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Holonomic non-linear modelling for the analysis of heterogeneously resisting structures

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A B S T R A C T

Failure analyses require more and more reliable and robust approaches to structural response prediction. This represents a quite complex task to be accomplished especially when dealing with historical or ancient constructions in seismic geographic areas. The paper is aimed at providing approaches to the analysis of buildings alternative to simplified methods usually adopted in professional practice, which do typically involve drastic simplifications of the fabric. The approach allows developing full holonomic non-linear analyses for forecasting the behavior of structural panels present in the structural system, and it results in higher reliability and flexibility than ordinary procedures that are generally adopted in commercial software and are typically based on gross simplifications of the real structure. The presented method allows taking into account the real geometry of the masonry building under analysis, also including a number of elements such as architraves, tendons or reinforcements possibly incorporated into the walls. Besides the consistent improved reliability in performing structural analyses under quasi-static loads, the main potential of the approach is related to the possibility of developing its extension for performing dynamic analyses quite easily.

1. Introduction

Generally, simplified methods are set up for developing static analyses of structures subjected to monotonically increased horizontal loads. Proposed methods for push-over analyses are typically based on extensions of pseudo spatial mathematical models of the structures into the nonlinear range, where the relationships between the global base shear of the structure and its top displacement are approximate and computed through step-by-step analyses, accounting for simplifications by macro-elements. Actually a higher degree of accuracy in the prediction of the structural behaviour would require more refined approaches, able to account for a number of features, such as the circumstance that usually constructions made of poor tensile resistant materials tend to collapse because of the activation of cracking mechanisms rather than local crisis ought to crushing in compression (Heyman, 1969). This observation would require to make recourse to material modelling able to include fracturing as an intrinsic pattern for the stress-strain relationships, and to adopt, even in more complex analyses, the assumption of No-Tension (NT) materials (see for relevant literature e.g. Heyman, 1969; Kooharian, 1952; Heyman, 1966; Pietruszczak & Ushaksaraei, 2003; Vintzileou, 2014; Baratta & Corbi, 2010;

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2015; Bazant & Li, 1995; Bažant, 1996; Corbi & Corbi, 2017; Furtmüller & Adam, 2011; Khludnev & Kovtunenko, 2000) or low resisting tension materials (Baratta et al., 2016, Corbi & Corbi, 2017).

In this case, solution paths developed through constrained optimisation of some suitably defined energetic functional (see e.g. Baratta & Corbi, 2010; 2015; Bazant & Li, 1995; Bažant, 1996) might be set up not only with reference to the single masonry monads such as panels, arches and vaults, but also for analysing the entire masonry building under static loads. Extensions can be provided as well for the analysis under quasi-static seismic-type actions, when the masonry structure exhibits some overall non-linearity properties, possibly entering a non-linear phase upon loading-unloading cycles.

In this paper an approach is presented where, after decomposing the 3D-structure in its elements, full analyses are performed through the adoption of proper modelling, allowing to treat them as a whole and also including elements such as architraves possibly present on the doorways or openings, which, on the contrary, are not considered in common practice. Moreover some strength in the cross direction of the walls can be accounted for as well by the assumption of elliptical resistance domain. Also collapse mechanism related to the overturning of the wall can be considered, which are absent as well in usual equivalent frame simplification adopted in the professional practice. Finally one should emphasize that the approach presented in the current form represents the conceptual basis for treating structures subject to a wider class of loads, and it can be potentially extended in a quite easy way for performing structural analyses under dynamic loads. This feature is pretty important for the forecast and design of possible control applications for the protection of monumental buildings, which appears especially useful in the absence of environmental forecasts.

2. A premise: Advantages and novelty of the method

As a premise one should emphasize that the mechanical model adopted in the paper for the masonry, under the NT hypothesis (some short summary is reported in the next Sect.3), is a model of the masonry material of macroscopic type, as well known. The main advantage in adopting such a model also lays in the circumstance that it appears a very powerful tool for capturing the essential behavior of the masonry, at the meanwhile based on a very few number of mechanical parameters and without the need of taking into account the bond between the bricks and the paste. This is due to the circumstance that the masonry is considered as a whole, neglecting any resisting tensile stresses, implicitly assuming that no glue exists between elements, which are kept together by contrast between each other. So, only fundamental mechanical phenomena, like impenetrability of bodies and friction contribute to structural strength. This assumption obviously results in an advantage in terms of safety evaluation, and, moreover, minimizes the uncertainties in the model (but also in the identification of the real structure mechanical parameters) related to the decay with time of the quality of mortar, in part responsible of the degradation of the masonry, and, therefore, of the bond capacity. In other words the model is drastically simple, and very clearly placed on the safe side as far as safety assessment is aimed at; moreover it is highly reliable and robust with reference to mechanical parameters that are practically reduced to the elastic moduli of homogenized masonry in compression. The results of the analysis, when transferred to single walls, include stresses, displacements and fractures. Numerical and experimental surveys prove that, despite of its (conceptual) simplicity, the NT model yields valuable results from the engineering point of view.

