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The effect of fractional derivative on photo-thermoelastic interaction in an infinite semiconducting medium with a cylindrical hole

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A B S T R A C T

In the present paper, the theory of generalized photo-thermoelasticity under fractional order derivative was used to study the coupled of thermal, plasma, and elastic waves on unbounded semiconductor medium with a cylindrical hole during the photo-thermoelastic process. The bounding surface of the cavity was traction free and loaded thermally by exponentially decaying pulse boundary heat flux. The medium was considered to be a semiconductor medium homogeneous, and isotropic. In addition, the elastic and thermal properties were considered without neglecting the coupling between the waves due to thermal, plasma and elastic conditions. Laplace transform techniques were used to obtain the exact solution of the problem in the transformed domain by the eigenvalue approach and the inversion of Laplace transforms were carried out numerically. The results were displayed graphically to estimate the effect of the thermal relaxation time and the fractional order parameters on the plasma, thermal and elastic waves.

Nomenclature

ρ	the medium density	τ_o	the thermal relaxation time
n_o	the equilibrium carrier concentration	T_o	the reference temperature
u_i	the displacement components	λ, μ	the Lame's constants
d_n	the electronic deformation coefficient	σ_{ij}	the stress components,
α_t	the coefficient of linear thermal expansion	K	the thermal conductivity
D_e	the carrier diffusion coefficient	E	the excitation energy
E_g	the semiconducting energy gap	τ	the photogenerated carrier lifetime
δ	the coupling parameter of thermal activation	σ_{ij}	the stress components
c_e	the specific heat at a constant strain	t	the time
$\boldsymbol{r}$	the position vector.		

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1. Introduction

During the last twenty-five years, great efforts have been carried out to investigate the structure of microelectronic and semiconductors through the technology of Photoacoustic (PA) and photothermal (PT). Both the PA and PT technology are considered as insignia modes which are highly sensitive to photoexcited carrier dynamics (Mandelis, 1987; Almond & Patel 1996). The absorption Laser beam with modulated intensity leads to the generation photo carriers namely electron-hole pairs. The carrier-diffusion wave or plasma wave plays a dominant role in the experiments of PA and PT for most semiconductors (Mandelis & Hess, 2000). Both the thermal and elastic waves produced as a contribution of the plasma waves depth-dependence that generates the periodic heat and mechanically vacillations. Thermoelastic (TE) mechanization of the elastic wave generation can be interpreted as a result of the propagation of elastic vacillations towards the material surface due to the thermal waves in that material. This mechanism (TE) depends on the generated heat in the material which may generate an elastic wave due to thermal expansion and bend that, in turn, produces a quantity of heat corresponding also to thermoelastic coupling. The electronic distortion (ED) was defined as a periodic elastic deformation in the material due to photoexcited carriers.

Many existing models of physical processes have been modified successfully by using the fractional calculus. We can say that the whole of integral theories and fractional derivatives was created in the last half of the last century. Various approaches and definitions of fractional derivatives have become the main object of numerous studies. Fractional order of weak, normal and strong heat conductivity under generalized thermoelastic theory was established by Youssef (Youssef, 2010; Youssef & Al-Lehaibi, 2010) who developed the corresponding variational theorem. The theory was then used to solve the problem of thermal shock in two dimensions using Laplace and Fourier transforms (Youssef, 2012). Based on a Taylor expansion of the order of time-fraction, a new model of fractional heat equation was established by Ezzatt and Karamany (Ezzat, 2011; Ezzat & El-Karamany 2011a,b). Also, Sherief et al. (2010) used the form of the law of heat conduction to depict a new model. Due to a thermal source, the effect of fractional order parameter on a deformation in a thermoelastic plane was studied by Kumar et al. (2013). Sherief and Abd El-Latief (2013) investigated the effect of the fractional order parameter and the variable thermal conductivity on a thermoelastic half-space. In the Laplace domain, the approach of eigenvalue gives an exact solution without any restrictions on the actual physical quantity assumption. Recently, Abbas (2014a,b, 2015a,b) investigated the fractional order effects on thermoelastic problems by using eigenvalues approach.

Understanding of transport phenomena is solid through the development spatially resolved in situ probes has recently received a great attention. In the present work the measuring of transport processes based on the principle of optical beam deflection through a photo-thermal approach is carried out. It can be considered as an expansion of the photo-thermal deflection technique. Such a technique is characterized by the fact that it is contactless and directly yields the parameters of the electronic and thermal transport at the semiconductor surface or at the interface and within the inner bulk of a semiconductor. Pure silicon is intrinsic semiconducting and is used in wide range of semiconducting industry, for example, the monocrystalline Si is used to produce silicon wafers. In general, the conduction in semiconductor (pure Si) is not the same experienced in metals. Both the electrons and holes are responsible of the conduction value in semiconductors as well as the electrons that may be released from atoms due to the heating of the material. Therefore electric resistance for semiconductor decreases with increasing values of the temperature. The structures of the thermal, elastic and plasma fields in one dimension was analyzed experimentally and theoretically by some researchers (Todorović, 2003a,b; Song et al., 2008). The effects of thermoelastic and electronic deformations in semiconductors without considering the coupled system of the equations of thermal, elastic and plasma have been studied in the past (McDonald and Wetsel 1978, Jackson and Amer 1980, Stearns and Kino 1985). Opsal and Rosencwaig (1985) introduced their research on semiconducting material based on the results shown by Rosencwaig et al. (1983). Abbas (2016) studied a dual phase lag model on photothermal interaction in an unbounded semiconductor medium with a cylindrical cavity. Hobiny and Abbas (2017) investigated the photothermal waves in an infinite semiconducting medium with a cylindrical cavity.

