

## Model of contact interaction in threaded joint equipped with spring-loaded collet

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### ABSTRACT

Loosening of threaded joints, especially those operating under dynamic loading conditions, is a common and traditional problem for machines, mechanisms and structures. This study develops scientific approaches to the frictional lock of bolted joints to prevent unintentional self-unscrewing. An analytical version of the theory of threaded joint, which is equipped with a spring-loaded collet, is developed in the paper. The mixed contact problem of the interaction of the cyclic-symmetric system of the collet nut blades with the inner bolt and the outer spring, which is fitted with tightness, was set up and solved. In order to obtain analytical solutions to the problem, original one-dimensional models of structural elements of the connection were constructed. Based on the solutions of the contact problem, the moment of friction force is calculated from the initial gap between the blades and the thread, from the value of the spring tightness, from the parameters of the rigidity of the blade and the clamping spring. Finally, the value of the maximum friction torque that counteracts the self-unscrewing of the collet nut is evaluated, and some features of the threaded joint with the collet nut design are considered.

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## 1. Introduction

The key advantage of threaded joints over other connection methods is that they can be taken apart and reused. However, such an advantage can be a source of problems for the structural integrity of the machine or structure due to unintentional self-unscrewing (Chen et al., 2017; Zhu et al., 2015; Fort et al., 2019). Such self-loosening of bolted joints is a common problem in machines, mechanisms and structures. This phenomenon can lead to significant losses in the industry due to the need for regular maintenance of connections, as well as cause extraordinary situations. To increase the reliability of threaded joints, especially those operating under dynamic loading conditions, they must be locked to eliminate self-unscrewing. Failure to comply with this instruction may result in emergencies and malfunction of expensive units and machines. This is especially true for robotics (Plooij et al., 2015), aviation technology (Tronci, 2017), oil and gas equipment (Velychkovich, 2005; Ropyak & Ostapovych, 2016; Panevnik & Velychkovich, 2017). The methods of locking threaded joints can be divided into rigid and friction (Plooij et al., 2015; Tronci, 2017). The rigid method of locking the threaded joints uses a rigid connection – a stopper (splints; tab washers; flat washers with a cutout for nuts that are screwed to

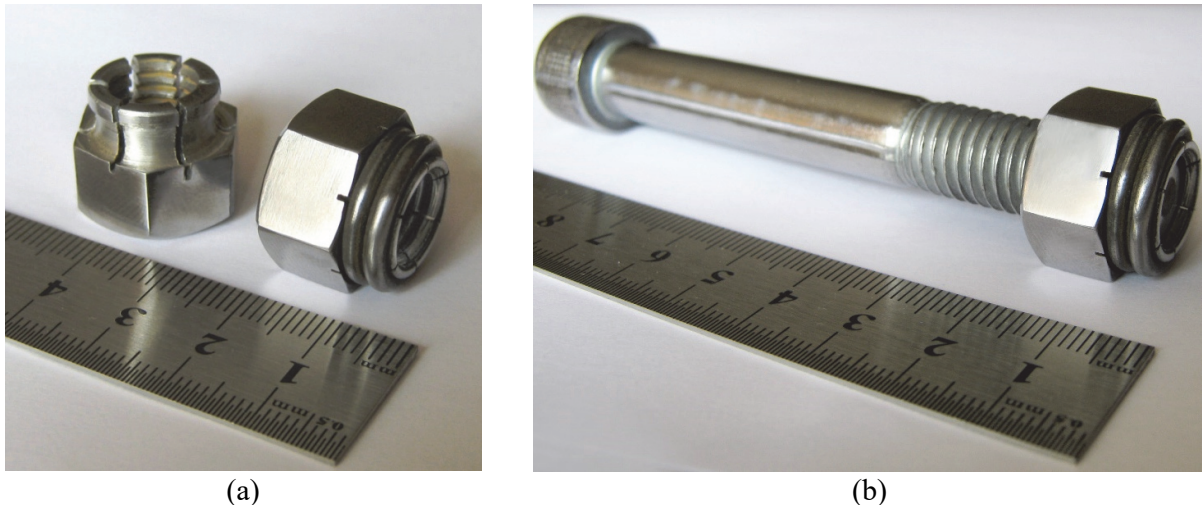
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the body; binding wires, etc.). A variant of this method is dead locking, at which the nut on the bolt is fixed forever (by welding; soldering; riveting; flaring). With the friction method, the locks create increased friction between the threaded connection elements (locking by lock nuts; spring washers; self-locking nuts, etc.). A variant of this method is elastic locking in which a threaded element is attached to the threaded connection, which constantly maintains tightness in the system (use of long bolts; elastic fastening elements; use of springs).