One should then emphasize that, in the paper, a method much more advanced with respect to the usually adopted approaches is proposed for analyzing the behavior of 3D masonry structures. Whilst in common practice, one usually adopts an approach that assembles together the walls within the 3D model, where the walls are modeled by forcing them to comply with their equivalent frame, in the paper the panels are taken into account by really performing their detailed analysis and getting their real response under a more appropriate behavior assumption, including the geometry, the presence of openings, reinforcements, architraves, beams and so on.

The result is that the obtained 3-D model is much more reliable and sharp (one is able to know the response of the structure at any point) and, at the same time, the entity of the introduced approximations is deeply decreased, since gross simplifications usually adopted are overpassed, and much more credible assumptions about the mechanical model are introduced.

Finally one should also mention that the major computational effort required by the approach is anyway kept limited and completely acceptable also considering the much higher complexity, reliability and richness of the model. In conclusion the impact of the method should not be ignored. Additionally, one may observe that, as well known, push-over is a measure of the lateral load-carrying capacity of a structure in elevation, and the related limit load is a parameter significant for seismic assessment. Of course, it is intended that the assessment includes stress, strain, and fracture, whose admissibility must be compared with the material strength, ductility and toughness. When Push-Over is mentioned, often it is intended that a *curve of push-over* is also required, yielding the proceeding of stress, strain and fractures in the walls with increasing the lateral load. That is exactly what is illustrated in the paper. So, frame-equivalent schemes, FEM approaches (often performed by commercial software) and other are all tools for the numerical estimate of these parameters. The method proposed in the paper is a mixed method, that conjugates NT FEM models with structural assemblage, taking advantage of the fact, already mentioned, that the NT structures are very robust with reference to material properties.

3. The approach to the analysis of 3-D structures

3.1. Material modelling

In the proposed approach, one refers to a full mathematical modelling through the NT material assumption, as mentioned in the above. The formulation of analysis tools for NT bodies basically stands on the possibility, initially pointed out by Kooharian (1952) and Heyman (1966), of performing some extension of the plastic theory to masonry gravity structures such as arches and panels, once made certain assumptions. The underlying premise is that masonry structures can conceptually be considered as possessing an overall ‘ductile’ capacity; this standpoint is largely supported by the experimental evidence (Pietruszczak & Ushaksaraei 2003; Vintzileou 2014). Some clear advantages can be immediately observed, like, as an example, that in the extension of plastic limit analysis methods, the structure can be considered solely in relation to its ultimate state, simplifying matters considerably.

This also implies that few material parameters and no prior knowledge of the initial stress state are required, as emphasized in the previous Sect.2. Therefore, structural analysis under ordinary loading conditions as well as for collapse load evaluation can be performed by means of suitable extensions to NT structures of theoretical approaches for elastic-plastic structures.

The results that can be obtained by performing a numerical investigation on NT structures are able to capture the essential behaviour of the analogous masonry structure, keeping in mind that in general, for structural assessment in particular of a masonry building, it is necessary to evaluate both the behaviour under loading patterns corresponding to the operation of the building and the safety margins with respect to ultimate limit states like structural collapse.

Analysis of NT bodies proves that the stress, strain and displacement fields obey extremum principles of the basic energy functional. Therefore the behaviour of NT solids under ordinary loading conditions can be investigated by means of some extensions of energy approaches to the case at hand (Baratta & Corbi, 2010, 2015; Bazant & Li, 1995). The solution of the related problems can be numerically pursued by means of Operational Research methods (see e.g. Rockafellar, 1970; Fletcher, 2013) suitably setting up discrete models of the analysed continua. Actually analytical solutions for the problem of equilibrium of bodies exhibiting a limited tensile and compressive resistance have been widely investigated, finally producing a limited number of solutions for a restricted class of problems, mainly exploited for validating the proposed techniques and for numerically approaching the analysis of complex structures.

