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DQM modeling of rectangular plate resting on two parameter foundation

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This paper presents two parameter foundation models for free vibration analysis of non-homogeneous orthotropic rectangular plate resting on elastic foundation whose concept is extensively used in engineering practice. Following Lévy approach i.e. the two parallel edges are simply supported, the fourth order differential equation governing the motion of such plates of non-linear varying thickness in one direction has been solved by using an efficient and rapid convergent numerical approximation technique that is called differential quadrature method (DQM). Appropriate boundary conditions accompany the differential quadrature method to transform the resulting differential equation into an eigenvalue problem. The effects of thickness variation, foundation parameters and other plate parameters with boundary conditions on frequency are examined. The numerical results show that the method converges significantly irrespective of parameters involved.

1. Introduction

The analysis of vibration of a plate on elastic foundation is of considerable interest and widely used in engineering structures such as railroad, pipeline, aerospace, biomechanics, petrochemical, marine industry, civil and mechanical engineering applications. Many problems in the engineering related to soil-structure interaction can be modeled by means of a beam and plate on an elastic foundation. In this context, Hetenyi (1966), Vlasov and Leontev (1966) investigated the effect of elastic foundation on the dynamic behavior of beams and plates. Various models approximating the supporting medium (i.e. foundation) such as Vlasov by Bhattacharya (1977), Pasternak by Wang and Stephens (1977) and Winkler by Chonan (1980) were proposed in the literature. Winkler foundation model is extensively used by engineers and researchers because of its simplicity and are reported in references (Gupta & Lal, 1978; Selvadurai, 1979; Liew et al., 1996; Gupta et al., 2012; Samaei et al., 2015).

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The free vibration analysis of rectangular orthotropic non- homogeneous plate on elastic foundation has been investigated by many researchers for the past forty years. Most of the studies on the dynamic behavior of rectangular plates resting on elastic foundation are devoted to Winkler foundation (Lal et al., 2001; Gupta et al., 2012). In Winkler model the foundation is assumed to be replaced by a series of unconnected closely spaced vertical elastic springs, but the main disadvantage of the Winkler model is the displacement discontinuity. To overcome the deficiency of Winkler model, various models have been proposed in the literature by different researchers. Kerr (1964) gave an excellent discussion about these models. Of these, the most natural extension of the Winkler model is the Pasternak model (two parameter foundation) as it takes into account not only its transverse reaction but also the shear interaction between the spring elements.

Numerous studies have appeared in the literature to analyze the effect of two parameter foundation on the static and dynamic behavior of rectangular plates. The prominent references are: Xiang et al. (1994), Omurtag and Kadioglu (1998) and Gupta et al. (2014). The numerical methods to study the vibrational behavior of uniform/variable thickness plates resting on elastic foundation have been discussed in prominent references are: Frobenious method presented by Jain and Soni (1973), finite difference for rectangular plates of exponentially varying thickness by Sonzogni et al. (1990), Rayleigh-Ritz method for free and forced vibration analysis of moderately thick isotropic rectangular plates resting on Pasternak foundation employing by Shen et al. (2001). Malekzadeh and Karami (2004) obtained a differential quadrature solution for free vibration analysis of isotropic non-uniform thick rectangular plates resting on Pasternak foundation. Civalek and Acar (2007) used discrete singular convolution method for the bending analysis of Mindlin plates on Pasternak foundation. Lal and Dhanpati (2007) applied Quintic spline technique to study the transverse vibration of non-homogeneous orthotropic rectangular plates of variable thickness. Furthermore, a global transfer matrix and Durbin's numerical Laplace inversion algorithm were employed by Hasheminejad and Gheshlaghi (2012) to study the transient vibration of simply supported, functionally graded rectangular plates resting on a linear Winkler–Pasternak viscoelastic foundation. Differential quadrature method (DQM) requires less grid points for desired accuracy as compare to finite difference method, finite element method, quintic splines, and characteristic orthogonal polynomials and Frobenius method. DQM was introduced by Bellman et al. (1972) and generalized and simplified subsequently by Quan and Chang (1989).

In the present study, differential quadrature method (DQM) is applied for computation of the free vibration analysis of rectangular orthotropic non-homogeneous plate of non-linear thickness variation embedded in two parameter foundation. The choice of Lévy approach reduces the complexity of governing fourth order differential equation with variable coefficients to one dimension. The effect of various plate parameters for a Huber type orthotropic plate material 'ORTHO1' (Biancolini et al., 2005) has been studied on the natural frequencies for the first three modes of vibration for different boundary conditions. Convergence studies have also been made to achieve four decimal place exactitude in frequencies. Frequencies and mode shapes for the first three modes of vibration are computed for specified plates. A close agreement of our results with those available in the literature shows the versatility of the DQM.

2. Mathematical Formulation

Following Gupta et al. (2014), the differential equation describing the motion of a non-homogeneous orthotropic rectangular plate of linear variation in thickness resting on two parameter foundation is given as follows:

$$\bar{h}^3 E_x W^{iv} + [2(\bar{h}^3 E_x' + 3\bar{h}^2 \bar{h}' E_x)]W'''$$
$$+[(6\bar{h}\bar{h}'^2 + 3\bar{h}^2 \bar{h}'')E_x + 6\bar{h}^2 \bar{h}' E_x' + \bar{h}^3 E_x'' - 2(1-\nu_x\nu_y)\{\lambda^2 \bar{h}^3 (E^* + 2G_{xy}) + 6(G_f/a)\}]W''$$
$$-[2\lambda^2\{3\bar{h}^2 \bar{h}'(\nu_y E_x + 2(1-\nu_x\nu_y)G_{xy}) + \bar{h}^3(\nu_y E_x' + 2(1-\nu_x\nu_y)G_{xy}')\}]W' \tag{1}$$
$$+[\lambda^4 \bar{h}^3 E_y - \lambda^2 \nu_y\{\bar{h}^3 E_x'' + 6\bar{h}^2 \bar{h}' E_x' + (6\bar{h}\bar{h}'^2 + 3\bar{h}^2 \bar{h}'')E_x\} + 12(1-\nu_x\nu_y)(ak_f + \lambda^2(G_f/a) - \rho\bar{h}a^2\omega^2)]W = 0,$$

where $\lambda^2 = p^2 a^2 \pi^2 / b^2$ and primes denote differentiation with respect to X.