Also known are combination locking methods (Rudawska et al., 2015) that combine the principles of rigid and friction locking, for example, ratchet washers, which create a rigid connection between the nut and the workpiece by cutting the washer teeth into the end face of the nut and the support surface of the workpiece and increase friction in the thread due to elastic deformation of the washer. Also known are nuts with a plug-in ring (usually plastic one) that prevents self-unscrewing of such nuts. However, the use of plastic does not make the connection feasible at high temperatures.

The analysis of the listed locking methods allows to conclude that it is advisable to use friction locking when assembling elements of steel and aluminum alloys for steel threaded joints (bolt – nut). For example, this type of connection in the form of collet nuts is used in the design of the impellers of axial fans of the gas pumping units. A comparative analysis of the self-locking nuts designs indicates that the most secure connection is provided by nuts equipped with elastic blades (Fig. 1).



**Fig. 1.** Collet nut with spring-loaded blades (a) and bolt with collet nut (b)

In the process of tightening such a nut, the bolt thread will enter the compressed area (collet), leading to an increased friction in the threads, the value of which can be adjusted by the choice of the necessary design parameters of the nut and the physical and mechanical characteristics of the materials from which the bolt, nut and spring are made. The more widespread and rational use of collet nuts in engineering is constrained by the lack of well-grounded methods in the literature for calculating the characteristics of such nuts – the friction torque in the turns of the threaded joint, the necessary geometric parameters of the collet mechanism, and the like. Obviously, when calculating a collet nut with spring-loaded blades (Fig. 1a), the engineer first faces the task of contact interaction of the spring, collet blades and bolt.

Separate problems with respect to collet connections in chucks have been considered in (Blinov & Shatilov, 2013; Viral & Chetankumar, 2017). The contact problem of the interaction of a spring, which is press-fitted on a shaft, is considered in article (Van der Heijden et al., 2006). A class of problems concerning the modeling of the contact of the edges of cuts or cracks in cylindrical shells, including the arbitrary combination of tensile and bending loads, is presented in (Shats'kyi, 2005; Shats'kyi & Makoviichuk, 2005; Shats'kyi & Makoviichuk, 2009; Shatskii & Makoviichuk, 2011). The problems of

contact interaction of cut shells with elastic body were considered in (Velichkovich et al., 2018; Shatskyi et al., 2018; Velichkovich, 2007), and of contact of deformable filler with cylindrical shell – in works (Popadyuk et al., 2016; Shopa et al., 1989; Velichkovich et al., 1991). From the point of view of engineering mechanics, a set of blades of a collet nut (Fig. 1) can be considered as a cyclic-symmetrical system of short rods. The experience of modeling cyclic rod structures with respect to vibration isolation techniques is presented in (Bedzir et al., 1995; Shopa et al., 2015), and with respect to the calculation of casing centralizers – in articles (Vytvytskyi et al., 2017; Shatskyi et al., 2019; Shatskyi et al., 2019). The papers (Ropyak et al., 2017; Ropyak et al., 2019) laid the foundations for controlling the performance properties of contact pairs by controlling the friction characteristics of surfaces. However, despite the variety of approaches, it is impossible to directly adapt the known results of contact problems and the results of studies of cyclic-symmetric rod structures to the calculation of a collet nut with spring-loaded blades. It is necessary to develop an engineering model of the contact interaction of the threaded connection elements equipped with a collet nut with spring-loaded blades. We can reach this goal of the paper by building one-dimensional models of structural elements of a connection, formulating a contact problem, and analyzing its solutions.

## 2. Main Content of the Paper

### 2.1. Problem statement and models of structural elements

Let us consider a collet nut (Fig. 2), which consists of a nut proper 1 and a six-blade collet (split cantilevered sleeve) 2. A two-coil spring 3 is put on the collet with tightness. Under the action of contact stresses, the blades of the collet are displaced to the axis of cyclic symmetry, thereby reducing the internal diameter of the collet. When screwing the nut on the bolt 4 in the threaded joint in the collet area, when eliminating the backlash, the contact stresses and the corresponding friction stresses will appear that must be overcome. The task is to evaluate the torque dependence on the tightness and other design parameters of the collet nut. Let us introduce a cylindrical coordinate system  $r\beta z$ .