3.2. Structural FEM modelling and energy approach

The approach to the analysis of 3D structures is set up by coupling structural assemblage and FEM analyses of structural components modelled under the NT hypothesis and developed through an energetic formulation of the solution pattern. Un-assemblage of the structure is firstly performed in macro-elements, whence FEM modelling through adoption of meshes follows. At any point, displacements and fractures are assumed as independent variables; the solution displacement and fracture strain fields, $\mathbf{u}_o$ and ε_{fo} respectively are found as the constrained minimum point of energy functional depending on those variables and the resistance of material at any point is ensured. To this purpose some constraints are introduced on the sign of strains, which cannot admit any contraction because of the material assumption.

Let then consider to decompose the spatial structure in a number of panels loaded by in-plane forces, and let introduce the FE-discretization of the single panel as shown in Fig. 1, through a number M of plane adjacent elements, jointed to each other at a number N of defined points, the nodes, which coincide with the edges of the considered element. Let introduce the quantities governing the problem, presented in vector form, in details, the overall nodal load vector $\mathbf{q}$, the nodal fracture (or mechanism) displacement $\mathbf{u}_f$ directly compatible with the fracture strain $\boldsymbol{\varepsilon}_f$ and the stress $\boldsymbol{\sigma}$. All of these quantities relevant to the original structure, i.e. loads $\mathbf{q}$, displacements $\mathbf{u}_f$, strains $\boldsymbol{\varepsilon}_f$ and stresses $\boldsymbol{\sigma}$, can be built up by suitably collecting the analogous quantities $\mathbf{q}_e$, $\mathbf{u}_{fe}$, $\boldsymbol{\varepsilon}_{fe}$ and $\boldsymbol{\sigma}_e$ relevant to the single elements *e*. Under the constant stress/strain assumption in the single element of the mesh, one assumes that both the strain vector $\boldsymbol{\varepsilon}_{fe}$ and stress vector $\boldsymbol{\sigma}_e$ are kept constant in the generic element. One can, thus, write with reference to the overall structure, with reference to nodes

$$\mathbf{q}_{[2N\times 1]} = \begin{bmatrix} q_1 \\ \vdots \\ q_{2N} \end{bmatrix}, \quad \mathbf{u}_{f\,[2N\times 1]} = \begin{bmatrix} u_{f\,1} \\ \vdots \\ u_{f\,2N} \end{bmatrix} \tag{1}$$

and to elements of the mesh

$$\boldsymbol{\varepsilon}_{f\,[3M\times 1]} = \begin{bmatrix} \varepsilon_{f\,1} \\ \vdots \\ \varepsilon_{f\,3M} \end{bmatrix}, \quad \boldsymbol{\sigma}_{[3\times 1]} = \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_{3M} \end{bmatrix} \tag{2}$$

As regards to the single *e*-th element of the mesh, quantities identifying the *e*-th vector components in Eqs. (1) are

$$\mathbf{q}_{e[2N_e\times 1]} = \begin{bmatrix} q_1^{(e)} \\ \vdots \\ q_{2N_e}^{(e)} \end{bmatrix}, \quad \mathbf{u}_{f\,e[2N_e\times 1]} = \begin{bmatrix} u_{f\,1}^{(e)} \\ \vdots \\ u_{f\,2N_e}^{(e)} \end{bmatrix} \tag{3}$$

where N_e is the number of nodes characterizing the element *e*; $u_{f\,r}^{(e)}, q_r^{(e)}$, with r =1…2$N_e$, denote respectively the in-plane components (along the two plane co-ordinate directions x,y) of the element displacement and load vectors;

$$\boldsymbol{\varepsilon}_{f\,\mathrm{e}[3\times1]}=\begin{bmatrix}\varepsilon_{f1}^{(\mathrm{e})}\\ \varepsilon_{f2}^{(\mathrm{e})}\\ \varepsilon_{f3}^{(\mathrm{e})}\end{bmatrix}=\begin{bmatrix}\varepsilon_{f\,x\mathrm{e}}\\ \varepsilon_{f\,y\mathrm{e}}\\ \gamma_{f\,\mathrm{e}}\end{bmatrix},\quad \boldsymbol{\sigma}_{\mathrm{e}[3\times1]}=\begin{bmatrix}\sigma_{1}^{(\mathrm{e})}\\ \sigma_{2}^{(\mathrm{e})}\\ \sigma_{3}^{(\mathrm{e})}\end{bmatrix}=\begin{bmatrix}\sigma_{x\mathrm{e}}\\ \sigma_{y\mathrm{e}}\\ \tau_{\mathrm{e}}\end{bmatrix}, \tag{4}$$