For non-linear (parabolic) variation in thickness i.e. $\bar{h} = h_0(1+\alpha X^2)$ and following references (Jain & Soni; 1973; Malekzadeh & Karami, 2004; Gupta et al., 2014) for non-homogeneity of the plate material in X direction as follows:

$$E_x = E_1 e^{\mu X}, \quad E_y = E_2 e^{\mu X}, \quad \rho = \rho_0 e^{\beta X}, \tag{2}$$

where $(h_0, \rho_0) = (h, \rho)\big|_{X=X_0}$, μ is the non-homogeneity parameter, α is the taper parameter, β is the density parameter and E_1, E_2 are Young's moduli in proper directions at X=0.

Eq. (1) now reduces to

$$A_0 W^{iv} + A_1 W''' + A_2 W'' + A_3 W' + A_4 W = 0, \tag{3}$$

where

$$A_0 = 1, \qquad A_1 = 2\left(\mu + \frac{6\alpha X}{(1+\alpha X)}\right),$$

$$A_2 = \frac{24\alpha^2 X^2}{(1+\alpha X^2)^2} + \frac{6\alpha}{(1+\alpha X^2)} + \frac{12\mu\alpha X}{(1+\alpha X^2)} + \mu^2 - 2\{\lambda^2(\nu_y + \frac{\sqrt{E_2/E_1}}{1+\sqrt{\nu_x\nu_y}}(1-\nu_x\nu_y)) + \frac{6G}{h_0^3(1+\alpha X^2)^3} e^{-\mu X}\}$$

$$A_3 = -2\lambda^2\left(\frac{6\alpha X}{(1+\alpha X^2)} + \mu\right)\left(\nu_y + \frac{\sqrt{E_2/E_1}}{1+\sqrt{\nu_x\nu_y}}(1-\nu_x\nu_y)\right),$$

$$A_4 = \frac{\lambda^4 E_2}{E_1} - \lambda^2\nu_y\{\mu^2 + \frac{12\mu\alpha X}{(1+\alpha X^2)} + \frac{24\alpha^2 X^2}{(1+\alpha X^2)^2} + \frac{6\alpha}{(1+\alpha X^2)}\} - \frac{\Omega^2}{(1+\alpha X^2)^2} e^{(\beta-\mu)X} + \frac{12K}{h_0^3(1+\alpha X^2)^3} e^{-\mu X} + \frac{12\lambda^2 G}{h_0^3(1+\alpha X^2)^3} e^{-\mu X}$$

$$\Omega^2 = \frac{12\rho_0(1-\nu_x\nu_y)a^2\omega^2}{E_1 h_0^2}, \quad K = \frac{ak_f(1-\nu_x\nu_y)}{E_1} \text{ and } G = \frac{G_f(1-\nu_x\nu_y)}{aE_1}$$

The solution of Eq. (3) in conjunction with boundary conditions at the edges $X = 0$ and $X = 1$ yields a two-point boundary value problem with variable coefficients whose close form solution is not possible. An approximate solution is obtained by employing Differential Quadrature Method.

3. Method of Solution: Dirrerential Quadrature Method

Let $X_1, X_2, \ldots, X_m$ be the m grid points in the applicability range [0, 1] of the plate. According to the DQM, the n^{th} order derivative of $W(X)$ w.r.t. X can be expressed discretely at the point X_i as

$$\frac{d^n W(X_i)}{dX^n} = \sum_{j=1}^{m} c_{ij}^{(n)} W(X_j), \qquad n = 1, 2, 3, 4 \text{ and } i = 1, 2, \ldots, m \tag{4}$$

where $c_{ij}^{(n)}$ are the weighting coefficients associated with the n^{th} order derivative of W(X) with respect to X at discrete point X_i. Following Shu (2000), the weighting coefficients in Eq. (4) are given by

$$c_{ij}^{(1)} = \frac{M^{(1)}(X_i)}{(X_i - X_j)M^{(1)}(X_j)} \quad , \quad i,\ j=1,2,\ldots,m\ ; \quad i \neq j \tag{5}$$

where

$$M^{(1)}(X_i) = \prod_{\substack{j=1 \\ j \neq i}}^{m} (X_i - X_j), \tag{6}$$

and

$$c_{ij}^{(n)} = n\left(c_{ii}^{(n-1)} c_{ij}^{(1)} - \frac{c_{ij}^{(n-1)}}{x_i - x_j} \right), \quad for\ \ i, j = 1,\ 2, \ldots, m; j \neq i; \quad and\, n = 2, 3, 4. \tag{7}$$

$$c_{ii}^{(n)} = -\sum_{\substack{j=1 \\ j \neq i}}^{m} c_{ij}^{(n)} \qquad for\ \ i = 1,\ 2\ , \ldots\ldots\ldots, m \qquad and\ \ n = 1,\ 2,\ 3, 4 \tag{8}$$

Discretizing Eq. (3) at grid points X_i, i = 3, 4,…, m-2, it reduces to,

$$A_{0,i} W^{iv}(X_i) + A_{1,i} W'''(X_i) + A_{2,i} W''(X_i) + A_{3,i} W'(X_i) + A_{4,i}\, W(X_i) = 0. \quad i{=}3,4,\ldots\ldots,(m\text{-}2) \tag{9}$$