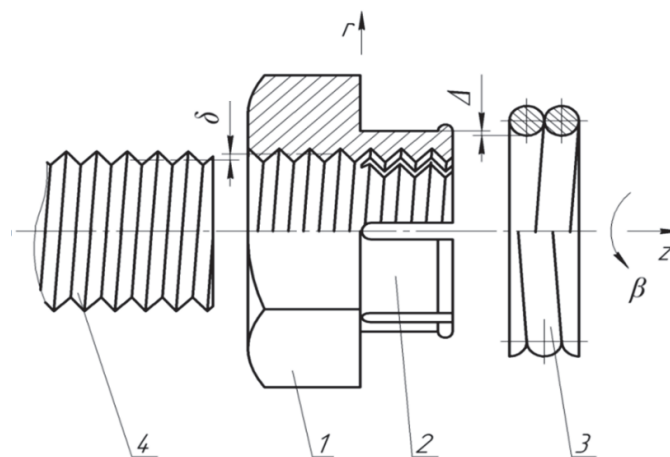


Fig. 2. Schematic diagram of threaded joint with collet nut

Here is the basic idea of modeling. It is intuitively understood that under spring contact with the collet, the spring diameter increases due to the deformation of the bend (decrease in curvature) of its coils in the planes  $z = const$ , and the reduction of the collet diameter is achieved due to the displacement of short blades in the radial planes. All other deformations – the yielding of the thread, the changes in the cross-sections of the coils of the spring, the blades of the collet and the bolt – are neglected in comparison with the aforementioned.

Let  $D_s^\pm(R_s^\pm)$  be the outer and inner diameters (radii) of the spring, and  $d = R_s^+ - R_s^-$  the diameter

of the cross-section of its coil. Let  $D_c^\pm(R_c^\pm)$  be the outer and inner diameters (radii) of the collet, and  $h$  the thickness (netto) of its blade. Because  $d/D_s^\pm \ll 1$ ;  $d/D_c^\pm \ll 1$ , when writing key equations we will not distinguish between outer, inner and middle radii, but will work with a single radius of the conjugation surface  $R = R_c^+$ , which coincides with the radius of the outer surface of the collet in an undeformed state. The special case when this assumption does not work will be conditioned. For similar reasons, the width of the sections is assumed to be zero. A closed momentless cylindrical shell, which is under the influence of the axis-symmetric contact load dependent on the coordinate,  $z$  will be put into conformity with discretely located coils of the spring, which bend under conditions of non-axisymmetric contact load. We choose the equivalent rigidity of this shell such that it on average identifies the mechanical properties of the spring and its continuous model. To do this, let us do an imaginary experiment (Shopa et al., 2015). Let  $E_s$  and  $[\sigma]_s$  are the Young's modulus and admissible stress of spring material. Let us introduce  $E_{eq}$  and  $[\sigma]_{eq}$  – the required Young's modulus and the allowable stress for the equivalent model material. Let us expose the open ring to the influence of distributed radial force  $qd$ , and the shell will be exposed to the internal pressure  $q$ . We calculate the average deflections and compare the results. Let us consider the equilibrium of the open ring:

$$\frac{dN_\beta}{d\beta} + Q_\beta = 0, \quad \frac{dQ_\beta}{d\beta} - N_\beta = -qdR, \quad \frac{dM_\beta}{d\beta} + Q_\beta R = 0, \quad \beta \in (0, 2\pi), \quad (1)$$

where  $N_\beta$  is a tangential force,  $M_\beta$ ,  $Q_\beta$  stand for bending moment and shearing force respectively. Integrating the Eq. (1) under boundary conditions

$$N_\beta = 0, \quad M_\beta = 0, \quad Q_\beta = 0, \quad \beta = 0, 2\pi,$$

which yields  $N_\beta = 2qaR \sin^2(\beta/2)$ ,  $Q_\beta = -qaR \sin \beta$ ,  $M_\beta = -2qaR^2 \sin^2(\beta/2)$ . The highest tensile stress in the spring is achieved at  $\beta = \pi$ :

$$\sigma_{\max}^{(1)} = \max_{\beta \in [0, 2\pi]} \left( \frac{N_\beta}{F_s} - \frac{M_\beta d}{2I_s} \right) = \frac{2qdR}{F_s} + \frac{2qdRd}{2I_s}.$$

Here  $F_s = \pi d^2/4$ ,  $I_s = \pi d^4/64$  are respectively the area and the moment of inertia of the cross-section of the coil of the spring. Finally

$$\sigma_{\max}^{(1)} = q \frac{8R}{\pi d} \left( 1 + 8 \frac{R}{d} \right) \approx q \frac{64}{\pi} \frac{R^2}{d^2} = q \frac{d^2 R^2}{I_s}. \quad (2)$$

The radial deflection of the spring coil is found on the basis of the differential equation:

$$\frac{d^2 w}{d\beta^2} + w = -\frac{R^2}{E_s I} M_\beta, \quad \beta \in (0, 2\pi). \quad (3)$$

Solving Eq. (3), we find:

$$w(\beta) = A \cos \beta + B \sin \beta + \frac{qdR^4}{E_s I_s} \left( 1 - \frac{1}{2} \beta \sin \beta \right).$$

