of inputs and outputs from two different viewpoints, the same ranking orders are not guaranteed. Thus, the idea of generating the alternative models for ranking all DMUs is attractive.

2.4 RC model

Based on the ideas of Wang, Chin and Luo (Wang et al., 2011), An IDMU and an ADMU are defined as follows:

**Definition 1.** A virtual DMU can be defined as an IDMU if it consumes the least inputs to generate the most outputs. While a virtual DMU can be defined as ADMU if it consumes the most inputs only to produce the least outputs.

Let  $x_i^{\min}$  and  $y_i^{\max}$  be the ideal input and ideal output of the IDMU<sub>k</sub>, and  $x_i^{\max}$  and  $y_i^{\min}$  be the anti-ideal input and anti-ideal output of the ADMU<sub>k</sub>, respectively.

By Definition 1, the inputs and outputs of an IDMU can be determined as

$$x_i^{\min} = \min_j (x_{ij}), i=1,2,\dots,n, y_r^{\max} = \max_j (y_{rj}), r=1,2,\dots,s$$

The inputs and outputs of an ADMU can be determined as

$$x_i^{\max} = \max_j (x_{ij}), i=1,2,\dots,n, y_r^{\min} = \min_j (y_{rj}), r=1,2,\dots,s$$

**Definition 2.** The distances between IDMU, ADMU and DMU<sub>k</sub> can be defined as

$$D_k^+ = \sum_{r=1}^s u_{rk} (y_r^{\max} - y_{rk}) + \sum_{i=1}^m v_{rk} (x_{ik} - x_i^{\min}), k=1,2,3,\dots,n$$

$$D_k^- = \sum_{r=1}^s u_{rk} (y_{rk} - y_r^{\min}) + \sum_{i=1}^m v_{rk} (x_i^{\max} - x_{ik}), k=1,2,3,\dots,n$$

$$D_k = D_k^+ + D_k^-$$

$$D_k = \sum_{r=1}^s u_{rk} (y_r^{\max} - y_r^{\min}) + \sum_{i=1}^m v_{rk} (x_i^{\max} - x_i^{\min}), k=1,2,3,\dots,n$$

where  $u_{rk} (r=1, \dots, s)$  and  $v_{ik} (i=1, \dots, m)$  are the weights for outputs and inputs, respectively.

**Definition 3.** The relative closeness of DMU<sub>k</sub> ( $RC_k$ ) with respect to IDMU and ADMU is defined as

$$RC_k = \frac{D_k^-}{D_k^- + D_k^+} = \frac{\sum_{r=1}^s u_{rk} (y_{rk} - y_r^{\min}) + \sum_{i=1}^m v_{rk} (x_i^{\max} - x_{ik})}{\sum_{r=1}^s u_{rk} (y_r^{\max} - y_r^{\min}) + \sum_{i=1}^m v_{rk} (x_i^{\max} - x_i^{\min})}, k=1,2,3,\dots,n$$

The relative closeness model (RC model) for cross efficiency evaluation is as follows (Wang et al., 2011):

$$\max \sum_{r=1}^s u_{rk} (y_{rk} - y_r^{\min}) + \sum_{i=1}^m v_{rk} (x_i^{\max} - x_{ik}) = RC_k$$

subject to:

$$\sum_{r=1}^s u_{rk} (y_r^{\max} - y_r^{\min}) + \sum_{i=1}^m v_{ik} (x_i^{\max} - x_i^{\min}) = 1, \tag{6}$$

$$\sum_{r=1}^s \mu_{rk} \cdot Y_{rk} - \theta_{kk} \sum_{i=1}^m v_{ik} \cdot X_{ik} = 0,$$

$$\sum_{r=1}^s \mu_{rk} \cdot Y_{rj} - \sum_{i=1}^m v_{ik} \cdot X_{ij} \leq 0, j=1,2,3,\dots,n,$$

$$u_{rk} \geq 0, \quad \forall r, \quad r = 1, 2, \dots, s,$$

$$v_{ik} \geq 0, \quad \forall i, \quad i = 1, 2, \dots, m;$$

### 3. The proposed method

There are many different DEA cross-efficiency models that have been proposed for ranking DMUs. Unfortunately, the ranking results obtained for each cross-efficiency model may differ for similar problems. Hence, it is wise to try the new effective methods that provide more reliable results in ranking DMUs effectively. In this section, a new hybrid method is offered for ranking DMUs. The framework for our method is shown in Fig. 1.