Substituting for $W(X)$ and its derivatives at the i^{th} grid point in the Eq. (9) and using Eq. (4) to Eq. (8), the Eq. (9) becomes

$$\sum_{j=1}^{m} (A_{0,i} c_{ij}^{(4)} + A_{1,i} c_{ij}^{(3)} + A_{2,i} c_{ij}^{(2)} + A_{3,i} c_{ij}^{(1)}) W(X_j) + A_{4,i} W(X_i) = 0. \qquad i{=}3,4,\ldots,(m\text{-}2) \tag{10}$$

The satisfaction of Eq. (10) at (m-4) nodal points X_i, i = 3, 4 …... (m-2) provides a set of (m-4) equations in terms of unknowns $W_j (\equiv W(X_j)),\ j = 1,2,\cdots, m,$ which can be written in the matrix form as

$$[B][W^*]=[0], \tag{11}$$

where B and W^* are matrices of order (m-4) × m and (m × 1) respectively.

Here, the (m-2) internal grid points chosen for collocation, are the zeros of shifted Chebyshev polynomial of order (m-2) with orthogonality range [0, 1] given by

$$X_{k+1} = \frac{1}{2}[1 + \cos(\frac{2k-1}{m-2}\frac{\pi}{2})] \ , \qquad k = 1,\ 2,\ \ldots\ldots\ldots,\ m\text{-}2 \tag{12}$$

4. Boundary Conditions and Frequency Equations

The two different combinations of boundary conditions namely, C-C, C-S have been considered here, where C, S stand for clamped and simply supported respectively and first symbol denotes the condition at the edge X=0 and second symbol at the edge X=1.By satisfying the relations.

$$W = \frac{dW}{dX} = 0\,,$$

$$W = \frac{d^2W}{dX^2} - (E^* / E_x^*)\lambda^2 W = 0\,,$$

for clamped and simply supported conditions, respectively, a set of four homogeneous equations in terms of unknown W_j are obtained. These equations together with field Eq. (11) give a complete set of m homogeneous equations in m unknowns. For C-C plate this set of equations can be written as

$$\begin{bmatrix} B \\ B^{CC} \end{bmatrix} \left[W^* \right] = [0], \tag{13}$$

where B^{CC} is a matrix of order $4 \times m$.
For a non-trivial solution of Eq. (13), the frequency determinant must vanish and hence,

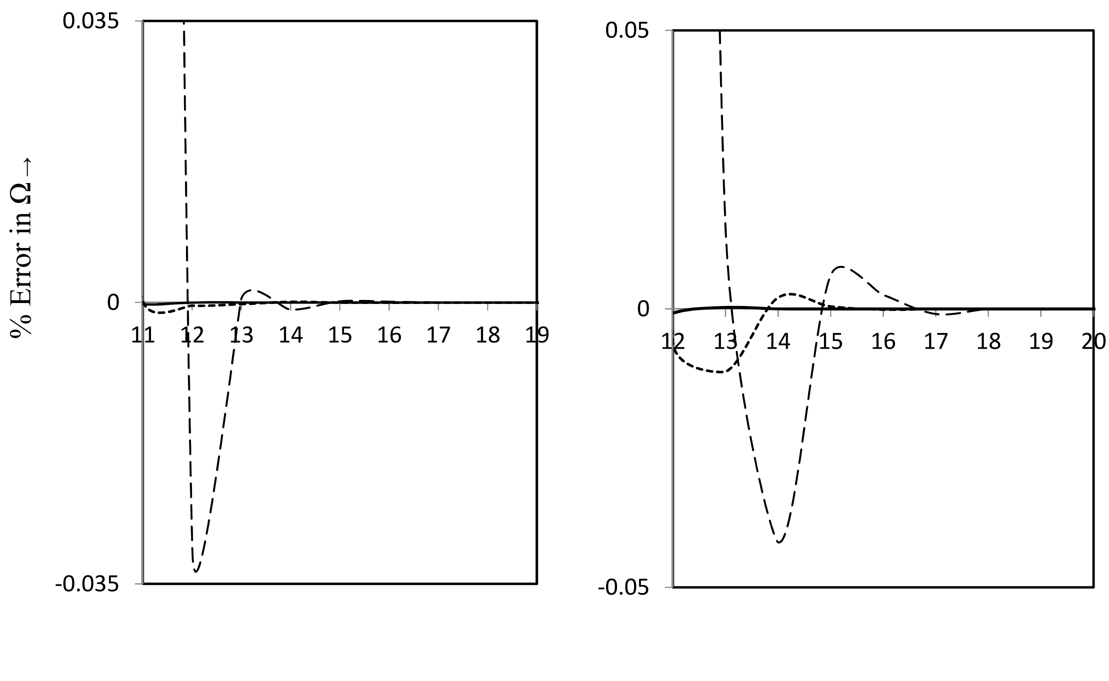
$$\begin{vmatrix} B \\ B^{CC} \end{vmatrix} = 0 . \tag{14}$$

Similarly for C-S plate, the frequency determinants can be written as

$$\begin{vmatrix} B \\ B^{CS} \end{vmatrix} = 0 . \tag{15}$$

5. Numerical Results and Discussion

The frequency Eqs. (14-15) have been solved to obtain the values of the frequency parameter Ω for C-C and C-S plates vibrating in first three modes of vibration. The effect of non-homogeneity together with foundation, orthotropy, thickness variation and aspect ratio on the frequency parameter Ω for p = 1 has been investigated. The values of various plate parameters are taken as follows: Winkler stiffness parameter K = 0.0 (0.01) 0.1, shear stiffness parameter G = 0.0 (0.001) 0.01, non-homogeneity parameter μ = -0.5 (0.1) 1.0, density parameter β = -0.5 (0.1) 1.0, taper parameter α = -0.5 (0.1) 1.0 and aspect ratio a/b = 0.5 (0.5) 2.0. The elastic constants for the plate material 'ORTHO1' are taken as $E_1 = 1\times10^{10}\, MPa,\ E_2 = 5\times10^{9}\, MPa,\ \nu_x = 0.2,\ \nu_y = 0.1$. The thickness h_0 at the edge $X = 0$ has been taken as 0.1. To choose the appropriate number of collocation points m, convergence studies have been carried out for different sets of parameters. For a specified plate, graphs are shown in figures 1(a, b) for μ = 0.5, α = 0.5, β = -0.5, K = 0.02, G = 0.001 and a/b = 1 for C-C and C-S plates, respectively. For this data, maximum deviations were observed. In all the computations we have fixed m = 18 because further increase in m does not improve the results even in the fourth place of decimal in the third mode of vibration for all the plates.