The present paper is an attempt to get a new picture of photothermoelastic theory with one relaxation time using the fractional calculus theory. Based on the fractional order theory, the photo-thermo-elastic interaction in an infinite semiconducting material containing a cylindrical hole is investigated herein. By using the eigenvalue approach and Laplace transform, the governing non-homogeneous equations are processed using a proper analytical-numerical technique. From the obtained results, the physical interpretation of the physical parameters involved in the problem is provided in this study. The numerical solutions are carried out by considering a silicon-like semiconducting medium and the results are verified numerically and are shown graphically in detail.

2. Basic equations

The theoretical analysis of the transport processes in a semiconductor material involves in the study coupled elastic, thermal and plasma waves simultaneously. A homogeneous semiconducting material is considered in the present work. The main physical quantities involved in the problem are the distribution of the temperature $T(\boldsymbol{r},t)$, the density of carriers $n(\boldsymbol{r},t)$ and the components of elastic displacement $u_i(\boldsymbol{r},t)$. For an isotropic, elastic and homogeneous semiconductor the governing equations of motion, plasma and heat conduction under fractional order theory can be described as follows according to previous researches (Lord & Shulman, 1967; Todorović, 2003; Todorović, 2005; El-Karamany & Ezzat 2011a,b):

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \lambda u_{j,ij} + \mu\left(u_{i,jj} + u_{j,ij}\right) - \gamma_n N_{,i} - \gamma_t \Theta_{,i}, \tag{1}$$

$$\frac{\partial N}{\partial t} = D_e N_{,jj} - \frac{N}{\tau} + \delta \frac{\Theta}{\tau}, \tag{2}$$

$$K\Theta_{,jj} = -\frac{E_g}{\tau} N + \left(1 + \frac{\tau_o^\alpha}{\Gamma(\alpha+1)} \frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho c_e \frac{\partial \Theta}{\partial t} + \gamma_t T_o \frac{\partial u_{j,j}}{\partial t}\right), 0 < \alpha \leq 1. \tag{3}$$

The stress-strain relations can be then expressed as

$$\sigma_{ij} = \mu\left(u_{i,j} + u_{j,i}\right) + \left(\lambda u_{k,k} - \gamma_n N - \gamma_t \Theta\right)\delta_{ij}, \tag{4}$$

where $N = n - n_o$, $\Theta = T - T_o$, $\gamma_n = (3\lambda + 2\mu)d_n$ $\gamma_t = (3\lambda + 2\mu)\alpha_t$, $\delta_E = E - E_g$ and $\delta = \frac{\partial n_o}{\partial \Theta}$ (Mandelis et al. 1997).

By taking into consideration the above definition it is possible to write:

$$\frac{\partial^\alpha h(\boldsymbol{r},t)}{\partial t^\alpha} = \begin{cases} h(\boldsymbol{r},t) - h(\boldsymbol{r},0), & \alpha \to 0, \\ I^{\alpha-1} \dfrac{\partial h(\boldsymbol{r},t)}{\partial t}, & 0 < \alpha < 1, \\ \dfrac{\partial h(\boldsymbol{r},t)}{\partial t}, & \alpha = 1, \end{cases} \tag{5}$$

where I^ν is the fraction of Riemann-Liouville integral introduced as a natural generalization of the well-known integral $I^\nu h(\boldsymbol{r},t)$ that can be written in the form of convolution type:

$$I^\nu h(\boldsymbol{r},t) = \int_0^t \frac{(t-s)^\nu}{\Gamma(\nu)} h(\boldsymbol{r},s) ds, \nu > 0, \tag{6}$$

In Eq. (6) $\Gamma(\nu)$ is the Gamma function and $h(\boldsymbol{r},t)$ is a Lebesgue's integrable function. In the case $h(\boldsymbol{r},t)$ is absolutely continuous, then it is possible to write

$$\lim_{\nu \to 1} \frac{\partial^\nu h(r,t)}{\partial t^\nu} = \frac{\partial h(r,t)}{\partial t}, \tag{7}$$

The whole spectrum of local heat conduction is described through the standard heat conduction to ballistic thermal conduction as shown in Eq. (5). The different values of fractional parameter $0<\alpha\leq 1$ cover two types of conductivity, $\alpha=1$ for normal conductivity and $0<\alpha<1$ for low conductivity. Let us consider a homogeneous isotropic infinite semiconducting medium containing a cylindrical hole. Its state can be expressed in terms of the space variable r and the time t which occupying the region $a \leq r < \infty$. The cylindrical coordinates (r, θ, z) are taken with z-axis aligned along the cylinder axis. Due to symmetry involved in the problem, only the radial displacement $u_r = u(r,t)$ is different from zero. Therefore Eqs. (1-4) can be expressed according to the following forms:

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) - \gamma_n \frac{\partial N}{\partial r} - \gamma_t \frac{\partial \Theta}{\partial r}, \tag{8}$$

$$\frac{\partial N}{\partial t} = D_e\left(\frac{\partial^2 N}{\partial r^2} + \frac{1}{r}\frac{\partial N}{\partial r}\right) - \frac{N}{\tau} + \frac{\delta}{\tau}\Theta, \tag{9}$$

$$K\left(\frac{\partial^2 \Theta}{\partial r^2} + \frac{1}{r}\frac{\partial \Theta}{\partial r}\right) = -\frac{E_g}{\tau}N + \left(1 + \frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho c_e \frac{\partial \Theta}{\partial t} + \gamma_t T_o \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right)\right), \tag{10}$$

$$\sigma_{rr} = (\lambda + 2\mu)\frac{\partial u}{\partial r} + \lambda \frac{u}{r} - \gamma_n N - \gamma_t \Theta, \tag{11}$$

$$\sigma_{\theta\theta} = (\lambda + 2\mu)\frac{u}{r} + \lambda \frac{\partial u}{\partial r} - \gamma_n N - \gamma_t \Theta. \tag{12}$$

3. Application

The initial conditions are assumed homogeneous and can be written as follows,

$$\Theta(r,0) = \frac{\partial \Theta(r,0)}{\partial t} = 0\,,\ u(r,0) = \frac{\partial u(r,0)}{\partial t} = 0, N(r,0) = \frac{\partial N(r,0)}{\partial t} = 0\,. \tag{13}$$

The traction free on the internal surface of cavity leads to the following condition

$$\sigma_{rr}(a,t) = 0. \tag{14}$$

The inner surface of the cavity $r = a$ is subjected to a heat flux with exponentially decaying pulse (Zenkour & Abouelregal 2015).

$$-K \frac{\partial T(r,t)}{\partial r}\bigg|_{r=a} = q_o \frac{t^2 e^{-\frac{t}{t_p}}}{16 t_p^2}, \tag{15}$$

In Eq. (15) t_p is the pulse heat flux characteristic time and $q_o$is a constant. During recombination and transport processes (surface and bulk) of the photogenerated carriers at the inner surface of cavity, the boundary condition of the density of carrier may be exepresed according to the following expression:

$$D_e \frac{\partial N(r,t)}{\partial r}\bigg|_{r=a} = S_a N(a,t)\ , \tag{16}$$

In Eq. (16) S_a is the velocity of recombination on the inner surface of hole. It is convenient to transform the governing equations with the initial and boundary conditions into the forms of dimensionless. Thus, the following non-dimensional quantities are introduced as

$$N' = \frac{N}{n_o}, \Theta' = \frac{\Theta}{T_o}, (r', u') = \xi c(r,u)\,, (\sigma'_{rr}, \sigma'_{\theta\theta}) = \frac{(\sigma_{rr}, \sigma_{\theta\theta})}{\lambda + 2\mu}, \left(t', \tau', \tau'_o, t'_p\right) = \xi c^2\left(t, \tau, \tau_o, t_p\right), q'_o = \frac{q_o}{\xi c T_o K}, \tag{17}$$

where $\xi = \frac{\rho c_e}{K}$ and $c^2 = \frac{\lambda + 2\mu}{\rho}$.

In terms of these non-dimensional form of variables in Eq. (17), Eqs. (8-16) can be re-converted in the following form (for convenience the primes has been dropped)

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} - s_1 \frac{\partial N}{\partial r} - s_2 \frac{\partial T}{\partial r}, \tag{18}$$

$$n_1\frac{\partial N}{\partial t}=\frac{\partial^2 N}{\partial r^2}+\frac{1}{r}\frac{\partial N}{\partial r}-\frac{n_1}{\tau}N+\frac{\beta}{\tau}\Theta, \tag{19}$$

$$\frac{\partial^2 \Theta}{\partial r^2}+\frac{1}{r}\frac{\partial \Theta}{\partial r}=-\frac{t_1}{\tau}N+\left(1+\frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\frac{\partial \Theta}{\partial t}+t_2\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)\right), \tag{20}$$

$$\sigma_{rr}=\frac{\partial u}{\partial r}+s_3\frac{u}{r}-s_1N-s_2\Theta, \tag{21}$$

$$\sigma_{\theta\theta}=s_3\frac{\partial u}{\partial r}+\frac{u}{r}-s_1N-s_2\Theta, \tag{22}$$

$$\left.\frac{\partial \Theta(r,t)}{\partial r}\right|_{r=a}=-q_o\frac{t^2e^{-\frac{t}{t_p}}}{16t_p^2}, \tag{23}$$

$$\left.\frac{\partial N(r,t)}{\partial r}\right|_{r=a}=\varepsilon N(a,t)\ , \tag{24}$$

$$\sigma_{rr}(a,t)=0, \tag{25}$$

where $s_1=\frac{n_o\gamma_n}{\lambda+2\mu}$, $s_2=\frac{T_o\gamma_t}{\lambda+2\mu}$, $s_3=\frac{\lambda}{\lambda+2\mu}$, $\beta=\frac{kT_o}{n_o\varpi D_e}$, $n_1=\frac{1}{\xi D_e}$, $t_1=\frac{n_oE_g}{\rho c_eT_o}$, $t_2=\frac{\gamma_t}{\rho c_e}$, $\varepsilon=\frac{S_s}{\xi cD_e}$.
Let us define the transformation of Laplace for a function $\Phi(r,t)$ by

$$L[\Phi(r,t)]=\bar{\Phi}(r,s)=\int_0^\infty \Phi(r,t)e^{-st}dt\,,s>0\,. \tag{26}$$