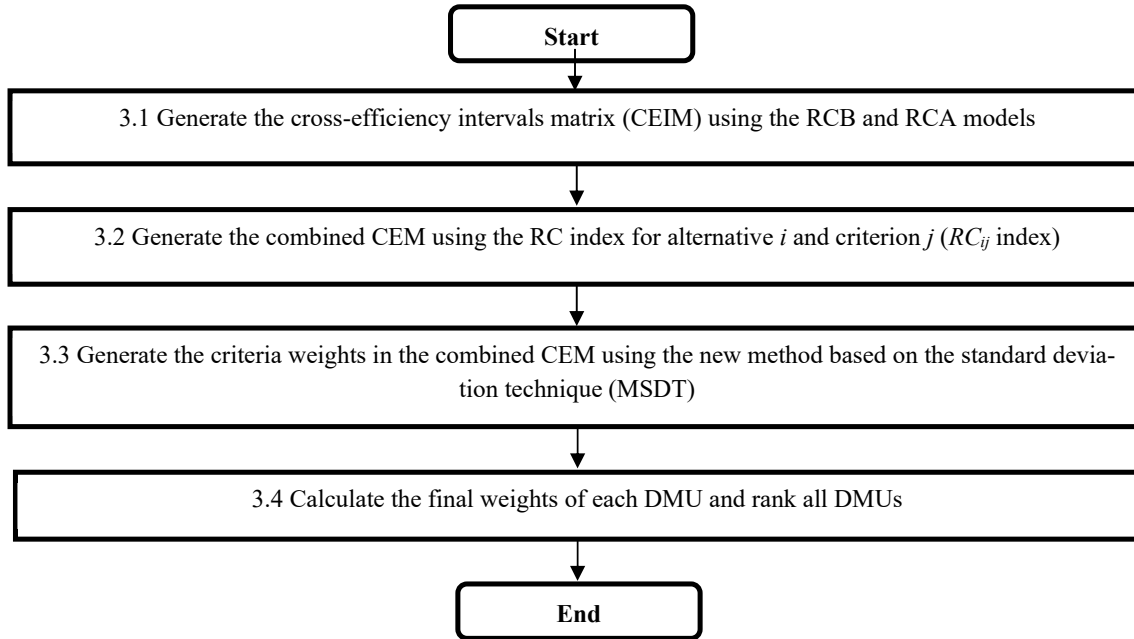


Fig. 1. Conceptual framework for the proposed method

#### 3.1 Generating the cross-efficiency intervals matrix (CEIM) using the RCB and RCA models

According to the ideas of Wang’s RC model (Wang et al., 2011) in Eq. (4) and the concept of the well-known aggressive and benevolent models (Doyle & Green, 1994), the relative closeness benevolent (RCB) and relative closeness aggressive (RCA) models can be determined as Eq. (7) and Eq. (8) respectively.

$$\max \sum_{r=1}^s u_{rk} \sum_{j=1, j \neq k}^n (y_{rj} - y_r^{\min}) + \sum_{i=1}^m v_{rk} \sum_{j=1, j \neq k}^n (x_i^{\max} - x_{ij}) = \theta_{RCB}^*$$

subject to:

$$\sum_{r=1}^s u_{rk} (y_r^{\max} - y_r^{\min}) + \sum_{i=1}^m v_{ik} (x_i^{\max} - x_i^{\min}) = 1, \quad \forall k = 1, 2, 3, \dots, n,$$

$$\sum_{r=1}^s \mu_{rk} \cdot Y_{rk} - \theta_{kk} \sum_{i=1}^m v_{ik} \cdot X_{ik} = 0, \quad \forall k = 1, 2, 3, \dots, n, \tag{7}$$

$$\sum_{r=1}^s \mu_{rk} \cdot \sum_{j=1, j \neq k}^n y_{rj} - \sum_{i=1}^m v_{ik} \sum_{j=1, j \neq k}^n x_{ij} \leq 0, \quad \forall k = 1, 2, 3, \dots, n,$$

$$u_{rj} \geq 0, \quad \forall r, \quad r = 1, 2, \dots, s, \quad \forall j = 1, 2, 3, \dots, n,$$

$$v_{ij} \geq 0, \quad \forall i, \quad i = 1, 2, \dots, m, \quad \forall j = 1, 2, 3, \dots, n;$$

and

$$\min \sum_{r=1}^s u_{rk} \sum_{j=1, j \neq k}^n (y_{rj} - y_r^{\min}) + \sum_{i=1}^m v_{rk} \sum_{j=1, j \neq k}^n (x_i^{\max} - x_{ij}) = \theta_{RCA}^* \tag{8}$$

subject to: the same constraints as in Eq. (7)

According to the RCB and RCA models, two evaluation matrices can be obtained as follows:

RCB cross-efficiency matrix (RCB-CEM) is

$$E^{RCB*} = \begin{bmatrix} \theta_{11}^{RCB*} & \theta_{12}^{RCB*} & \theta_{13}^{RCB*} & \dots & \theta_{1n}^{RCB*} \\ \theta_{21}^{RCB*} & \theta_{22}^{RCB*} & \theta_{23}^{RCB*} & \dots & \theta_{2n}^{RCB*} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \theta_{n1}^{RCB*} & \theta_{n2}^{RCB*} & \theta_{n3}^{RCB*} & \dots & \theta_{nn}^{RCB*} \end{bmatrix}. \tag{9}$$

RCA cross-efficiency matrix (RCA-CEM) is

$$E^{RCA*} = \begin{bmatrix} \theta_{11}^{RCA*} & \theta_{12}^{RCA*} & \theta_{13}^{RCA*} & \dots & \theta_{1n}^{RCA*} \\ \theta_{21}^{RCA*} & \theta_{22}^{RCA*} & \theta_{23}^{RCA*} & \dots & \theta_{2n}^{RCA*} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \theta_{n1}^{RCA*} & \theta_{n2}^{RCA*} & \theta_{n3}^{RCA*} & \dots & \theta_{nn}^{RCA*} \end{bmatrix}. \tag{10}$$

Cross-efficiency intervals matrix (CEIM) is

$$E^* = \begin{bmatrix} [\theta_{11}^{\min}, \theta_{11}^{\max}] & [\theta_{12}^{\min}, \theta_{12}^{\max}] & [\theta_{13}^{\min}, \theta_{13}^{\max}] & \dots & [\theta_{1n}^{\min}, \theta_{1n}^{\max}] \\ [\theta_{21}^{\min}, \theta_{21}^{\max}] & [\theta_{22}^{\min}, \theta_{22}^{\max}] & [\theta_{23}^{\min}, \theta_{23}^{\max}] & \dots & [\theta_{2n}^{\min}, \theta_{2n}^{\max}] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ [\theta_{n1}^{\min}, \theta_{n1}^{\max}] & [\theta_{n2}^{\min}, \theta_{n2}^{\max}] & [\theta_{n3}^{\min}, \theta_{n3}^{\max}] & \dots & [\theta_{nn}^{\min}, \theta_{nn}^{\max}] \end{bmatrix}. \tag{11}$$

where

$$\theta_{ij}^{\min} = \min\{\theta_{ij}^{RCB*}, \theta_{ij}^{RCA*}\}, \forall i, \forall j = 1, 2, 3, \dots, n$$

$$\theta_{ij}^{\max} = \max\{\theta_{ij}^{RCB*}, \theta_{ij}^{RCA*}\}, \forall i, \forall j = 1, 2, 3, \dots, n$$

### 3.2 Generating the combined CEM using the relative closeness index

The distance measures for alternative  $i$  ( $DMU_i$ ) and criterion  $j$  (target  $DMU_j$ ) can use the relative closeness formula to generate a combined CEM. There are three calculation steps for generating the combined CEM as follows.