Here, the first two summands represent the displacement as a solid whole. Now, let us calculate the average deflection value

$$w^{(1)} = \frac{1}{2\pi} \int_0^{2\pi} w(\beta) d\beta = q \frac{3}{2} \frac{dR^4}{E_s I_s}. \quad (4)$$

In a momentless shell with thickness  $d$  under internal pressure we will have  $N_\beta = qR$ . For maximum stress and deflection, we have the appropriate expressions

$$\sigma_{\max}^{(2)} = \frac{N_\beta}{d} = q \frac{R}{d}, \quad (5)$$

$$w^{(2)} = \frac{N_\beta R}{E_{eq}} = q \frac{R^2}{E_{eq} d}. \quad (6)$$

In pursuit of the equality of the average deflections in the left-hand sides of Eq. (4) and Eq. (6), we find an equivalent modulus of elasticity of the continuous shell:

$$E_{eq} = \frac{2}{3} \frac{E_s I_s}{d^2 R^2} = \frac{\pi}{96} \frac{d^2}{R^2} E_s = \frac{\pi}{24} \frac{d^2}{D^2} E_s.$$

Based on the strength criterion for maximum tensile stresses, according to the results (2) and Eq. (5) we have

$$q \frac{d^2 R^2}{I_s} = [\sigma]_s, \quad q \frac{R}{d} = [\sigma]_{eq}.$$

It is easy to find an equivalent strength characteristic

$$[\sigma]_{eq} = \frac{I_s}{d^3 R} [\sigma]_s = \frac{\pi}{64} \frac{d}{R} [\sigma]_s = \frac{\pi}{32} \frac{d}{D} [\sigma]_s. \quad (7)$$

In this situation, the shell can still be considered as the elastic basis of Winkler's type (a set of radially located springs)

$$q = cw, \quad (8)$$

with coefficient of subgrade reaction:

$$c = \frac{E_{eq} d}{R^2} = \frac{\pi}{96} \frac{d^3}{R^4} E_s = \frac{\pi}{6} \frac{d^3}{D^4} E_s. \quad (9)$$

Now, let us consider the deformation of the collet blades as a pure shear of short panels in the radial plane  $\beta = const$ . Equilibrium equations in the projection on the axis  $z$  will be as follows:

$$h \frac{d\tau_{rz}}{dz} + \sigma^+ - \sigma^- = 0, \quad z \in (0, l),$$

where  $\tau_{rz}$  is a tangential stress,  $\sigma^\pm$  are contact stresses on the outer and inner surfaces of the sleeve,  $l = 2d$  is the length of the blade. Given Hooke's law

$$\tau_{rz} = G du_r / dz$$

( $G$  – modulus of collet material displacement), we get the equation for radial displacement of the blades

$$Gh \frac{d^2 u_r}{dz^2} + \sigma^+ - \sigma^- = 0, \quad z \in (0, l). \quad (10)$$

## 2.2. Contact problems and analysis of results

First, let's solve the problem press-fit of spring on a collet with some tightness  $\Delta$  in the absence of a bolt ( $\sigma^- = 0, z \in (0, l)$ ). Then we will formulate the contact relations

$$q = -\sigma^+ > 0, R_s^- + w = R_y^+ + u_r, \quad (11)$$

or

$$w = u_r + \Delta, \quad (12)$$

where  $\Delta = R_y^+ - R_s^- > 0$  (the only case where the radii of the surfaces should be distinguished!). Taking into consideration Eq. (8), Eq. (11), Eq. (12), from Eq. (10) we obtain the key equation in displacements:

$$\frac{d^2 u_r}{dz^2} - \lambda^2 (u_r + \Delta) = 0, \quad z \in (0, l), \quad (13)$$

where

$$\lambda = \sqrt{\frac{c}{Gh}} = \frac{d}{D^2} \sqrt{\frac{\pi E_s d}{6 Gh}}.$$

Eq. (13) should be supplemented by conditions for fixing the left edge and the absence of stresses on the right edge:

$$u_r(0) = 0, \quad \frac{du_r}{dz}(l) = 0. \quad (14)$$

The solution of boundary value problem (13), (14) has the form

$$u_r(z) = -\Delta \left( 1 - \frac{\text{ch } \lambda(l-z)}{\text{ch } \lambda l} \right).$$

For contact stress and tangential stress in the panel we obtain

$$\sigma^+(z) = -c\Delta \frac{\text{ch } \lambda(l-z)}{\text{ch } \lambda l}, \quad \sigma^-(z) = 0,$$

$$\tau_{rz}(z) = -G\Delta\lambda \frac{\text{sh } \lambda(l-z)}{\text{sh } \lambda l} = -\frac{c\Delta l}{h} \frac{\text{sh } \lambda(l-z)}{\text{sh } \lambda l}. \quad (15)$$