(a) m (number of nodes) → (b) m (number of nodes) →

Fig. 1. Percentage error in frequency parameter Ω; (a) C-C plate, and (b) C-S plate, for *a/b*= 1.0, *K*= 0.02, g = 0.001, μ= 0.5, β= -0.5, α = 0.5, ———, first mode, ············, second mode, ----------, third mode. % error = [$(\Omega_m - \Omega_{18})/\Omega_{18}$] × 100.

0.0, 0.02 and shear stiffness parameter G = 0.0, 0.002 for C-C and C-S plates vibrating in fundamental mode. The frequency parameter Ω is found to increase with the increasing values of non-homogeneity parameter μ . The rate of increase of Ω with μ is smaller for a C-S plate than that for a C-C plate. This rate decreases with the increase in the value of density parameter β and foundation parameters *K* as well as *G*. A similar behavior is observed for the plate vibrating in second and third modes of vibration (Figs. 2(b, c)). The rate of increase of Ω with μ gets pronounced with the increasing number of modes.

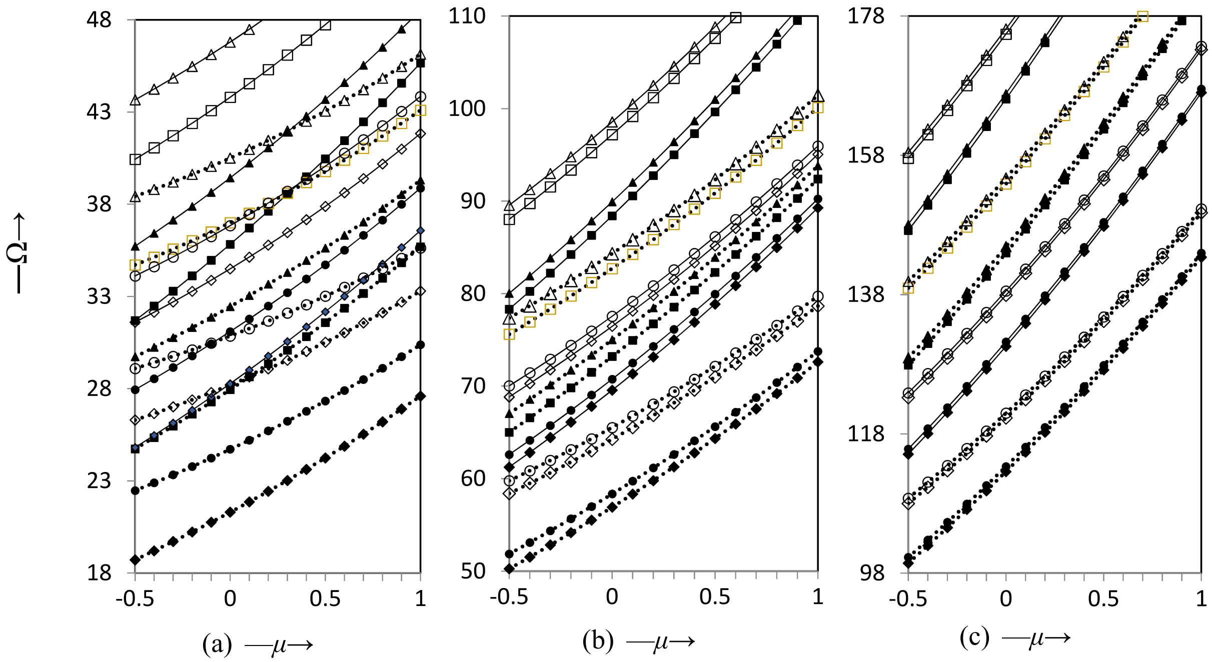
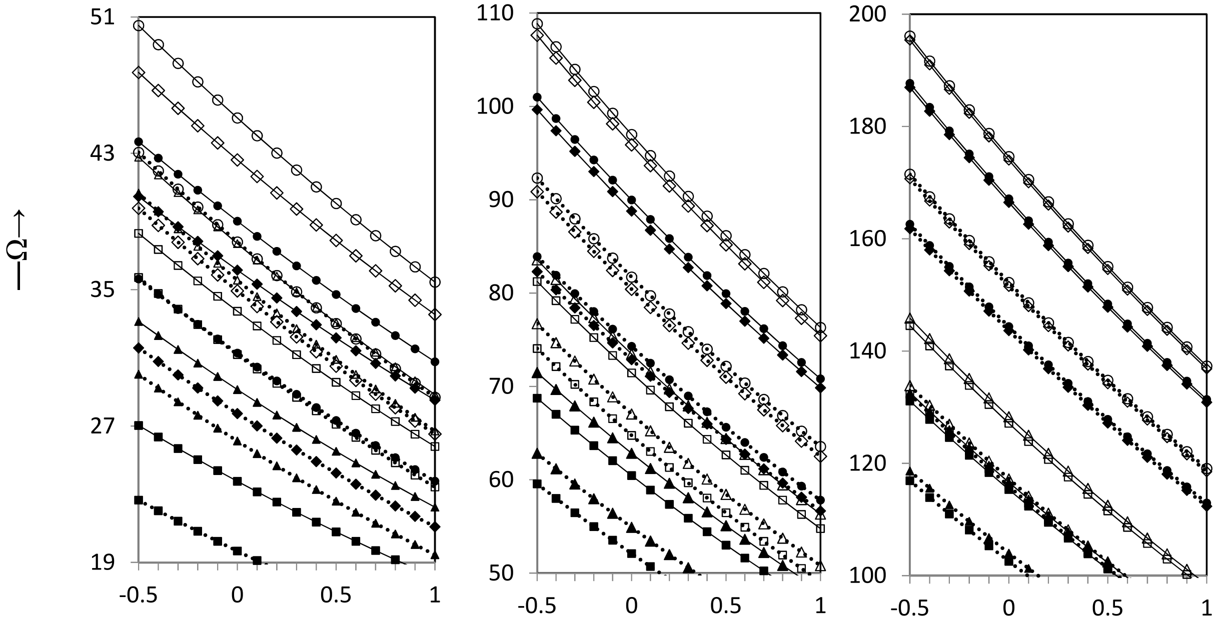