Eqs. (18-25) by using the initial conditions (13) can be rewritten as follows

$$s^2\bar{u}=\frac{d^2\bar{u}}{dr^2}+\frac{1}{r}\frac{d\bar{u}}{dr}-\frac{\bar{u}}{r^2}-s_1\frac{d\bar{N}}{dr}-s_2\frac{d\bar{\Theta}}{dr}, \tag{27}$$

$$sn_1\bar{N}=\frac{d^2\bar{N}}{dr^2}+\frac{1}{r}\frac{d\bar{N}}{dr}-n_1\frac{\bar{N}}{\tau}+\frac{\beta}{\tau}\bar{\Theta}, \tag{28}$$

$$\frac{d^2\bar{\Theta}}{dr^2}+\frac{1}{r}\frac{d\bar{\Theta}}{dr}=-\frac{t_1}{\tau}\bar{N}+s\left(1+\frac{s^\alpha\tau_o^\alpha}{\Gamma(\alpha+1)}\right)\left(\bar{\Theta}+t_2\left(\frac{d\bar{u}}{dr}+\frac{\bar{u}}{r}\right)\right), \tag{29}$$

$$\bar{\sigma}_{rr}=\frac{d\bar{u}}{dr}+s_3\frac{\bar{u}}{r}-s_1\bar{N}-s_2\bar{\Theta}, \tag{30}$$

$$\bar{\sigma}_{\theta\theta}=s_3\frac{d\bar{u}}{dr}+\frac{\bar{u}}{r}-s_1\bar{N}-s_2\bar{\Theta}, \tag{31}$$

$$\left.\frac{d\bar{\Theta}(r,t)}{dr}\right|_{r=a}=\frac{-q_ot_p}{8\left(st_p+1\right)^3}, \tag{32}$$

$$\left.\frac{d\bar{N}(r,t)}{dr}\right|_{r=a}=\varepsilon\bar{N}(a,t)\,, \tag{33}$$

$$\bar{\sigma}_{rr}(a,r)=0, \tag{34}$$

Differentiating Eq. (28) and Eq. (29) with respect to r and using in combination Eq. (27), it is possible to obtain the following expressions:

$$\frac{d^2\bar{u}}{dr^2}+\frac{1}{r}\frac{d\bar{u}}{dr}-\frac{\bar{u}}{r^2}=s^2\bar{u}+s_1\frac{d\bar{N}}{dr}+s_2\frac{d\bar{\Theta}}{dr}, \tag{35}$$

$$\frac{d^2}{dr^2}\left(\frac{d\bar{N}}{dr}\right)+\frac{1}{r}\frac{d}{dr}\left(\frac{d\bar{N}}{dr}\right)-\frac{1}{r^2}\left(\frac{d\bar{N}}{dr}\right)=n_1\left(s+\frac{1}{\tau}\right)\frac{d\bar{N}}{dr}-\frac{\beta}{\tau}\frac{d\bar{\Theta}}{dr}, \tag{36}$$

$$\frac{d^2}{dr^2}\left(\frac{d\bar{\Theta}}{dr}\right)+\frac{1}{r}\frac{d}{dr}\left(\frac{d\bar{\Theta}}{dr}\right)-\frac{1}{r^2}\left(\frac{d\bar{\Theta}}{dr}\right)=s^2t_2t_3\bar{u}+\left(t_2t_3s_1-\frac{t_1}{\tau}\right)\frac{d\bar{N}}{dr}+t_3(1+t_2s_2)\frac{d\bar{\Theta}}{dr}, \tag{37}$$

where $t_3=s\left(1+\frac{s^\alpha\tau_o^\alpha}{\Gamma(\alpha+1)}\right)$. Now, it is possible to solve the coupled differential Eqs (35), (36) and (37) by the eigenvalue approach proposed (Das et al., 1997; Abbas 2014a,b,c,d2015a,b,c). From Eqs. (35-37), the vector-matrix can be expressed in the following form

$$LV=BV, \tag{38}$$

where $=\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{1}{r^2}$, $\mathrm{V}=\begin{bmatrix}\bar{u} & \frac{d\bar{N}}{dr} & \frac{d\bar{\Theta}}{dr}\end{bmatrix}^T$ and $\mathrm{B}=\begin{bmatrix}b_{11} & b_{12} & b_{13}\\ 0 & b_{22} & b_{23}\\ b_{31} & b_{32} & b_{33}\end{bmatrix}$,

with $b_{11} = s^2, b_{12} = s_1, b_{13} = s_2, b_{22} = n_1\left(s + \frac{1}{\tau}\right), b_{23} = -\frac{\beta}{\tau}, b_{31} = s^2 t_2 t_3,$
$b_{32} = t_2 t_3 s_1 - \frac{t_1}{\tau}, b_{33} = t_3(1 + t_2 s_2).$

The matrix B has its characteristic equation which is as follows

$$\varpi^3 - \varpi^2(b_{11} + b_{22} + b_{33}) - \varpi(-b_{11}b_{22} + b_{13}b_{31} + b_{23}b_{32} - b_{11}b_{33} - b_{22}b_{33}) + b_{13}b_{22}b_{31} - b_{12}b_{23}b_{31} + b_{11}b_{23}b_{32} - b_{11}b_{22}b_{33} = 0, \quad (39)$$