The highest by module tensile stress and tangential stress in the collet are achieved at  $z = 0$ :

$$\sigma^+(0) = -c\Delta, \quad \tau_{rz}(0) = -c\Delta \frac{l}{h} \frac{\text{th } \lambda l}{\lambda l}.$$

The highest tensile stresses are achieved in the left coil of the spring. Taking advantage of strength conditions

$$|\sigma^+|_{\max} \frac{D}{2d} \leq [\sigma]_{eq}, \quad |\tau_{rz}|_{\max} \leq [\tau],$$

( $[\tau]$  is a safe tangential stress for the collet material) and by relations (7), (9), we obtain inequalities for the starting tightness:

$$\Delta \leq \frac{2d}{D} \frac{[\sigma]_{eq}}{c} = \frac{3}{8} d \frac{D^3}{d^3} \frac{[\sigma]_s}{E_s}, \quad (16)$$

$$\Delta \leq \frac{h}{l} \frac{\lambda}{\text{th } \lambda l} \frac{[\tau]}{c} = \frac{3}{\pi} h \frac{D^4}{d^4} \frac{[\tau]}{E_s}.$$

The analysis showed that the first inequality is stronger, that is, the spring is the weaker link in the design. Now let the collet spring nut be threaded to the bolt. To begin assembling such a structure, some radial play between the threaded nut and the bolt is required. Let us mark it  $\delta$ . The conditions for connecting the spring to the collet Eq. (11), Eq. (12) remain unchanged. Contact conditions on the inner surface of the collet with the bolt will be mixed:

$$\sigma^- = 0, \quad u_r > -\delta, \quad z \in (0, a), \quad (17)$$

$$\sigma^- < 0, \quad u_r = -\delta, \quad z \in (a, l).$$

The demarcation point  $a$  of the free area and the contact area is the required parameter. Contact ratios (11), (12) and (17) lead to a mixed boundary value problem:

$$\frac{d^2 u_r}{dz^2} - \lambda^2 (u_r + \Delta) = 0, \quad z \in (0, a),$$

$$u_r = -\delta, \quad z \in (a, l),$$

$$u_r(0) = 0, \quad u_r(a) = -\delta, \quad \frac{du_r}{dr}(a) = 0.$$

The solution to this problem is as follows

$$u_r(z) = \begin{cases} -\Delta + (\Delta - \delta) \text{ch } \lambda(a-z), & z \in (0, a); \\ -\delta, & z \in (a, l), \end{cases} \quad a = \frac{1}{\lambda} \text{arch} \frac{\Delta}{\Delta - \delta}. \quad (18)$$

For contact and tangential stresses on the basis of the solution (18) we obtain expressions:

$$\sigma^+(z) = \begin{cases} -c(\Delta - \delta) \text{ch } \lambda(a-z), & z \in (0, a); \\ -c(\Delta - \delta), & z \in (a, l), \end{cases}$$

$$\sigma^-(z) = \begin{cases} 0, & z \in (0, a); \\ -c(\Delta - \delta), & z \in (a, l), \end{cases}$$

$$\tau_{rz}(z) = \begin{cases} -c(\Delta - \delta) \frac{l \operatorname{ch} \lambda(a-z)}{h \lambda l}, & z \in (0, a); \\ 0, & z \in (a, l). \end{cases} \quad (19)$$

Based on the result (19), we find the moment of friction forces that occur in a loose connection when rotating the collet relative to the bolt:

$$M = -f\sigma^- \pi D(l-a)D/2 = M_0 \frac{\Delta - \delta}{D} \left( 1 - \frac{1}{\lambda l} \operatorname{arch} \frac{\Delta}{\Delta - \delta} \right), \quad (20)$$

where  $M_0 = f \frac{\pi D^3 l}{2} c$ .

By the Eq. (20) it is possible to estimate the influence of mechanical and structural parameters of a collet nut on the torque value. In particular, this value is proportional to  $M_0$  and depends on three dimensionless complexes:  $\delta/D$ ,  $\Delta/D$  та  $\lambda l$ . For backlash that is small compared to the spring tightness ( $\delta/\Delta \rightarrow 0$ , or  $\Delta/\delta \rightarrow \infty$ ), the contact area occupies almost the entire blade ( $a/l \rightarrow 0$ ). In this case, formula (20) is simplified and an approximation takes place

$$M = M_0 \frac{\Delta}{D} \left( 1 - \frac{1}{\lambda l} \sqrt{2 \frac{\delta}{\Delta}} + O\left(\frac{\delta}{\Delta}\right) \right). \quad (21)$$