(1) Calculate the distance from positive ideal solution for alternative  $i$  and criterion  $j$  ( $d_{ij}^+$ )

Let positive ideal solution (PIS) and negative ideal solution (NIS) be 1 and 0 respectively. Then the distance from PIS for alternative  $i$  and criterion  $j$  ( $d_{ij}^+$ ) can be defined as follows.

$$d_{ij}^+ = (1 - \theta_{ij}^{\min}) + (1 - \theta_{ij}^{\max}), \forall i, \forall j = 1, 2, 3, \dots, n \tag{12}$$

(2) Similarly, the distance from NIS ( $d_{ij}^-$ ) is

$$d_{ij}^- = \theta_{ij}^{\min} + \theta_{ij}^{\max}, \forall i, \forall j = 1, 2, 3, \dots, n \tag{13}$$

(3) Then a relative closeness index for alternative  $i$  and criterion  $j$  ( $rc_{ij}$ ) can be determined as follows.

$$rc_{ij} = \frac{d_{ij}^-}{d_{ij}^- + d_{ij}^+}, \forall i, \forall j = 1, 2, 3, \dots, n \tag{14}$$

The combined CEM will be generated using the results of the Eq. (11) to Eq. (14). Details are shown in Table 1.

**Table 1**  
The combined CEM for alternative  $i$  and criterion  $j$

Combined CEM	Target DMU				
	1	2	3	...	$n$
DMU1	$rc_{11}$	$rc_{12}$	$rc_{13}$	...	$rc_{1n}$
DMU2	$rc_{21}$	$rc_{22}$	$rc_{23}$	...	$rc_{2n}$
DMU3	$rc_{31}$	$rc_{32}$	$rc_{33}$	...	$rc_{3n}$
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
DMU $n$	$rc_{n1}$	$rc_{n2}$	$rc_{n3}$	...	$rc_{nm}$

3.3 Generating the criteria weights in the combined decision matrix using MSDT

In Table 1, consider a combined CEM ( $X$ ),  $X = [rc_{ij}]_{n \times n}$ , where  $rc_{ij}$  is the relative closeness index with respect to alternative  $i$  (DMU $i$ ) and criterion  $j$  (target DMU $j$ ) and  $n$  is the number of DMUs respectively. The new weighting method is formulated for addressing the ranking problem in this paper. Since a higher standard deviation means that the factor is worse, the standard deviation must be converted to a value with an opposite trend for calculating the final weight in the next step. Based on ideas of He et al. (2012) and Wichapa et al. (2018), the standard deviations of each criterion ( $\sigma_j$ ) in the combined CEM can be converted to the weighting factors ( $\omega_j$ ) as in Eq. (15).

$$\omega_j = \frac{\sum_{j=1}^n \sigma_j - \sigma_j}{\sum_{j=1}^n \sigma_j \cdot (n - 1)} \tag{15}$$

After obtaining weighting factors, the higher value of weighting factor is the better criterion.

3.4 Calculating the final weights of DMUs and ranking all DMUs

After obtaining  $\omega_j$ , the final weights of each DMU ( $w_i$ ) can be obtained by multiplying the weight of each criterion by the corresponding combined CEM using Eq. (16).

$$w_i = \sum_{j=1}^m (\omega_j \cdot x_{ij}), \quad \forall i, i = 1, 2, 3, \dots, n \tag{16}$$

After obtaining  $w_i$ , all DMUs can be ranked so that a higher value of  $w_i$  means that the DMU's ranking is higher.

4. Numerical examples

In this section, the validity of the proposed method is examined with three numerical examples. The first is the performance assessment problem investigated by Andersen and Petersen (1993), the second is six nursing homes investigated by Sexton et al. (1986), and the third is fourteen international passenger airlines by Tofallis (1997a). Details of the calculation steps are shown in Sections 4.1, 4.2 and 4.3 respectively.

4.1 Performance assessment problem

Consider a performance assessment problem, investigated by Andersen and Petersen (Andersen & Petersen, 1993), which has five DMUs to be evaluated in the light of two inputs ( $x_1$  and  $x_2$ ) and one output ( $y_1$ ). The data set of the performance assessment problem is provided in Table 2, together with the CCR-efficiency scores of the five DMUs.

**Table 2**

Data set of performance assessment problem

DMUs	$x_1$	$x_2$	$y_1$	CCR
DMU1	2	12	1	1.000
DMU2	2	8	1	1.000
DMU3	5	5	1	1.000
DMU4	10	4	1	1.000
DMU5	10	6	1	0.750

**Step1:** Generate the cross-efficiency intervals matrix (CEIM) using the RCB and RCA models

The efficiencies based on the CCR model must be evaluated first. After that, the RCB model as in Equation (7) and the RCA model as in Equation (8) were coded using LINGO software. Based on Equation (9) and Equation (10), the results of the RCA and RCB models can be obtained as RCB-CEM and RCA-CEM, as listed in Table 3 and Table 4 respectively.

**Table 3**

RCB cross-efficiency matrix (RCB-CEM) of performance assessment problem

DMU	1	2	3	4	5
DMU1	<b>1.0000</b>	0.7143	0.4839	0.4839	0.4839
DMU2	1.0000	<b>1.0000</b>	0.7143	0.7143	0.7143
DMU3	0.4000	1.0000	<b>1.0000</b>	1.0000	1.0000
DMU4	0.2000	0.7143	1.0000	<b>1.0000</b>	1.0000
DMU5	0.2000	0.6250	0.7500	0.7500	<b>0.7500</b>

**Table 4**

RCA cross-efficiency matrix (RCA-CEM) of performance assessment problem

DMU	1	2	3	4	5
DMU1	<b>1.0000</b>	1.0000	0.7143	0.3333	0.4839
DMU2	1.0000	<b>1.0000</b>	1.0000	0.5000	0.7143
DMU3	0.4000	0.4000	<b>1.0000</b>	0.8000	1.0000
DMU4	0.2000	0.2000	0.7143	<b>1.0000</b>	1.0000
DMU5	0.2000	0.2000	0.6250	0.6667	<b>0.7500</b>

After obtaining the RCB-CEM and RCA-CEM, the cross-efficiency intervals matrix (CEIM) can be generated using Eq. (11) as listed in Table 5.