where $\varepsilon_{f\ell}^{(e)},\sigma_{\ell}^{(e)}$ with ℓ=1…3 represent respectively the in-plane strain and stress element components in the co-ordinate reference axes, i.e. the normal ε_{fke}, σ_{ke} (k=x,y) and tangential components τ_{fe}, γ_{fe} of stress and strain.
One should notice that the components $\varepsilon_{f\ell}^{(e)},\sigma_{\ell}^{(e)}$ of the element strain and stress vectors correspond to the components $\varepsilon_{ft},\sigma_{t}$ of the overall vectors, according to the relation between indexes t=3(e-1)+ ℓ.
Under these assumptions, one can write general equalities and inequalities governing the whole problem, i.e. all of those equations that should be satisfied by the solution. For the FE-model, one should consider compatibility equations between elements' fracture strains and nodal displacements on one side, and equilibrium conditions between elements' stresses and nodal loads on the other side, in the form

$$\mathbf{f}_{\varepsilon}\left(\mathbf{u}_{f},\boldsymbol{\varepsilon}_{f}\right)=\mathbf{B}\,\mathbf{u}_{f}-\boldsymbol{\varepsilon}_{f}=\mathbf{0}, \tag{5}$$

$$\mathbf{f}_{\sigma}\left(\boldsymbol{\sigma}\right)=-\mathbf{q}+\mathbf{A}\boldsymbol{\sigma}=\mathbf{0}, \tag{6}$$

where **B** is the [3M×2N] compatibility matrix with elements b_{ij}, and **A** is the [2N×3M] equilibrium matrix with elements a_{ij} , and relationships between each other being ruled by the conditions $b_{ij}=a_{ji}$.
The material, compatibility requires that the additional fracture field does not admit contraction in any point and along any direction, which implies that the fracture strain field $\boldsymbol{\varepsilon}_f$ is positive semi-definite and, therefore, with reference to the finite element (FE) model, that the element strain components satisfy the inequalities (with J_{1f}^{e}, J_{2f}^{e} the first and second invariant of the fracture strain tensor)

$$\mathbf{h}_{\varepsilon}\left(\boldsymbol{\varepsilon}_{\mathrm{f}}\right)=\begin{cases}J_{1\mathrm{f}}^{e}\geq 0\\ J_{2\mathrm{f}}^{e}\geq 0\end{cases},\quad e=1...\mathrm{M} \tag{7}$$

where synthetically expressed by the set of inequalities $\mathbf{h}_{\varepsilon}\left(\boldsymbol{\varepsilon}_{\mathrm{f}}\right)\geq\mathbf{0}$.
The solution [$\mathbf{u}_o(\mathbf{x})$, $\boldsymbol{\varepsilon}_{fo}(\mathbf{x})$] under applied loads may be searched for as the minimum of an energetic functional depending on displacement and fracture strains, thus attaining

$$\mathcal{E}\left[\mathbf{u}_{\mathrm{o}}(\mathbf{x}),\boldsymbol{\varepsilon}_{\mathrm{fo}}(\mathbf{x})\right]=\min_{\substack{\mathbf{u}(\mathbf{x})\in\mathbf{U}\\ \boldsymbol{\varepsilon}_{\mathrm{f}}(\mathbf{x})\in\mathbf{F}}}\mathcal{E}\left[\mathbf{u}(\mathbf{x}),\boldsymbol{\varepsilon}_{\mathrm{f}}(\mathbf{x})\right]=\mathcal{E}_{\mathrm{o}}, \tag{8}$$

where $\mathbf{U}$ denotes the set of admissible displacements (i.e. such that the displacement functions are continuous, derivable and compatible with the external constraints on the solid) and $\mathbf{F}$ the variety of admissible fracture fields (i.e. positive semi-defined tensor fields $\boldsymbol{\varepsilon}_f(\mathbf{x})$, such that $\boldsymbol{\varepsilon}_f(\mathbf{x})$ belongs to the set Φ_f in every point of the body).
Finally the problem is set in the form