The eigenvalues of matrix B are the three roots of Eq. (39) which are named here $\varpi_1, \varpi_2, \varpi_3$. Thus, the corresponding eigenvector $X = [X_1, X_2, X_3]$ can be calculated as:

$$X_1 = b_{13}(-\varpi + b_{22}) - b_{12}b_{23}, X_2 - (\varpi - b_{11})b_{23}, X_3 - (\varpi - b_{11})(\varpi - b_{22}). \quad (40)$$

The solution of Eq. (38) which is bounded as $r \to \infty$ can be written as

$$\mathrm{V}(r, s) = \sum_{i=1}^{3} A_i X_i K_1(m_i r), \quad (41)$$

In Eq. (41) $m_i = \sqrt{\varpi_i}$, $K_1$is the modified of Bessel's function of order one, $A_1, A_2$and $A_3$are constants that can be calculated by using the problem boundary conditions. Hence, the field variables have the solutions with respect to r and s in the forms:

$$\bar{u}(r, s) = \sum_{i=1}^{3} A_i U_i K_1(m_i r), \quad (42)$$

$$\bar{N}(r, s) = -\sum_{i=1}^{3} A_i \frac{N_i}{m_i} K_0(m_i r), \quad (43)$$

$$\bar{T}(r, s) = -\sum_{i=1}^{3} A_i \frac{T_i}{m_i} K_0(m_i r), \quad (44)$$

$$\bar{\sigma}_{rr}(r, s) = \sum_{i=1}^{3} A_i \left(\frac{-m_i^2 U_i + s_1 N_i + s_2 T_i}{m_i} K_0(m_i r) + \frac{(s_3 - 1)U_i}{r} K_1(m_i r)\right), \quad (45)$$

$$\bar{\sigma}_{\theta\theta}(r, s) = \sum_{i=1}^{3} A_i \left(\frac{-s_3 m_i^2 U_i + s_1 N_i + s_2 T_i}{m_i} K_0(m_i r) - \frac{(s_3 - 1)U_i}{r} K_1(m_i r)\right). \quad (46)$$

4. Numerical inversions and discussions of the results

For the general solution of temperature, density of carrier, displacement, and stress distribution, a numerical inversion method was adopted based on Stehfest's derivation (Stehfest 1970). In this method, the inverse $f(r, t)$ of the Laplace transform $\bar{f}(r, p)$ is approximated by the relation

$$f(r, t) = \frac{ln2}{t} \sum_{j=1}^{M} V_j f\left(r, j\frac{ln2}{t}\right), \quad (47)$$

where V_j is given by the following equation:

$$V_j = (-1)^{\frac{M}{2}+1} \sum_{k=\frac{i+1}{2}}^{min\left(i, \frac{M}{2}\right)} \frac{k^{\frac{M}{2}+1}(2k)!}{\left(\frac{M}{2}-k\right)! k! (i-k)! (2k-1)!}. \quad (48)$$

Now, we consider a numerical example for the computational purpose, silicon (Si) like material has been considered. The main material constants are taken from a recent and update reference (Song et al. 2014):

$\rho = 2330(kg)(m^{-3}), \mu = 5.46 \times 10^{10}(N)(m^{-2}), \lambda = 3.64 \times 10^{10}(N)(m^{-2})$,
$d_n = -9 \times 10^{-31}(m^3), S_s = 2\ (m)(s^{-1}), n_o = 10^{20}(m^{-3}), c_e = 695(J)(kg^{-1})(k^{-1}),$
$\alpha_t = 3 \times 10^{-6}(k^{-1}), D_e = 2.5 \times 10^{-3}(m^2)(s^{-1}), E_g = 1.11\ (eV),$
$T_o = 300(k)$, $t_p = 2(ps), \tau = 5 \times 10^{-5}(s), \tau_o = 0.2(ps).$

The numerical techniques, described above, have been used for the variations of the carrier density n, the displacement u, the temperature T, the radial and hoop stresses σ_{rr}, $\sigma_{\theta\theta}$ with respect to r-direction in the context of generalized photothermal theory with a relaxation time under a fractional order derivative. By using the relationship between the variable and its non-dimensional form, here all the variables are expressed in the dimensional forms and displayed graphically as in Figs. 1-10. The calculations were performed for the time $t = 4.4470\ ps$.

From Fig. 1 and Fig. 6, the density of carrier have their maximum values on the surface of the hole $(r = a = 0.3\ \mu m)$ and decreases with the radial distance up to near the equilibrium carrier concentration n_o $(n_o = 10^{20}\ m^{-3} = 10^2\ \mu m^{-3})$. Fig. 2 and Fig. 7 display the variation of temperature along the radial distance r. It can be observed that starting from the heights values on the surface of cavity then the value decreases gradually by increasing the radial distance r. This happens in the proximity of the reference temperature $(T_o = 300\ K^{\circ})$ beyond a wave front for the generalized photo-thermal model, which satisfies the theoretical boundary conditions of the problem. The radial displacement varies as a function of r as shown in Fig. 3 and Fig. 8. It is noticed that the displacement attains a maximum negative value then it increases gradually up to a peak value in a particular location proximately close to the surface and then continuously decreases to zero. Fig. 4 and Fig. 9 show the variation of radial stress with respect to r. It can be observed that the radial stress, always starts from zero and drops to zero to obey the boundary conditions. The variation of hoop stress along the radial distance r was shown in Fig. 5 and Fig. 10. It is noticed that the hoop stress attains a maximum negative values and then continuously increases to zero.