**Table 5**

Cross-efficiency intervals matrix (CEIM) of performance assessment problem

DMU	1	2	3	4	5
DMU1	[ <b>1.000, 1.000</b> ]	[0.714, 1.000]	[0.484, 0.714]	[0.333, 0.484]	[0.484, 0.484]
DMU2	[1.000, 1.000]	[ <b>1.0000, 1.000</b> ]	[0.714, 1.000]	[0.500, 0.714]	[0.714, 0.714]
DMU3	[0.400, 0.400]	[0.400, 1.000]	[ <b>1.0000, 1.000</b> ]	[0.8000, 1.000]	[1.000, 1.000]
DMU4	[0.200, 0.200]	[0.200, 0.714]	[0.714, 1.000]	[ <b>1.000, 1.000</b> ]	[1.000, 1.000]
DMU5	[0.200, 0.200]	[0.200, 0.625]	[0.625, 0.750]	[0.667, 0.750]	[ <b>0.750, 0.750</b> ]

**Step2:** Generate the combined CEM using the relative closeness index

(1) The distances from PIS for alternative  $i$  and criterion  $j$  ( $d_{ij}^+$ ) for the performance assessment problem were obtained using Eq. (12). For example,  $d_{11}^+ = (1-1) + (1-1) = 0.000$ ,  $d_{12}^+ = (1-0.714) + (1+1) = 0.286$  and  $d_{13}^+ = (1-0.484) + (1-0.714) = 0.802$ . The details of the distance from PIS for alternative  $i$  and criterion  $j$  are shown in Table 6.

**Table 6**

The distance from PIS for alternative  $i$  and criterion  $j$  for a performance assessment problem

$d_{ij}^+$	1	2	3	4	5
DMU1	0.000	0.286	0.802	1.183	1.032
DMU2	0.000	0.000	0.286	0.786	0.571
DMU3	1.200	0.600	0.000	0.200	0.000
DMU4	1.600	1.086	0.286	0.000	0.000
DMU5	1.600	1.175	0.625	0.583	0.500

(2) The distances from NIS for alternative  $i$  and criterion  $j$  ( $d_{ij}^-$ ) were obtained using Eq. (13). For example,  $d_{11}^- = 1+1 = 2$ ,  $d_{12}^- = 0.714 + 1 = 1.174$  and  $d_{13}^- = 0.484 + 0.714 = 1.198$ . The details of the distance from NIS for alternative  $i$  and criterion  $j$  are shown in Table 7.



**Table 7**  
The distance from NIS for alternative *i* and criterion *j* for a performance assessment problem

$d_{ij}^-$	1	2	3	4	5
DMU1	2.000	1.714	1.198	0.817	0.968
DMU2	2.000	2.000	1.714	1.214	1.429
DMU3	0.800	1.400	2.000	1.800	2.000
DMU4	0.400	0.914	1.714	2.000	2.000
DMU5	0.400	0.825	1.375	1.417	1.500

(3) Then the relative closeness index for alternative *i* and criterion *j* ( $rc_{ij}$ ) can be obtained using Equation (14). For example,  $rc_{11} = 2 / (2.000 + 0.000) = 1.0000$ ,  $rc_{12} = 1.7143 / (1.7143 + 0.2857) = 0.8571$  and  $rc_{13} = 1.1982 / (1.1982 + 0.8018) = 0.5991$ . Finally, the combined CEM will be generated as in Table 8.

**Table 8**  
The combined CEM for a performance assessment problem

$RC_{ij}$	1	2	3	4	5
DMU1	<b>1.0000</b>	0.8571	0.5991	0.4086	0.4839
DMU2	1.0000	<b>1.0000</b>	0.8571	0.6071	0.7143
DMU3	0.4000	0.7000	<b>1.0000</b>	0.9000	1.0000
DMU4	0.2000	0.4571	0.8571	<b>1.0000</b>	1.0000
DMU5	0.2000	0.4125	0.6875	0.7083	<b>0.7500</b>
$\sigma_j$	0.40988	0.25262	0.15776	0.23480	0.21751
$\omega_j$	<b>0.16948</b>	<b>0.20037</b>	<b>0.21901</b>	<b>0.20387</b>	<b>0.20387</b>

**Step3:** Calculate the criteria weights in combined CEM using MSDT

In Table 8,  $\sigma_j$  must be calculated first. Then the weights of each criterion in the combined CEM were determined using Eq. (15). For example,  $\omega_1 = (1.2726 - 0.40988) / ((5 - 1)(1.2726)) = 0.16948$ ,  $\omega_2 = (1.2726 - 0.25262) / ((5 - 1)(1.2726)) = 0.20037$  and  $\omega_3 = (1.2726 - 0.15776) / ((5 - 1)(1.2726)) = 0.21901$ . Details are shown in Table 8.

**Step4:** Calculate the final weights of each DMU and rank all DMUs

After obtaining the  $\omega_j$ , the  $w_i$  is obtained by multiplying the weight value by the corresponding combined CEM using Equation (16). For example,  $w_1 = (0.16948)(1.0000) + (0.20037)(0.8571) + (0.21901)(0.5991) + (0.20387)(0.4086) + (0.20387)(0.4839) = 0.6560$ . As a result, all DMUs were ranked as listed in Table 9.