$$\begin{cases}\text{Search}\quad \mathcal{E}\left[\mathbf{u}_{\mathrm{o}}(\mathbf{x}),\boldsymbol{\varepsilon}_{\mathrm{fo}}(\mathbf{x})\right]=\min\limits_{\substack{\mathbf{u}(\mathbf{x})\in\mathbf{U}\\ \boldsymbol{\varepsilon}_{\mathrm{f}}(\mathbf{x})\in\mathbf{F}}}\mathcal{E}\left[\mathbf{u}(\mathbf{x}),\boldsymbol{\varepsilon}_{\mathrm{f}}(\mathbf{x})\right]=\mathcal{E}_{\mathrm{o}}\\ \text{Sub}\quad \mathbf{h}_{\varepsilon}\left[\boldsymbol{\varepsilon}_{\mathrm{f}}(\mathbf{x})\right]\geq\mathbf{0}\end{cases} \tag{9}$$

The solution of the non-linear problem in Eq. (9) can be numerically pursued through implementation of Operational Research methods (see e.g. Rockafellar, 1970). As an example, some computational results are reported referred to the panel in Fig. 1, which is assumed to be subject to vertical self-weight forces and horizontal seismic loads. The wall is characterized by geometric dimensions of 5.60×8.40 m with a thickness of 0.5 m; the storey fasciae are assumed to be connected by stringcourses and the openings to be reinforced by architraves. The range of variation of the seismic coefficient is assumed between 0.0 and 0.6, with the application of distribution coefficients on the height according to technical standards.

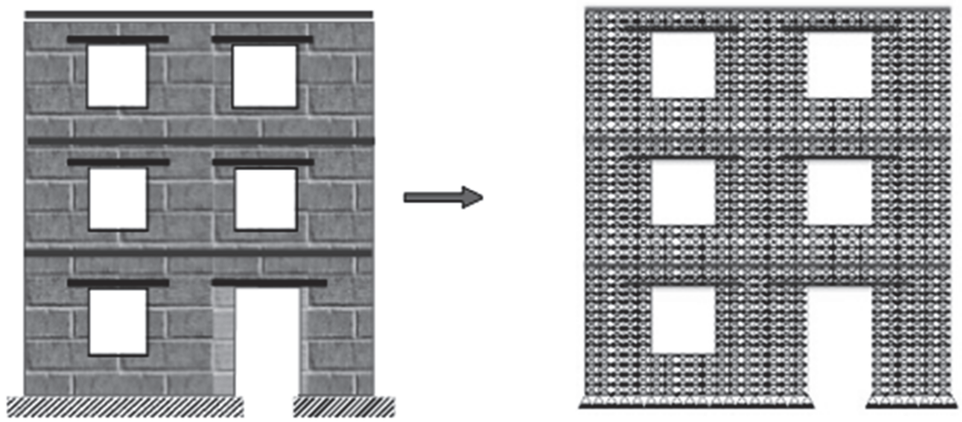


Fig. 1. FE-wall model

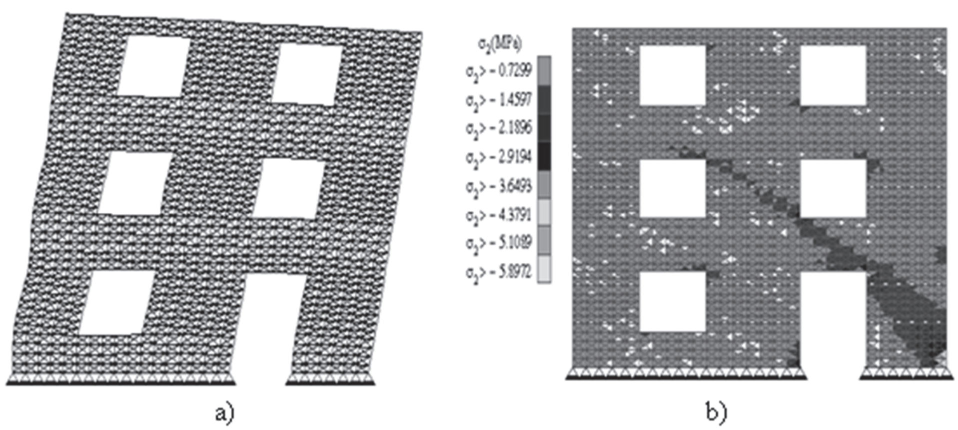


Fig. 2. (a) FEM deformed configuration; (b) distribution of stresses

The deformed configuration of the panel relevant to a seismic coefficient of 0.3 is depicted in Fig. 2a, where an amplification factor of 100 has been adopted in order to emphasize the induced displacements, with the relevant compressive stress distribution in the panel plotted in Fig. 2b.