Fig. 2. Frequency parameter for C-C and C-S plates vibrating in (a) first mode (b) second mode and (c) third mode for α = 0.5, *a/b* = 1. ————, C-C; ----------, C-S. ■, □, β = -0.5, *K*=0.0 ; ♦,◊, β = 0.5, *K*=0.0; ▲, △, β = -0.5, *K*=0.02; •,o, β = 0.5, *K*=0.02. ■, ♦,▲,•, *G*=0.0; □,◊, △,o, *G* = 0.002.

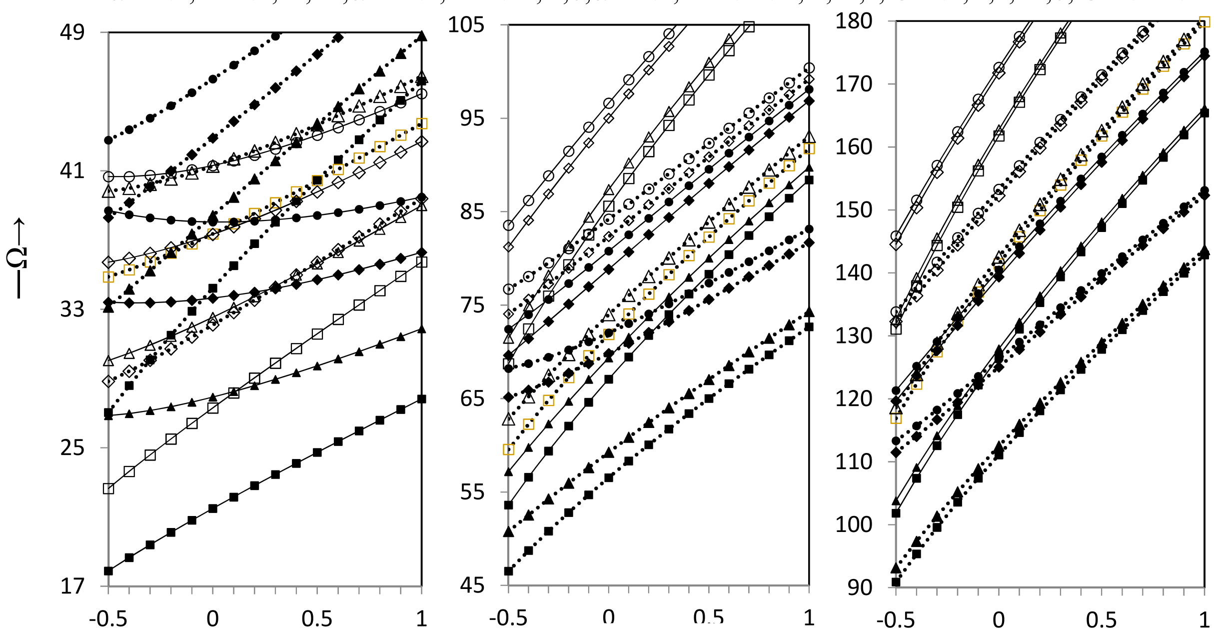
Figs. 3(a, b, c) show the variation of frequency parameter Ω with density parameter β for μ = 0.5, *a*/*b* =1, α = -0.5, 0.5, *K*= 0.0, 0.02 and *G* = 0.00, 0.002 for C-C and C-S plates vibrating in the fundamental, second and third modes, respectively. It is observed that frequency parameters (Ω) decreases with the increasing values of density parameter β irrespective of the values of other plate parameters. The rate of decrease of frequency parameter Ω with β increases with the increase in the values of α, *K* as well as *G*. This rate of decrease is greater for a C-C plate than that for a C-S plate. Also, the rate of decrease in frequency parameter Ω increases with the increase in the number of modes. The effect of taper parameter α on the frequency parameter Ω for C-C and C-S plates has been shown in Figs. 4(a-c) for *a*/*b* = 1, β = -0.5, μ = -0.5, 0.5, *K* = 0.0, 0.02 and *G* =0.0, 0.002 for fundamental, second and third modes of vibration, respectively. It is observed that the frequency parameter Ω increases with the increasing values of taper parameter α for C-C and C-S plates for all the three modes except in case of C-S plate vibrating in fundamental mode for μ = 0.5, *K* = 0.02 and *G* = 0.00. In this case the frequency parameter Ω first decreases and then increases with the increasing values of α with a local minima in the vicinity of α = 0.1. The rate of increase of frequency parameter Ω increases with the increasing values of μ, *K* as well as *G*. This rate of increase of Ω is more prominent in case of

C-C plate as compared to C-S plate in all the modes of vibrations. Also, this rate of increase of Ω with α increases with the increasing number of modes.