**Table 9**  
The rating and ranking of proposed method for performance assessment problem

$RC_{ij}$	1	2	3	4	5	$w_i$	Rank
DMU1	0.1695	0.1717	0.1312	0.0833	0.1003	0.6560	4
DMU2	0.1695	0.2004	0.1877	0.1238	0.1480	0.8294	1
DMU3	0.0678	0.1403	0.2190	0.1835	0.2073	0.8178	2
DMU4	0.0339	0.0916	0.1877	0.2039	0.2073	0.7244	3
DMU5	0.0339	0.0827	0.1506	0.1444	0.1555	0.5670	5

Next a rating and ranking comparison of the proposed methods is shown in Table 10. Finally, spearman's rank correlation was used for testing the correlation of each method ( $r_s$ ). The details of each  $r_s$  value are shown in Table 11.

**Table 10**  
The rating and ranking comparison of the proposed method for a performance assessment problem

DMU	Benevolent	Rank	Aggressive	Rank	Wang's RC model	Rank	Proposed	Rank
DMU1	0.6793	4	0.6602	4	0.6602	4	0.6697	4
DMU2	0.8857	1	0.7857	1	0.7857	1	0.8357	1
DMU3	0.8800	2	0.7200	2	0.7200	2	0.8000	2
DMU4	0.7257	3	0.6800	3	0.6800	3	0.7029	3
DMU5	0.5900	5	0.5133	5	0.5133	5	0.5517	5

**Table 11**  
Spearman's rank correlation test for performance assessment problem

Correlation test	Benevolent	Aggressive	Wang's RC model	Proposed model
Benevolent	<b>1.000</b>	1.000	1.000	1.000
Aggressive	1.000	<b>1.000</b>	1.000	1.000
Wang's model	1.000	1.000	<b>1.000</b>	1.000
Proposed model	1.000	1.000	1.000	<b>1.000</b>

As seen in Table 10, the rating and ranking comparison of the proposed method for a performance assessment problem was obtained. The proposed method, benevolent model, aggressive model and Wang’s RC model as in Equation (6) (Wang et al., 2011) assess that  $DMU_2 > DMU_3 > DMU_4 > DMU_1 > DMU_5$ . As seen in Table 11, the correlation coefficients for the proposed method and all models are the same value with  $r_s = 1.000, 1.000, 1.000$  respectively.

4.2 Six nursing homes

In Table 12, the six nursing homes, proposed by Sexton et al. (Sexton et al., 1986), have two inputs ( $x_1$  and  $x_2$ ) and two outputs ( $y_1$  and  $y_2$ ). Let  $x_1$  be staff hours per day, including nurses, physicians, etc. Let  $x_2$  be the supplies per day, measured in thousands of dollars,  $y_1$  be the total Medicare-plus-Medicaid reimbursed patient days, and  $y_2$  be the total privately paid patient days.

**Table 12**  
Data set of six nursing homes

DMUs	$x_1$	$x_2$	$y_1$	$y_2$	CCR
DMU1	1.50	0.20	1.40	0.35	<b>1.0000</b>
DMU2	4.00	0.70	1.40	2.10	<b>1.0000</b>
DMU3	3.20	1.20	4.20	1.05	<b>1.0000</b>
DMU4	5.20	2.00	2.80	4.20	<b>1.0000</b>
DMU5	3.50	1.20	1.90	2.50	<b>0.9775</b>
DMU6	3.20	0.70	1.40	1.50	<b>0.8675</b>

**Step1:** Generate the CEIM using the RCB and RCA models

Based on the same calculation steps in Section 4.1, the RCB and RCA models were coded using LINGO software. Based on Eq. (7) to Eq. (10), the results of RCB-CEM and RCA-CEM can be obtained as listed in Table 13 and Table 14 respectively.

**Table 13**  
RCB cross-efficiency matrix (RCB-CEM) of six nursing homes

DMU	1	2	3	4	5	6
DMU1	<b>1.0000</b>	0.5833	1.0000	0.4977	1.0000	1.0000
DMU2	1.0000	<b>1.0000</b>	0.8640	1.0000	1.0000	1.0000
DMU3	0.5000	0.2917	<b>1.0000</b>	0.4129	0.8295	0.8295
DMU4	0.7000	0.7000	1.0000	<b>1.0000</b>	1.0000	1.0000
DMU5	0.7083	0.6944	0.9676	0.9506	<b>0.9775</b>	0.9775
DMU6	0.7551	0.7143	0.8046	0.8027	0.8675	<b>0.8675</b>

**Table 14**  
RCA cross-efficiency matrix (RCA-CEM) of six nursing homes

DMU	1	2	3	4	5	6
DMU1	<b>1.0000</b>	1.0000	0.7111	0.7111	1.0000	1.0000
DMU2	0.3505	<b>1.0000</b>	0.2667	0.6500	1.0000	1.0000
DMU3	1.0000	0.8295	<b>1.0000</b>	1.0000	0.8295	0.8295
DMU4	0.4056	1.0000	0.4103	<b>1.0000</b>	1.0000	1.0000
DMU5	0.4301	0.9775	0.4136	0.9205	<b>0.9775</b>	0.9775
DMU6	0.4099	0.8675	0.3333	0.6482	0.8675	<b>0.8675</b>

After obtaining the RCB-CEM and RCA-CEM, the cross-efficiency intervals matrix (CEIM) of six nursing homes can be generated using Eq. (11) as listed in Table 15.

**Table 15**  
Cross-efficiency intervals matrix (CEIM) of six nursing homes

DMU	1	2	3	4	5	6
DMU1	<b>[1.0000, 1.0000]</b>	[0.5833, 1.0000]	[0.7111, 1.0000]	[0.4977, 0.7111]	[1.0000, 1.0000]	[1.0000, 1.0000]
DMU2	[0.3505, 1.0000]	<b>[1.0000, 1.0000]</b>	[0.2667, 0.8640]	[0.6500, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]
DMU3	[0.5000, 1.0000]	[0.2917, 0.8295]	<b>[1.0000, 1.0000]</b>	[1.0000, 1.0000]	[0.8295, 0.8295]	[0.8295, 0.8295]
DMU4	[0.4056, 0.7000]	[0.7000, 1.0000]	[0.4103, 1.0000]	<b>[1.0000, 1.0000]</b>	[1.0000, 1.0000]	[1.0000, 1.0000]
DMU5	[0.4301, 0.7083]	[0.6944, 0.9775]	[0.4136, 0.9676]	[0.9205, 0.9506]	<b>[0.9775, 0.9775]</b>	[0.9775, 0.9775]
DMU6	[0.4099, 0.7551]	[0.7143, 0.8675]	[0.3333, 0.8046]	[0.6482, 0.8027]	[0.8675, 0.8675]	<b>[0.8675, 0.8675]</b>

**Step 2:** Generate the combined CEM using the relative closeness index

(1) Based on the same calculation steps as in Section 4.1,  $d_{ij}^+$  were obtained using Eq.n (12). The details of each  $d_{ij}^+$  are shown in Table 16.