In the absence of a backlash on the thread ( $\delta/D = 0$ ) the contact is along the entire length of the blade ( $a/l = 0$ ). Then the formulas (20), (21) give a rough upper estimate for the moment of friction:

$$M_{\max} \approx M_0 \frac{\Delta}{D} = f \frac{\pi^2}{6} \frac{d^4}{D^4} E_s D^2 \Delta \approx 1.64 f E_s \frac{d^4 \Delta}{D^2},$$

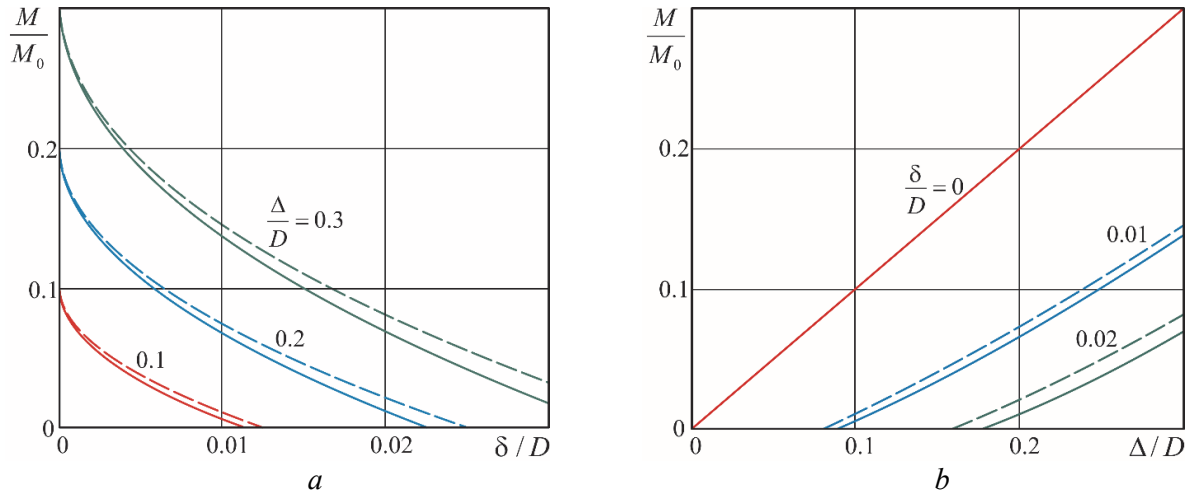
which does not depend on the properties of the collet's blades. Due to the limited strength of the spring, on the basis of inequality (16) we get the limitation on the maximum possible moment of friction, which can be achieved by using a two-turn spring:

$$M_{\max}^* < f \frac{\pi^2}{16} [\sigma]_s D d^2 \approx 0.62 f [\sigma]_s D d^2.$$

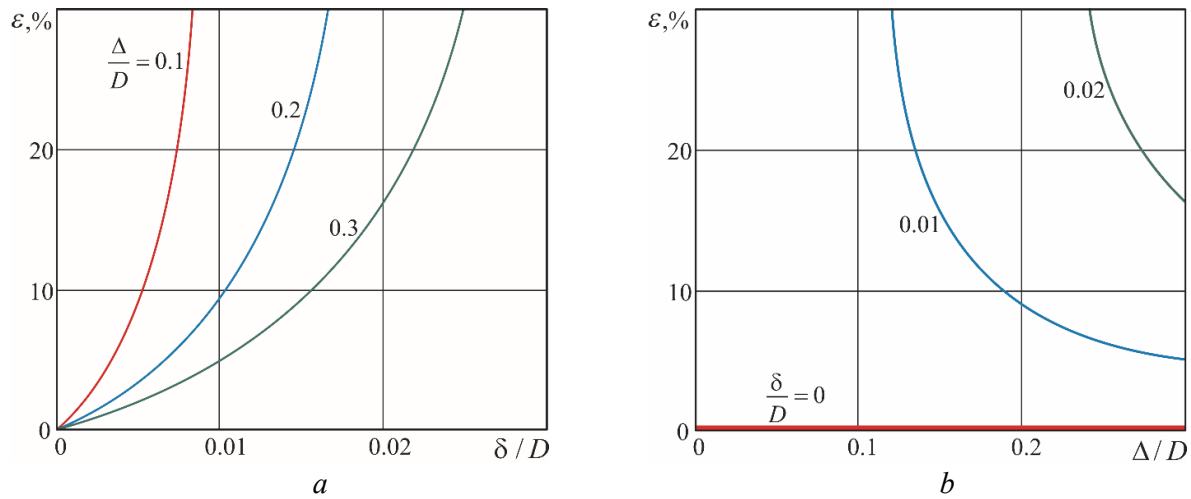
With the fixed value of the dimensionless parameter of stiffness (length) of the blade  $\lambda l = 0.5$ , we investigated the dependences of the moment of friction forces on the backlash on the thread and on the geometric tightness of the spring on the blades (Fig. 3). Solid lines represent the exact result (20), and dashed lines stand for approximations (21).