An overall behaviour of panels of the type illustrated in Fig. 3 may be observed after performing numerical simulation on walls with a variety of openings and geometric characteristics. The overall non-linearity of the response of the panels may be approximated by the bi-linear dashed diagram in the figure, allowing to set up more simply the static analysis of complete 3D models through the adoption of the introduced set of holonomic non-linear equations. The full implementation of the problem is performed with reference to the 3D model in the following section where numerical investigation results are reported.

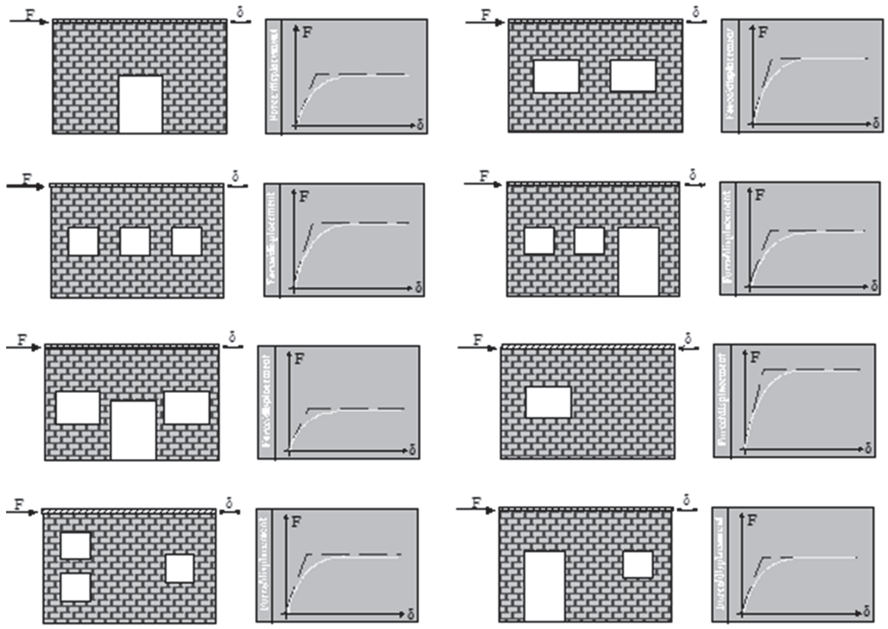
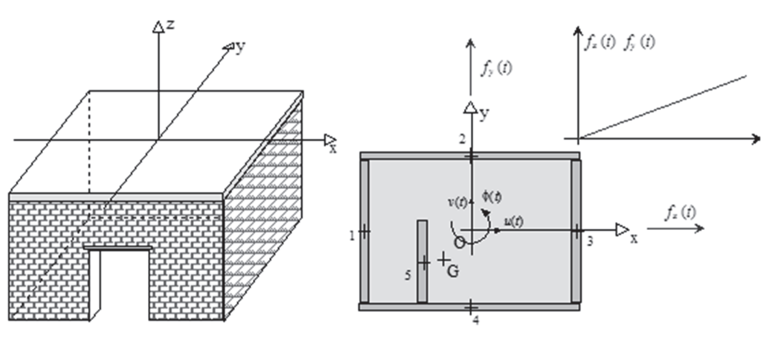


Fig. 3. Force-displacement F-δ diagrams under horizontal action

4. Numerical investigation

4.1. The reference model

In the following one refers to the structural model represented in Fig. 4a and b. It consists of a 3D 1-storey masonry structure with a rigid floor slab, four perimeter walls and a further internal wall. As concerns the loading condition, a monotonically increasing static load, expressed by two vector components along the two coordinate axes, is assumed to act on the considered 1-story frame for performing the push-over analysis. In the considered model, the Lagrangian coordinates are the two components of floor rigid translation, $u(t)$ and $v(t)$, that can be identified in the displacement components of the reference frame origin point G, and the xy-axes rotation $\phi(t)$. Treating the structure according to Sect.3 under the selected material assumption for masonry, the non-linear character of behavior of walls is accounted for, denoting by T_{xi}, T_{yi} the components of the shear absorbed by the i-th wall, by T_{oxi}, T_{oyi} its limit shears and by k_{xi}, k_{yi} its stiffnesses. The limit domain of each masonry wall is assumed to have an elliptical shape, with the principal axes parallel to the edges of the panel cross-sections in plant. The analysis of the NT 3-degrees of freedom structure is performed through implementation of the problem in an ad-hoc set up calculus code, and the numerical investigation is developed.