**Table 16**  
The distance from PIS for alternative  $i$  and criterion  $j$  for six nursing homes

$d_{ij}^+$	1	2	3	4	5	6
DMU1	0.0000	0.4167	0.2889	0.7912	0.0000	0.0000
DMU2	0.6495	0.0000	0.8693	0.3500	0.0000	0.0000
DMU3	0.5000	0.8788	0.0000	0.5871	0.3409	0.3409
DMU4	0.8944	0.3000	0.5897	0.0000	0.0000	0.0000
DMU5	0.8615	0.3281	0.6188	0.1288	0.0450	0.0450
DMU6	0.8350	0.4183	0.8620	0.5491	0.2651	0.2651

(2) Based on the same calculation steps as in Section 4.1,  $d_{ij}^-$  were obtained using Eq. (13). The details of each  $d_{ij}^-$  are shown in Table 17.

**Table 17**  
The distance from NIS for alternative  $i$  and criterion  $j$  for six nursing homes

$d_{ij}^-$	1	2	3	4	5	6
DMU1	2.0000	1.5833	1.7111	1.2088	2.0000	2.0000
DMU2	1.3505	2.0000	1.1307	1.6500	2.0000	2.0000
DMU3	1.5000	1.1212	2.0000	1.4129	1.6591	1.6591
DMU4	1.1056	1.7000	1.4103	2.0000	2.0000	2.0000
DMU5	1.1385	1.6719	1.3812	1.8712	1.9550	1.9550
DMU6	1.1650	1.5817	1.1380	1.4509	1.7349	1.7349

(3) Then  $RC_{ij}$  can be obtained using Equation (14). As a result, the combined CEMs were generated as in Table 18.

**Table 18**  
The combined CEMs for six nursing homes

$RC_{ij}$	1	2	3	4	5	6
DMU1	<b>1.0000</b>	0.7917	0.8556	0.6044	1.0000	1.0000
DMU2	0.6752	<b>1.0000</b>	0.5654	0.8250	1.0000	1.0000
DMU3	0.7500	0.5606	<b>1.0000</b>	0.7064	0.8295	0.8295
DMU4	0.5528	0.8500	0.7051	<b>1.0000</b>	1.0000	1.0000
DMU5	0.5692	0.8360	0.6906	0.9356	<b>0.9775</b>	0.9775
DMU6	0.5825	0.7909	0.5690	0.7254	0.8675	<b>0.8675</b>
$\sigma_j$	0.17022	0.14221	0.16962	0.14938	0.07677	0.07677
$\omega_j$	<b>0.15663</b>	<b>0.16377</b>	<b>0.15678</b>	<b>0.16194</b>	<b>0.18044</b>	<b>0.18044</b>

**Step3:** Generate the criteria weights in combined CEM using MSDT

Based on the same calculation steps in Section 4.1, the weights of each criterion in the combined CEM were determined using Eq. (15). Details are shown in Table 18.

**Step 4:** Calculate the final weights of each DMU and rank all DMUs

After obtaining the  $\omega_j$ , the  $w_i$  is obtained by multiplying the weight value by the corresponding combined CEM using Eq. (16). As a result, all DMUs are ranked as listed in Table 19.

**Table 19**  
The rating and ranking of the proposed method for six nursing homes

DMU weight	1	2	3	4	5	6	$cci$	Rank
DMU1	0.1566	0.1296	0.1341	0.0979	0.1804	0.1804	0.6987	1
DMU2	0.1058	0.1638	0.0886	0.1336	0.1804	0.1804	0.6722	3
DMU3	0.1175	0.0918	0.1568	0.1144	0.1497	0.1497	0.6301	5
DMU4	0.0866	0.1392	0.1106	0.1619	0.1804	0.1804	0.6787	2
DMU5	0.0892	0.1369	0.1083	0.1515	0.1764	0.1764	0.6622	4
DMU6	0.0912	0.1295	0.0892	0.1175	0.1565	0.1565	0.5840	6

The rating and ranking comparison of the proposed method is shown in Table 20. Finally, Spearman's rank correlation was used for testing the correlation of each method ( $r_s$ ). The details of each  $r_s$  value are shown in Table 21.

**Table 20**  
The rating and ranking comparison of the proposed method for six nursing homes

DMU	Benevolent	Rank	Aggressive	Rank	Wang's model	Rank	Proposed model	Rank
DMU1	1.0000	1	0.7639	1	0.7639	1	0.6987	1
DMU2	0.9773	3	0.7004	3	0.7004	3	0.6722	3
DMU3	0.8580	5	0.6428	5	0.6428	5	0.6301	5
DMU4	1.0000	1	0.7184	2	0.7184	2	0.6787	2
DMU5	0.9758	4	0.6956	4	0.6956	4	0.6622	4
DMU6	0.8570	6	0.6081	6	0.6081	6	0.5840	6

**Table 21**  
Spearman’s rank correlation test for six nursing homes

Correlation test	Benevolent	Aggressive	Wang’s model	Proposed model
Benevolent	1.000	0.986	0.986	0.986
Aggressive	0.986	1.000	1.000	1.000
Wang’s model	0.986	1.000	1.000	1.000
Proposed model	0.986	1.000	1.000	1.000

As seen in Table 20, the rating and ranking comparisons of the proposed method for six nursing homes were obtained. The proposed method, aggressive model and Wang’s RC model assess that  $DMU_1 > DMU_4 > DMU_2 > DMU_5 > DMU_3 > DMU_6$ .