The visual proximity of the dashed and solid lines in these graphs is sometimes misleading and does not always mean that formula (21) gives a good approximation. In fact, the relative error  $\mathcal{E}$  of the calculations by formula (21) (Fig. 4) is quite large everywhere, where the ratio  $\delta/\Delta$  is not sufficiently small and where the exact torque values  $M$  are close to zero.





**Fig. 3.** Dependences of friction torque on the gap in the thread (a) and spring tightness (b)



**Fig. 4.** Relative errors in the calculation of friction torque by the approximate formula (21)

The following example of calculation gives the sense of reality. Let  $E_s = 2.2 \cdot 10^{11} Pa$ ,  $[\sigma]_s = 10^9 Pa$ ,  $D = 14 \cdot 10^{-3} m$ ,  $d = 2 \cdot 10^{-3} m$ ,  $\Delta = 10^{-3} m$ ,  $f = 0.1$ . Then

$$M_{\max} \approx 1.64 \cdot 0.1 \cdot 2.2 \cdot 10^{11} \cdot \frac{(2 \cdot 10^{-3})^4 \cdot 10^{-3}}{(14 \cdot 10^{-3})^2} \approx 3 N \cdot m.$$

Moreover,

$$M_{\max}^* \approx 0.62 \cdot 0.1 \cdot 10^9 \cdot 14 \cdot 10^{-3} \cdot (2 \cdot 10^{-3})^2 \approx 3.5 N \cdot m,$$

therefore, the spring operates in the elastic range. Finally, it should be noted that when using a multi-turn spring, in the cases mentioned in the paper instead of the length of the blade  $l = 2d$  we should take  $l = nd$ , where  $n$  is the number of turns. The value  $M_{\max}$  will increase  $n/2$  times. However, increasing the length of the collet blades is not a good constructive solution because the stiffness is lost due to twisting the blades.

#### 4. Conclusions

An original version of the theory of threaded joint equipped with a collet nut with spring-loaded blades has been developed. The model contact problem for the interaction of the cyclic-symmetrical system of blades with the inner bolt and the outer spring, which was press-fitted in relation to the blade surface was formulated and solved. The model assumptions made about the contact interaction of the structural elements of the joint led to the mixed boundary value problem of the blades radial displacement. The analytical dependences of the moment of friction force on the initial gap between the blades and the thread, on the magnitude of the spring tightness and on the rigidity parameters of the connection components were obtained. The value of the maximum friction torque, which counteracts the self-unscrewing of the nut in the tight connection when rotating the collet nut relative to the bolt, is estimated. The results obtained were tested on a specific numerical example. Recommendations are given on the design features of the threaded joint with a collet nut. The task of the next stages of research is to develop the theory by using an elastic-plastic model to describe the behavior of the blades of the collet nut. The need for such a model arises when the spring is mounted on a high tightness collet.

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#### References

- Bedzir, A. A., Shatskii, I. P., & Shopa, V. M. (1995). Nonideal contact in a composite shell structure with a deformable filler. *International Applied Mechanics*, 31(5), 351–354.
- Blinov, D. S. & Shatilov, A. A. (2013). Stress-strain state of slotted grips. *Russian Engineering Research*, 33(4), 188–193.
- Chen, Y., Gao, Q., & Guan, Z. (2017). Self-loosening failure analysis of bolt joints under vibration considering the tightening process. *Shock and Vibration*, 2017.
- Fort, V., Bouzid, H. & Gratton, M. (2019). Analytical modeling of self-loosening of bolted joints subjected to transverse loading. *Journal of Pressure Vessel Technology*, 141, 1–11.
- Mevcha Viral B., & Patel Chetankumar M. (2017). Design and analysis of single angle expanding collets. *Journal of Experimental & Applied Mechanics*, 8(2), 1–12.
- Panevnik, D. A., & Velichkovich, A. S. (2017). Assessment of the stressed state of the casing of the above-bit hydroelevator. *Neftyanoe Khozyaystvo – Oil Industry*, 1, 70–73.
- Plooij, M., Mathijssen, G., Cherelle, P., Lefeber, D., & Vanderborght, B. (2015). Lock your robot: A review of locking devices in robotics. *IEEE Robotics & Automation Magazine*, 22 (1), 106–117.
- Popadyuk, I. Yo., Shats'kyi I. P., Shopa V. M., & Velychkovych A. S. (2016). Frictional interaction of a cylindrical shell with deformable filler under nonmonotonic loading. *Journal of Mathematical Sciences*, 215(2), 243–253.
- Ropyak, L., & Ostapovych, V. (2016). Optimization of process parameters of chrome plating for providing quality indicators of reciprocating pumps parts. *Eastern-European Journal of Enterprise Technologies*, 2(5 (80)), 50-62.
- Ropyak, L. Ya., Shatskyi, I. P. & Makoviichuk, M. V. (2017). Influence of the oxide-layer thickness on the ceramic–aluminium coating resistance to indentation. *Metallofizika i Noveishie Tekhnologii*, 39(4), 517–524.
- Ropyak, L. Ya., Shatskyi, I. P. & Makoviichuk, M. V. (2019). Analysis of interaction of thin coating with an abrasive using one-dimensional model. *Metallofizika i Noveishie Tekhnologii*, 41(5), 647–654.
- Rudawska, A., Cisz, S., & Warda, T. (2015). Selected methods for locking screw joints, including the use of adhesives, used in the helicopter construction. *Technological Engineering*, 11(2), 26–31.