a) b)

Fig. 4. (a) Spatial structure; (b) Plant with the applied monotonically increasing bi-variate load

Table 1. Geometric and mechanical characteristics of the walls of the 3D model

Wall "i"	x_i (cm)	y_i (cm)	λ_i (cm)	s_i (cm)	T_{oxi} (kg)	T_{oyi} (kg)	k_{xi} (kg·cm^{-1})	k_{yi} (kg·cm^{-1})
1	-500	0	500	40	860	10833	945	57000
2	0	250	1000	40	21666	1720	114000	1889
3	500	0	500	40	860	10833	945	57000
4	0	-250	1000	40	21666	1720	114000	1889
5	-250	-125	250	40	430	5416	472	28500

4.2. Numerical results

A forcing action inclined of 45° with respect to the reference system is considered, composed by two static loads $f_x(t)$, $f_y(t)$ monotonically increasing with the parameter t with final value T=12 and steps of Δt=0.02, and the push-over analysis is executed; the final value attained by each force component is equal to 48917 kg. As regards to the 3D model, the floor is assumed to transmit a vertical uniformly distributed load to the walls, of approximately 500 kg·m^{-2}. The geometric and mechanical characteristics of the walls as numbered in Fig. 4b are reported respectively in Table 1, where, with reference to the i-th wall, x_i , y_i, denote the position of its barycenter with respect to the reference system, λ_i its longitudinal dimension, s_i its thickness, T_{oxi}, T_{oyi} denote its limit shears and k_{xi}, k_{yi} its stiffness.

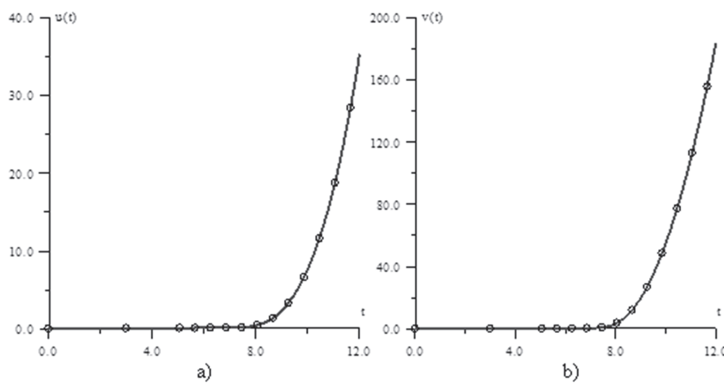


Fig. 5. Displacement (cm) vs force parameter t: (a) u-component and (b) v-component along the coordinate axes.

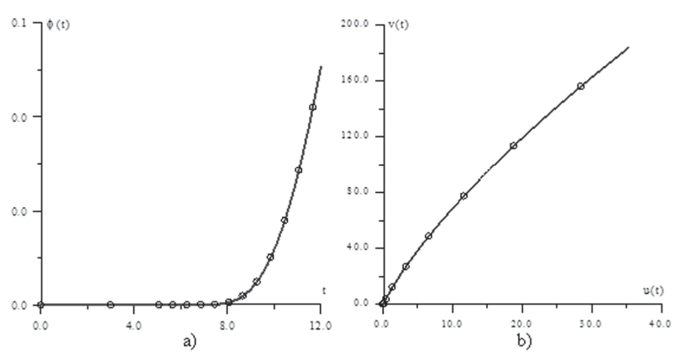


Fig. 6. (a) Rotation (°) vs force parameter t; (b) Phase plots of translational components under increasing force parameter t.

In Fig. 5 and Fig. 6a the diagrams are depicted representing the increasing in the generalized displacement components of the frame barycentre related to the monotonic variation of the load parameter up to the collapse condition of the structure. Fig. 6b reports the u-v phase diagram related to the followed load path. In Fig. 7 the forcing actions adopted for the simulations vs the relevant axis displacement components are depicted, allowing to follow the response path of the structure finally achieving the collapse condition and to check the limit load.