As seen in Table 21, the correlation coefficients for the proposed method and the benevolent model, the aggressive model and Wang’s RC model have values of  $r_s = 0.986, 1.000, 1.000, 1.000$  respectively. This is a guarantee that the proposed method is highly reliable.

4.3 Fourteen international passenger airlines

In Table 22, the data set of fourteen international passenger airlines, proposed by Tofallis (Tofallis, 1997b), has three inputs ( $x_1, x_2$  and  $x_3$ ) and two outputs ( $y_1$  and  $y_2$ ). Let  $x_1$  be the aircraft capacity in ton kilometers,  $x_2$  be operating cost and  $x_3$  be non-flight assets such as reservation systems, facilities and current assets. Let  $y_1$  be passenger kilometers and  $y_2$  be non-passenger revenue.

**Table 22**  
Data set of fourteen international passenger airlines

DMUs	$x_1$	$x_2$	$x_3$	$y_1$	$y_2$	CCR
1	5723	3239	2003	26677	697	<b>0.8684</b>
2	5895	4225	4557	3081	539	<b>0.3379</b>
3	24099	9560	6267	124055	1266	<b>0.9475</b>
4	13565	7499	3213	64734	1563	<b>0.9581</b>
5	5183	1880	783	23604	513	<b>1.0000</b>
6	19080	8032	3272	95011	572	<b>0.9766</b>
7	4603	3457	2360	22112	969	<b>1.0000</b>
8	12097	6779	6474	52363	2001	<b>0.8588</b>
9	6587	3341	3581	26504	1297	<b>0.9477</b>
10	5654	1878	1916	19277	972	<b>1.0000</b>
11	12559	8098	3310	41925	3398	<b>1.0000</b>
12	5728	2481	2254	27754	982	<b>1.0000</b>
13	4715	1792	2485	31332	543	<b>1.0000</b>
14	22793	9874	4145	122528	1404	<b>1.0000</b>

Based on the same calculation steps as in Section 4.1 and Section 4.2, the weights of each criterion in the combined CEM are obtained as in Table 23.

**Table 23**  
The combined CEM for fourteen international passenger airlines

DMU\Target DMU	1	2	3	4	5	6	7
DMU1	<b>0.8684</b>	0.4501	0.6225	0.8684	0.5965	0.4726	0.7893
DMU2	0.1719	<b>0.3379</b>	0.0472	0.1719	0.1141	0.0247	0.2625
DMU3	0.8826	0.1942	<b>0.9475</b>	0.8826	0.7206	0.6898	0.6850
DMU4	0.9581	0.4259	0.7034	<b>0.9581</b>	0.7398	0.6973	0.7928
DMU5	0.9653	0.3658	1.0000	0.9653	<b>1.0000</b>	1.0000	0.7357
DMU6	0.8818	0.1108	0.9563	0.8818	0.8404	<b>0.9766</b>	0.6180
DMU7	0.9211	0.7781	0.4773	0.9211	0.5458	0.3382	<b>1.0000</b>
DMU8	0.7813	0.6114	0.5162	0.7813	0.5107	0.2924	0.8436
DMU9	0.7855	0.7278	0.5075	0.7855	0.5415	0.2677	0.8832
DMU10	0.7821	0.6354	0.6520	0.7821	0.6669	0.3564	0.7715
DMU11	1.0000	1.0000	0.4287	1.0000	0.7101	0.4418	1.0000
DMU12	0.9462	0.6336	0.7500	0.9462	0.7042	0.4395	0.9222
DMU13	1.0000	0.4256	1.0000	1.0000	0.7013	0.4555	0.9756
DMU14	1.0000	0.2277	1.0000	1.0000	0.9187	1.0000	0.7357

**Table 23**  
(Continued)

DMU\Target DMU	8	9	10	11	12	13	14	$w_i$	Rank
DMU1	0.7881	0.7031	0.5780	0.4686	0.7261	0.6973	0.6609	0.6636	12
DMU2	0.2724	0.2808	0.2380	0.2202	0.2405	0.1591	0.0991	0.1886	14
DMU3	0.6833	0.6225	0.5313	0.3373	0.7095	0.8128	0.7871	0.6776	11
DMU4	0.7850	0.6991	0.5940	0.5658	0.7535	0.7346	0.8193	0.7305	8
DMU5	0.7359	0.7778	0.7726	0.6829	0.8910	0.8680	1.0000	0.8400	2
DMU6	0.6084	0.5099	0.4312	0.3610	0.6191	0.7770	0.9273	0.6785	10
DMU7	1.0000	0.8395	0.6631	0.6243	0.8048	0.7300	0.6088	0.7323	7
DMU8	<b>0.8588</b>	0.8208	0.6713	0.4913	0.7853	0.6820	0.5313	0.6555	13
DMU9	0.9072	<b>0.9477</b>	0.8086	0.5643	0.8901	0.7033	0.5283	0.7034	9
DMU10	0.7944	1.0000	<b>1.0000</b>	0.5960	1.0000	0.7257	0.5907	0.7395	6
DMU11	1.0000	1.0000	0.8985	<b>1.0000</b>	0.9882	0.7068	0.7209	0.8496	1
DMU12	0.9395	0.9998	0.8906	0.5859	<b>1.0000</b>	0.8413	0.6999	0.8071	4
DMU13	1.0000	1.0000	0.8079	0.4005	1.0000	<b>1.0000</b>	0.7278	0.8210	3
DMU14	0.7275	0.6478	0.5654	0.4869	0.7562	0.8573	<b>1.0000</b>	0.7802	5

After obtaining the combined CEM, DMU<sub>s</sub> ratings ( $w_i$ ) were obtained, and all DMUs were ranked as listed in Table 24. Finally, Spearman’s rank correlation was used for testing the correlation ( $r_s$ ). The details of each  $r_s$  value are shown in Table 25.