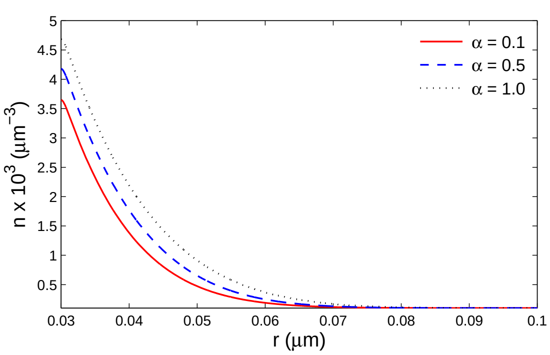

Fig. 1. The variation of carrier density with distance for different values of ν

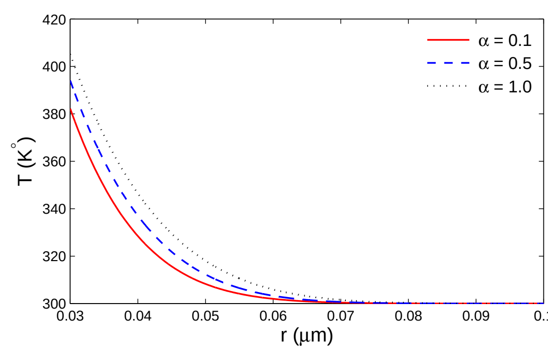

Fig. 2. The variation of temperature with distance for different values of ν

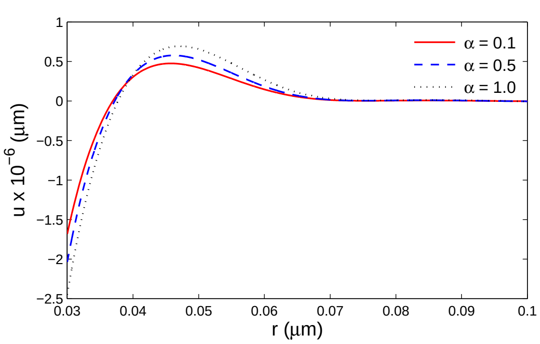

Fig. 3. The variation of displacement with distance for different values of ν

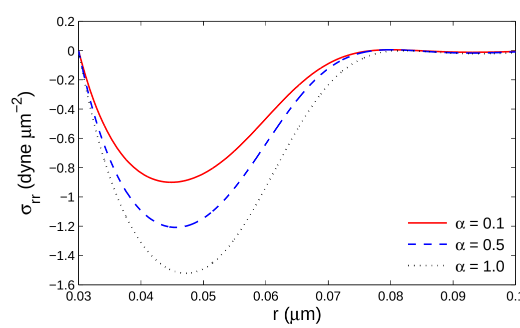

Fig. 4. The variation of radial stress with distance for different values of α

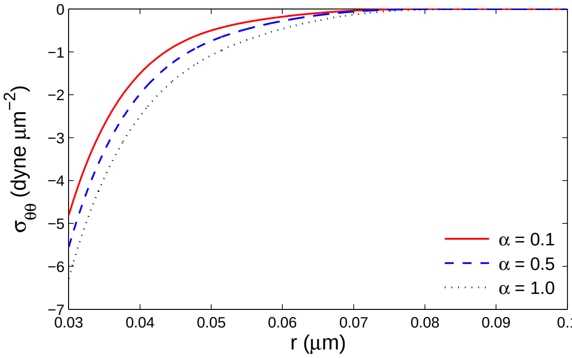

Fig. 5. The variation of hoop stress with distance for different values of α

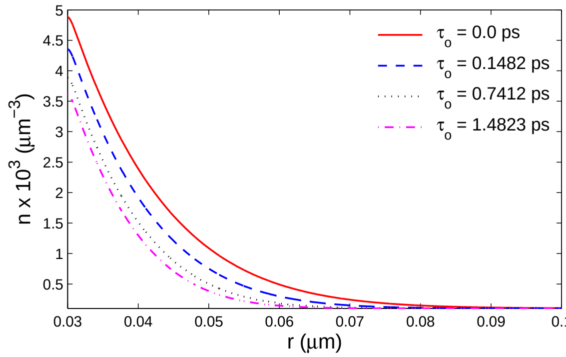

Fig. 6. The carrier density distribution for different values of relaxation time τ_o