(a) —β→ (b) —β→ (c) —β→

Fig. 3. Frequency parameter for C-C and C-S plates vibrating in (a) first mode (b) second mode and (c) third mode, for μ = 0.5, a/b = 1. ————, C-C; -----------, C-S; ■, □, α = -0.5, K=0.0 ; ♦,◊, α = 0.5, K=0.0; ▲, △ ,α = -0.5, K=0.02; •,o,α = 0.5, K=0.02. ■, ♦, ▲,•, G=0.0; □,◊, △ ,o, G=0.002.



(a) —α→ (b) —α→ (c) —α→

Fig. 4. Frequency parameter for C-C and C-S plates vibrating in (a) first mode (b) second mode and (c) third mode for β = 0.5, a/b = 1. ————, C-C; ----------, C-S; ■, □, μ = -0.5, K=0.0; ♦,◊, μ = 0.5, K=0.0; ▲, △ , μ= -0.5, K=0.02; •,o, μ= 0.5, K=0.02. ■, ♦, ▲,•, G=0.0; □,◊, △ ,o, G = 0.002.

Figs. 5(a-c) show the behavior of frequency parameter Ω with aspect ratio a/b for β = -0.5, 0.5, K =0.0, 0.02, G =0.0, 0.002, μ = 0.5 and α = -0.5 for C-C and C-S plates vibrating in fundamental, second and third mode of vibration, respectively. It is observed that the frequency parameter Ω increases with the increasing values of aspect ratio a/b whatever are other plate parameters. The rate of increase of Ω with a/b is much pronounced for a/b >1 than that for a/b < 1. This rate of increase decreases with the increasing values of β, K as well as G. Also, The rate of increase of Ω with a/b is greater in case of C-C plate as compared to C-S plate. The rate of increase decreases for higher and higher modes.

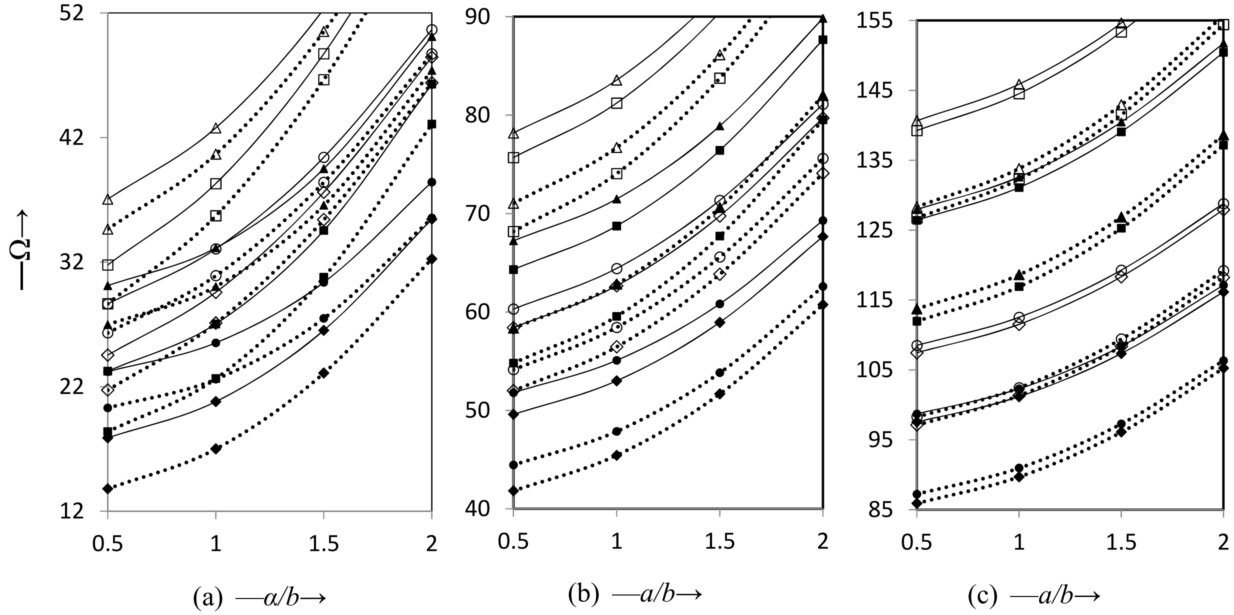


Fig. 5. Frequency parameter for C-C and C-S plates vibrating in (a) first mode (b) second mode and (c) third mode for α = -0.5, μ = 0.5. ————, C-C; ----------, C-S. ■, □, β = -0.5, K=0.0 ; ♦,◊, β = 0.5, K=0.0; ▲, △ , β = -0.5, K=0.02; •,o, β = 0.5, K=0.02. ■, ♦, ▲,•, G = 0.0; □,◊, △ ,o, G = 0.002.