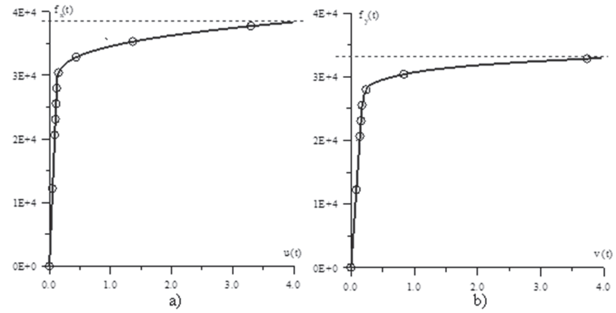


Fig. 7. Load components (kg) vs displacement (cm) components:
(a) x-components; (b) y-components

Finally Figs. 8-10 report the components of the shear absorbed by the single walls, as numbered in Fig. 4. The representation in the phase plane of the shears in the single panel composing the 3D structure allows following the loading path and the consequent increasing in the solicitation level of the panels by direct reference to their elliptical resistance domains. The exceeding of the resistance threshold of the domains can be observed related to the achievement of the collapse condition.

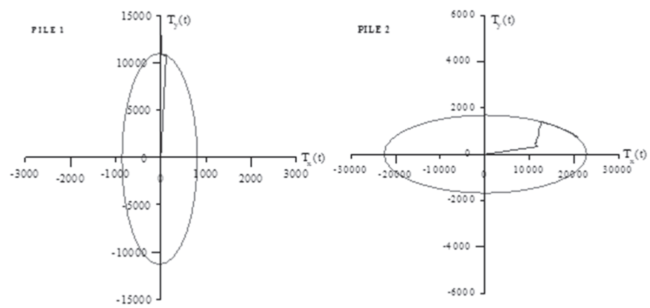


Fig. 8. Shear in the phase plane: the 1st and 2nd piles

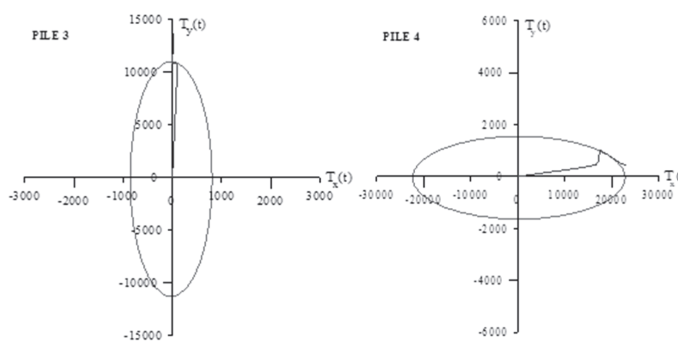


Fig. 9. Shear in the phase plane: the 3rd and 4th piles

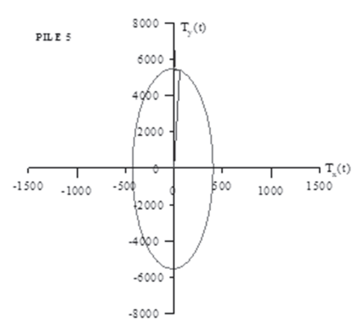


Fig. 10. Shear in the phase plane: the 5th pile

5. Conclusion

In the paper an approach is presented to full push-over analysis of spatial masonry constructions. The proposed formulation is based on a mixed method that conjugates NT FEM models with structural assemblage, in order to implement the overall holonomic non-linear relations ruling the behaviour of the structure. One should emphasize that part of the interest of the presented approach lays in its flexibility that allows figuring out some easy extension of the method to dynamic solicitations. The performance of the presented approach overcomes the usual POR analyses, showing great potentials in current applications. The main fundamental result lays in the capability of obtaining a more reliable as well as more flexible model, thus allowing to have at one's disposal a powerful tool to study a much wider variety of cases, also potentially embedding (in the model with a certain degree of simplicity) different types of reinforcements. A number of advantages directly derive from the adoption of the approach presented in the paper, such as: a more complete representation of the single wall behaviour is allowed, also including possible overturning; no need emerges of selecting single parts of the masonry panel between its openings, but it is treated as a whole; different members like architraves, tendons and reinforcements may be included; extension for dynamic analysis may be performed.

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