- Shats'kyi, I. P. (2005). Closure of a longitudinal crack in a shallow cylindrical shell in bending. *Materials Science*, 41(2), 186–191.
- Shats'kyi, I. P., & Makoviichuk, M. V. (2005). Contact interaction of crack lips in shallow shells in bending with tension. *Materials Science*, 41(4), 486–494.
- Shats'kyi, I. P., & Makoviichuk, M. V. (2009). Analysis of the limiting state of cylindrical shells with cracks with regard for the contact of crack lips. *Strength of Materials*, 41(5), 560–565.
- Shatskii, I. P., & Makoviichuk, N. V. (2011). Effect of closure of collinear cracks on the stress-strain state and the limiting equilibrium of bent shallow shells. *Journal of Applied Mechanics and Technical Physics*, 52(3), 464–470.
- Shatskyi, I., Velychkovych, A., Vytvytskyi, I., & Seniushkovych, M. (2019). Analytical models of contact interaction of casing centralizers with well wall. *Engineering Solid Mechanics*, 7(4), 355–366.
- Shatskyi, I., Vytvytskyi, I., Seniushkovych, M., Velychkovych, A. (2019). Modelling and improvement of the design of hinged centralizer for casing. *IOP Conference Series: Materials Science and Engineering*, 564, 012073.
- Shatskyi, I., Popadyuk, I., & Velychkovych, A. (2018). Hysteretic Properties of Shell Dampers. In: Awrejcewicz J. (eds) *Dynamical Systems in Applications. DSTA 2017. Springer Proceedings in Mathematics & Statistics*, 249. Springer, Cham, pp. 343–350.
- Shopa, V. M., Shatskii, I. P., & Popadyuk, I. I. (1989). Elementary calculation of structural damping in shell springs. *Soviet Engineering Research*, 9(3), 42–44.
- Shopa, V. M., Shatskyi, I. P., Bedzir, O. O., & Velychkovych, A. S. (2015). Contact Interaction of Cut Shells with Deformable Bodies. Ivano-Frankivsk: IFNTUOG (in Ukrainian).
- Tronci, Giuseppe (2017). Frictional Behaviour of Coated Self-locking Aerospace Fasteners. PhD thesis, University of Sheffield.
- Van der Heijden, G. H. M., Peletier, M. A., & Planqué, R. (2006). Self-contact for rods on cylinders. *Archive for Rational Mechanics and Analysis*, 182(3), 471–511.
- Velichkovich, A. S. (2005). Shock absorber for oil-well sucker-rod pumping unit. *Chemical and Petroleum Engineering*, 41(9–10), 544–546.
- Velichkovich, A., Dalyak, T., & Petryk, I. (2018). Slotted shell resilient elements for drilling shock absorbers. *Oil & Gas Science and Technology – Rev. IFP Energies nouvelles*, 73 (34), 1–8.
- Velichkovich, A. S. (2007). Design features of shell springs for drilling dampers. *Chemical and Petroleum Engineering*, 43(7-8), 458–461.
- Velichkovich, S. V., Popadyuk, I. I., Shatskii, I. P. & Shopa, V. M. (1991). Structural hysteresis in a shell-type vibration damper with distributed friction. *Strength of Materials*, 23(3), 279–281.
- Vytvytskyi, I. I., Seniushkovych, M. V., & Shatskyi, I. P. (2017). Calculation of distance between elastic-rigid centralizers of casing. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, 5, 28–35.
- Zhu, L., Hong, J., Yang, G., & Jiang, X. (2015). Experimental study on initial loss of tension in bolted joints. *Journal of Mechanical Engineering Science*, 230(10), 35–54.



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