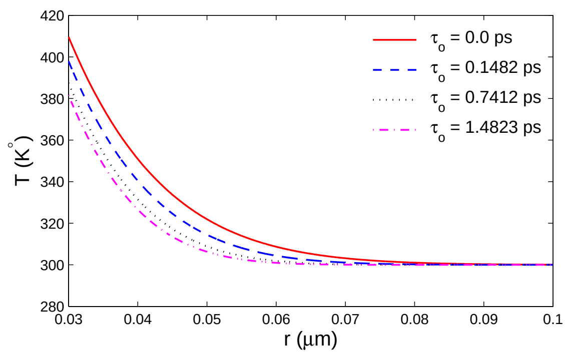

Fig. 7. The temperature distribution for different values of relaxation time τ_o

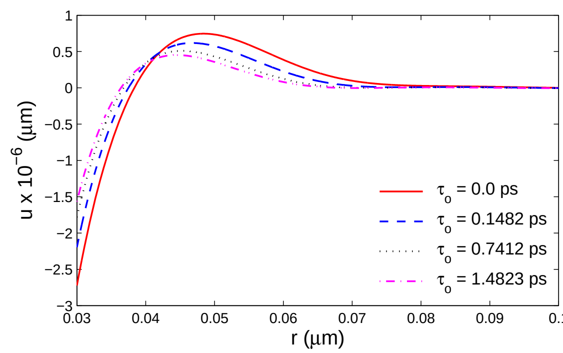

Fig. 8. The displacement distribution for different values of relaxation time τ_o

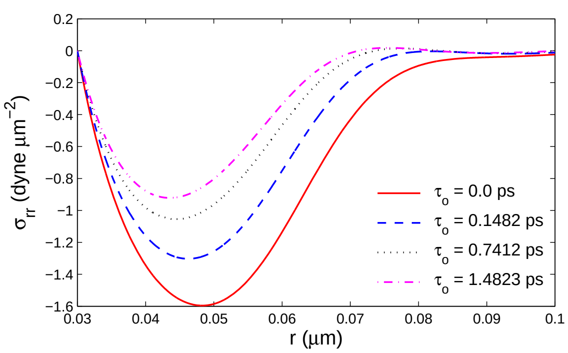

Fig. 9. The radial stress distribution for different values of relaxation time τ_o

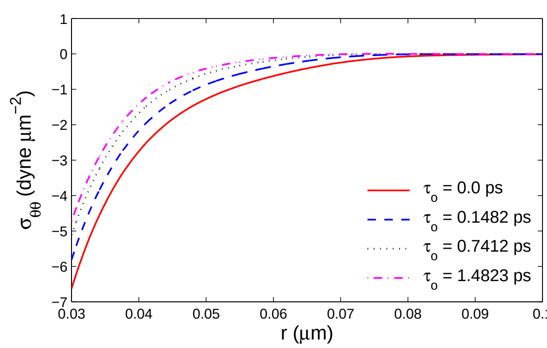

Fig. 10. The hoop stress distribution for different values of relaxation time τ_o

From the results, Figs. 1-5 show the variation of all physical quantities with respect to the radial distance r for different values of the fractional order parameter α when $\tau_o = 0.1432\, ps$. It can be observed that the dotted line refer to the normal conductivity while the solid and dashed lines refer to the low conductivity. From these results, the fractional parameter α has a significant effect on all the physical quantities. In the case of $\alpha = 0.5$, the effect of thermal relaxation time on the variation of all variables is depicted in Figs. 6-10. The results allow depicting the differences by using the coupled photothermoelastic theory and the generalized photothermoelastic theory for a specific value of the relaxation time.

5. Conclusion

In the present work, the effects of thermal relaxation time and fractional order parameters on the plasma, thermal, and elastic waves in semiconductor media with cylindrical holes has been studied. Analytical expressions for temperature, displacement, density of carrier, radial stress and hoop stress in the medium have been accurately derived. Results carried out in this paper can be used to design various semiconductor elements during the presence of coupled thermal, elastic and plasma and waves and can also be applied to other fields like in the material science, physical engineering. Moreover these findings can help the designers of new materials to meet special engineering requirements in very specific service conditions.

References

Abbas, I. A. (2014a). Eigenvalue approach for an unbounded medium with a spherical cavity based upon two-temperature generalized thermoelastic theory. *Journal of Mechanical Science and Technology*, *28*(10), 4193-4198.

Abbas, I. A. (2014b). Eigenvalue approach in a three-dimensional generalized thermoelastic interactions with temperature-dependent material properties. *Computers & Mathematics with Applications*, *68*(12), 2036-2056.

Abbas, I. A. (2014c). A problem on functional graded material under fractional order theory of thermoelasticity. *Theoretical and Applied Fracture Mechanics*, *74*, 18-22.

Abbas, I. A. (2014d). The effects of relaxation times and a moving heat source on a two-temperature generalized thermoelastic thin slim strip. *Canadian Journal of Physics*, *93*(5), 585-590.

Abbas, I. A. (2015a). Eigenvalue approach on fractional order theory of thermoelastic diffusion problem for an infinite elastic medium with a spherical cavity. *Applied Mathematical Modelling*, *39*(20), 6196-6206.

Abbas, I. A. (2015b). A dual phase lag model on thermoelastic interaction in an infinite fiber-reinforced anisotropic medium with a circular hole. *Mechanics Based Design of Structures and Machines*, *43*(4), 501-513.

Abbas, I. A. (2015c). Eigenvalue approach to fractional order generalized magneto-thermoelastic medium subjected to moving heat source. *Journal of Magnetism and Magnetic Materials*, *377*, 452-459.

Abbas, I. A. (2016). A dual phase lag model on photothermal interaction in an unbounded semiconductor medium with cylindrical cavity. *International Journal of Computational Materials Science and Engineering*, *5*(03), 1650016.

Almond, D. P., & Patel, P. (1996). *Photothermal science and techniques* (Vol. 10). Springer Science & Business Media.