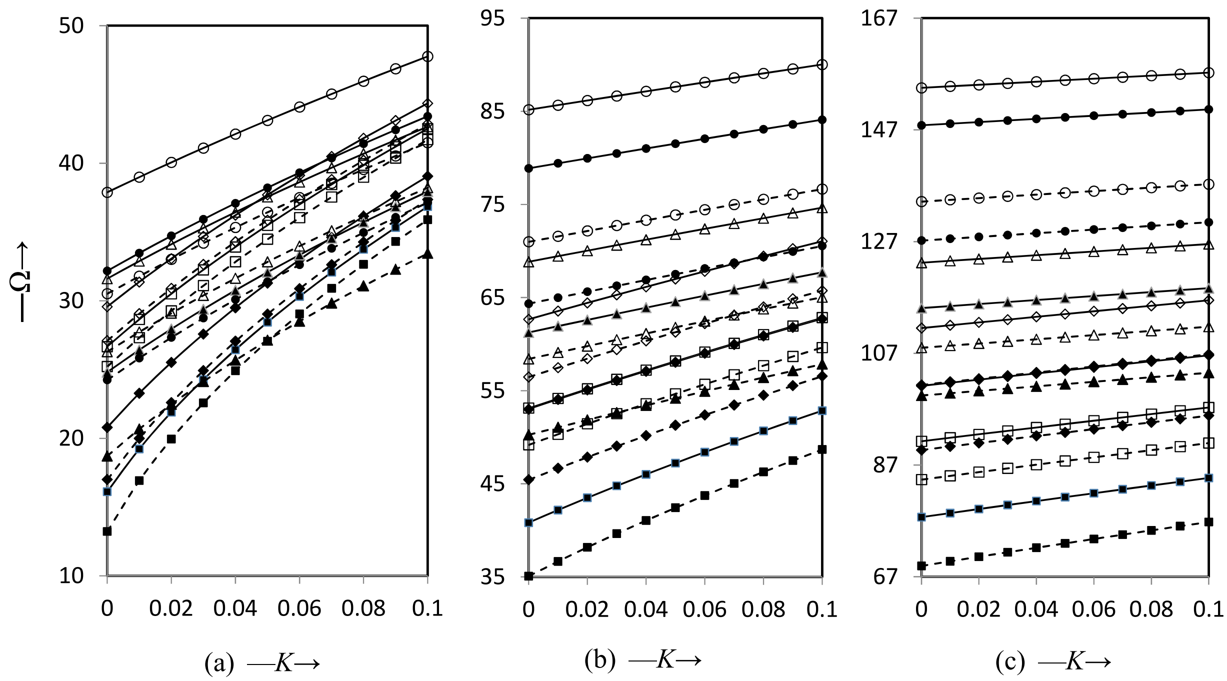


Fig. 6. Frequency parameter for C-C and C-S plates vibrating in (a) first mode (b) second mode and (c) third mode for β = 0.5, a/b = 1. ————, C-C; ----------, C-S; ■, □, α = -0.5, μ = -0.5 ; ♦,◊, α = -0.5, μ = 0.5; ▲, △ , α = 0.5, μ = -0.5; •,o, α = 0.5, μ = 0.5. ■, ♦, ▲,•, G=0.0; □,◊, △ ,o, G = 0.002.

Figs. 6(a-c) show the plots of the frequency parameter Ω versus Winkler foundation stiffness K for α = -0.5, 0.5, μ = -0.5, 0.5, G =0.00, 0.002, β =0.5 and a/b=1 for C-C and C-S plates vibrating in fundamental, second and third mode, respectively. It is seen that the frequency parameter Ω increases with the increasing values of K. The rate of increase of frequency parameter Ω with K is smaller for a C-S plate than that for a C-C plate. The rate of increase of Ω with K decreases with the increasing number of modes. Figure 7(a) depicts the variation of frequency parameter Ω with shear stiffness parameter G for α = -0.5, 0.5, μ = -0.5, 0.5, K=0.0, 0.02, β = 0.5 and a/b=1 for C-C and C-S plates vibrating in fundamental mode. The frequency parameter Ω increases with the increasing values of shear stiffness parameter G. The rate of increase of frequency parameter Ω with G is higher for a C-C plate than that for a C-S plate. A similar inference can be drawn from Figs. 7(b, c), when the plate is vibrating in second and third mode of vibration, respectively, with the exception that the rate of increase in Ω with shear stiffness parameter G increases with the increasing number of modes.

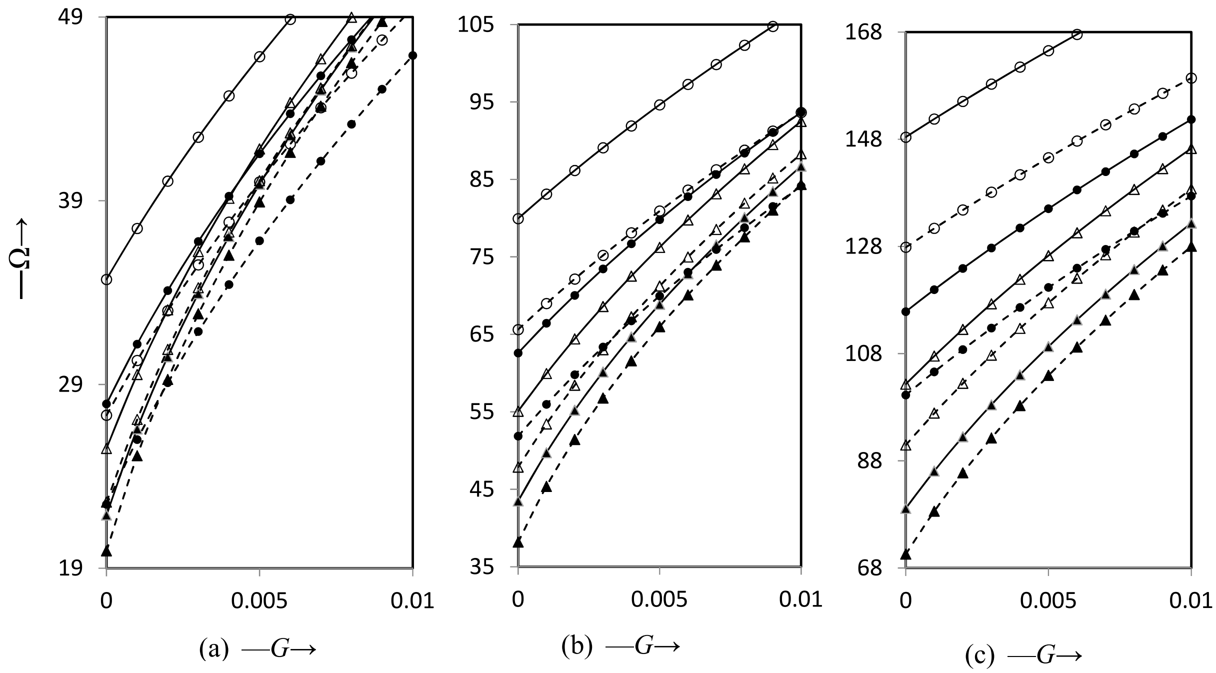


Fig. 7. Frequency parameter for C-C and C-S plates vibrating in (a) first mode (b) second mode and (c) third mode for β = 0.5, a/b = 1. ————, C-C; ----------, C-S; ▲, △, α = -0.5, K=0.02; ●,o, α = 0.5, K=0.02. ▲,●, μ = -0.5; △,o, μ = 0.5.