**Table 24**  
The rating and ranking comparison for fourteen international passenger airlines

DMU	Benevolent	Rank	Aggressive	Rank	Wang’s RC model	Rank	Proposed method	Rank
DMU1	0.7543	12	0.5990	12	0.6069	13	0.6643	12
DMU2	0.1894	14	0.1652	14	0.1811	14	0.1892	14
DMU3	0.7678	9	0.6226	11	0.6224	11	0.6773	11
DMU4	0.8222	6	0.6734	7	0.6699	8	0.7308	8
DMU5	0.8912	3	0.7983	1	0.7789	1	0.8396	2
DMU6	0.7554	11	0.6385	9	0.6342	10	0.6776	10
DMU7	0.8214	7	0.6478	8	0.6748	7	0.7338	7
DMU8	0.7242	13	0.5855	13	0.6077	12	0.6568	13
DMU9	0.7590	10	0.6309	10	0.6576	9	0.7049	9
DMU10	0.7803	8	0.6813	6	0.6914	6	0.7407	6
DMU11	0.9193	1	0.7742	2	0.7742	2	0.8514	1
DMU12	0.8850	4	0.7314	5	0.7464	4	0.8082	4
DMU13	0.9190	2	0.7503	3	0.7655	3	0.8219	3
DMU14	0.8659	5	0.7316	4	0.7243	5	0.7796	5

**Table 25**  
Spearman’s rank correlation test for fourteen international passenger airlines

Ranking model	Benevolent	Aggressive	Wang’s RC model	Proposed method
Benevolent	1.000	0.952	0.952	0.965
Aggressive	0.952	1.000	0.982	0.982
Wang’s model	0.952	0.982	1.000	0.991
Proposed method	0.965	0.982	0.991	1.000

As seen in Table 24, the proposed method was used to calculate the rating and ranking of all DMUs. The proposed method and the benevolent model agree that DMU<sub>11</sub> is the best DMU, but the aggressive model and Wang’s RC model indicate that DMU<sub>5</sub> is the best DMU. All of the methods agree that DMU<sub>2</sub> is the worst DMU.

As seen in Table 25, the Spearman’s rank correlation coefficients for the proposed method and the benevolent efficiency value, aggressive efficiency and RC efficiency values are calculated as  $r_s = 0.965, 0.982$  and  $0.991$  respectively. Based on the calculation results from the three numerical examples above, this is a guarantee that the proposed method is highly reliable.

As seen in Fig. 2, the correlation coefficient of the proposed method and Wang's RC efficiency for each problem were most closely related, while the correlation coefficient of the proposed method and benevolent were least related. Undoubtedly, the proposed method can contribute to achieving a more reliable result than the method which is based on a stand-alone existing model.

**5. Conclusions**

This research presents a novel aggregation method offered to combine the ideas of benevolent, aggressive and relative closeness (RC) models for ranking all DMUs. In this paper, the proposed method was tested with three numerical examples. We first formulated the new RC models, namely relative closeness benevolent model (RCB model) and relative closeness aggressive model (RCA model), to evaluate the rating of all DMUs. The results of the RCB and RCA models were utilized to

generate a cross-efficiency intervals matrix (CEIM). Secondly, the relative closeness index was used to generate a combined cross-efficiency matrix (combined CEM). In the combined CEM, a set of target DMUs was viewed as criteria and a set of DMUs was viewed as alternatives. Finally, all DMUs were ranked. In a comparative analysis, the proposed method shows potential in ranking DMUs, which differ from other models in the literature. Based on the results of this research, the proposed model can lead to achieving a more reliable result than the other existing methods.

For future research, in order to enhance the validity of the research output further, application of the proposed method should be tested with more cases. Although the inputs and outputs in this case are measured by exact values, we believe that the proposed method can be adapted to deal with fuzzy data.

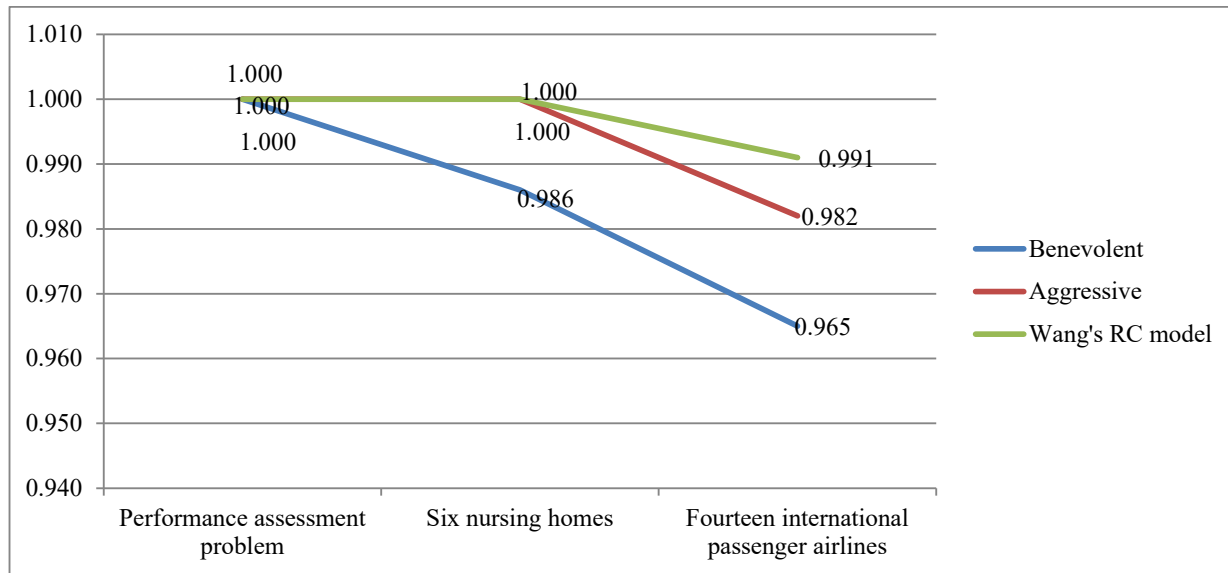


Fig. 2. Correlation coefficients of the proposed method and other methods

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