Das, N. C., Lahiri, A., & Giri, R. R. (1997). Eigenvalue approach to generalized thermoelasticity. *Indian Journal of Pure and Applied Mathematics*, *28*, 1573-1594.

El-Karamany, A. S., & Ezzat, M. A. (2011). On fractional thermoelasticity. *Mathematics and Mechanics of Solids*, *16*(3), 334-346.

Ezzat, M. A. (2011a). Magneto-thermoelasticity with thermoelectric properties and fractional derivative heat transfer. *Physica B: Condensed Matter*, *406*(1), 30-35.

Ezzat, M. A. (2011b). Theory of fractional order in generalized thermoelectric MHD. *Applied Mathematical Modelling*, *35*(10), 4965-4978.

Ezzat, M. A., & El-Karamany, A. S. (2011a). Fractional order theory of a perfect conducting thermoelastic medium. *Canadian Journal of Physics*, *89*(3), 311-318.

Ezzat, M. A., & El Karamany, A. S. (2011b). Theory of fractional order in electro-thermoelasticity. *European Journal of Mechanics-A/Solids*, *30*(4), 491-500.

Hobiny, A. D., & Abbas, I. A. (2017). A study on photothermal waves in an unbounded semiconductor medium with cylindrical cavity. *Mechanics of Time-Dependent Materials*, *21*(1), 61-72.

Jackson, W., & Amer, N. M. (1980). Piezoelectric photoacoustic detection: theory and experiment. *Journal of Applied Physics*, *51*(6), 3343-3353.

Kumar, R., Gupta, V., & Abbas, I. A. (2013). Plane deformation due to thermal source in fractional order thermoelastic media. *Journal of computational and theoretical nanoscience*, *10*(10), 2520-2525.

Lord, H. W., & Shulman, Y. (1967). A generalized dynamical theory of thermoelasticity. *Journal of the Mechanics and Physics of Solids*, *15*(5), 299-309.

Mandelis, A. (1987). Photoacoustic and thermal wave phenomena in semiconductors.

Mandelis, A., & Hess, P. (2000). *Semiconductors and electronic materials* (Vol. 4). Spie Press.

Mandelis, A., Nestoros, M., & Christofides, C. (1997). Thermoelectronic-wave coupling in laser photothermal theory of semiconductors at elevated temperatures. *Optical Engineering*, *36*(2), 459-469.

McDonald, F. A., & Wetsel Jr, G. C. (1978). Generalized theory of the photoacoustic effect. *Journal of Applied Physics*, *49*(4), 2313-2322.

Opsal, J., & Rosencwaig, A. (1985). Thermal and plasma wave depth profiling in silicon. *Applied Physics Letters*, *47*(5), 498-500.

Rosencwaig, A., Opsal, J., & Willenborg, D. L. (1983). Thin-film thickness measurements with thermal waves. *Le Journal de Physique Colloques*, *44*(C6), C6-483.

Sherief, H., & El-Latief, A. A. (2013). Effect of variable thermal conductivity on a half-space under the fractional order theory of thermoelasticity. *International Journal of Mechanical Sciences*, *74*, 185-189.

Sherief, H. H., El-Sayed, A. M. A., & El-Latief, A. A. (2010). Fractional order theory of thermoelasticity. *International Journal of Solids and structures*, *47*(2), 269-275.

Song, Y., Cretin, B., Todorovic, D. M., & Vairac, P. (2008). Study of photothermal vibrations of semiconductor cantilevers near the resonant frequency. *Journal of Physics D: Applied Physics*, *41*(15), 155106.

Song, Y., Todorovic, D. M., Cretin, B., Vairac, P., Xu, J., & Bai, J. (2014). Bending of semiconducting cantilevers under photothermal excitation. *International Journal of Thermophysics*, *35*(2), 305-319.

Stearns, R. G., & Kino, G. S. (1985). Effect of electronic strain on photoacoustic generation in silicon. *Applied Physics Letters*, *47*(10), 1048-1050.

Stehfest, H. (1970). Algorithm 368: Numerical inversion of Laplace transforms [D5]. *Communications of the ACM*, *13*(1), 47-49.

Todorović, D. M. (2003a). Photothermal and electronic elastic effects in microelectromechanical structures. *Review of Scientific Instruments*, *74*(1), 578-581.

Todorović, D. M. (2003b). Plasma, thermal, and elastic waves in semiconductors. *Review of Scientific Instruments*, *74*(1), 582-585.

Todorović, D. M. (2005, June). Plasmaelastic and thermoelastic waves in semiconductors. In *Journal de Physique IV (Proceedings)* (Vol. 125, pp. 551-555). EDP sciences.

Youssef, H. M. (2010). Theory of fractional order generalized thermoelasticity. *Journal of Heat Transfer*, *132*(6), 061301.

Youssef, H. M. (2012). Two-dimensional thermal shock problem of fractional order generalized thermoelasticity. *Acta Mechanica*, *223*(6), 1219-1231.

Youssef, H. M., & Al-Lehaibi, E. A. (2010). Variational principle of fractional order generalized thermoelasticity. *Applied Mathematics Letters*, *23*(10), 1183-1187.

Zenkour, A. M., & Abouelregal, A. E. (2015). Effect of temperature dependency on constrained orthotropic unbounded body with a cylindrical cavity due to pulse heat flux. *Journal of Thermal Science and Technology*, *10*(1), JTST0019-JTST0019.