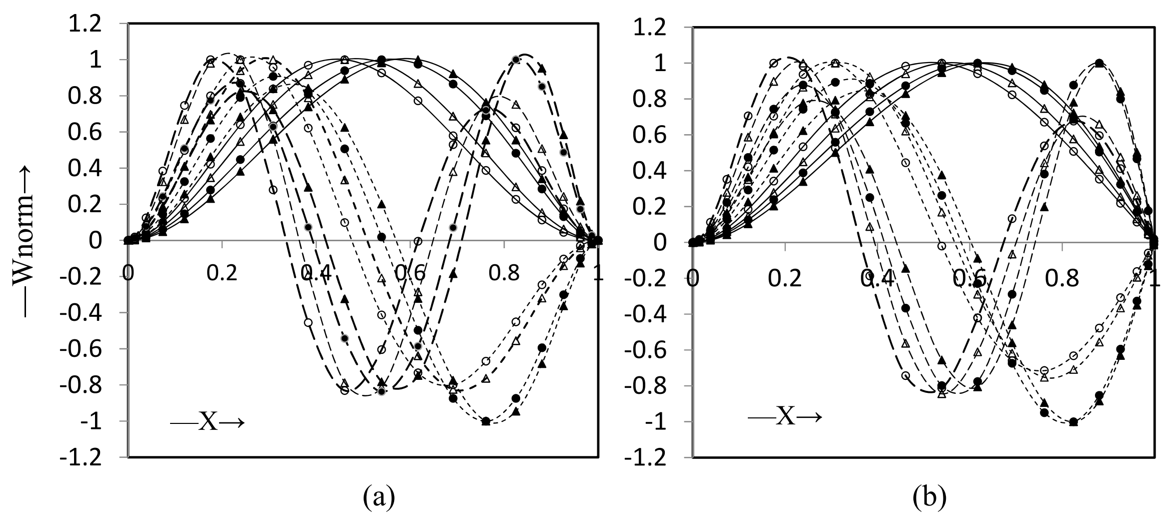


Fig. 8. Normalized displacements for the first three modes of vibration for (a) C-C and (b) C-S plates, for a/b=1.0, α =0.5, K = 0.02,G =0.001. ———— , first mode; ………, second mode;

----------, third mode. △, μ = -0.5; ○, μ = 0.5; ▲, ●, β = -0.5; △,○, β = 0.

Mode shapes for a square plate i.e. a/b=1 have been computed for β = 0.5, K = 0.02, G=0.001, μ = -0.5, 0.5 and α = -0.5, 0.5. Normalized displacements for first three modes of vibration are shown in Figs. 8 (a, b) for C-C and C-S plates, respectively. It is observed that the nodal lines shift towards the edge X = 0, as α increases from -0.5 to 0.5. Also, the radii of nodal circle decrease as μ increases from -0.5 to 0.5. A comparison of results for isotropic (E_2/E_1=1), homogeneous (μ=β= 0), uniform thickness (α = 0.0) plates with Chebyshev collocation technique (Lal et al. 2001), quintic splines technique (Lal & Dhanpati, 2007), finite element method, Frobenius method (Jain & Soni, 1973) and exact solutions by (Leissa, 1969) for two values of aspect ratio a/b=0.5 and 1.0, υ = 0.3 and p = 1 has been presented in Table 1. A close argument between the results is found, which shows the versatility of DQM.

Table 1. Comparison of frequency parameter Ω for isotropic (E_2/E_1=1), homogeneous (μ=β= 0), C-C and C-S plates for υ= 0.3.

Boundary Condition	Mode	K= 0.0 a/b = 0.5	K= 0.0 a/b = 1.0	K= 0.01 a/b = 0.5	K= 0.01 a/b = 1.0
		23.8156	**28.9509**	**26.2142**	**30.9540**
	I	23.820[a]	28.950[a]	26.219[a]	30.953[a]
		23.816[b]	28.951[b]	26.214[b]	30.954[b]
		—	28.946[c]	—	—
C-C		23.816[d]	28.951[d]	—	—
	II	**63.5345**	**69.3270**	**64.4720**	**70.1872**
		63.603[a]	69.380[a]	64.539[a]	70.239[a]
		63.635[b]	69.327[b]	64.472[b]	70.187[b]
		—	69.320[c]	—	—
		63.535[d]	69.327[d]	—	—
	I	**17.3318**	**23.6363**	**20.5034**	**26.0605**
		17.335[a]	23.647[a]	20.506[a]	26.061[a]
		17.332[b]	23.646[b]	20.503[b]	26.060[b]
		—	23.646[c]	—	—
C-S		17.332[d]	23.646[d]	—	—
	II	**52.0979**	**58.6464**	**53.2372**	**59.6607**
		52.150[a]	58.688[a]	53.288[a]	59.702[a]
		52.098[b]	58.646[b]	53.237[b]	59.661[b]
		—	58.641[c]	—	—
		52.097[d]	58.646[d]	—	—

(a). Values from Spline technique **(b).**Values by Chebyshev collocation technique **(c)**. Exact values from Liessa (1969) **(d).** Frobenius method **(e)**. Values from finite element method.

6. Conclusion

The present work emphases on the application of differential quadrature method. For this purpose, the effects of plate parameters on natural frequencies of rectangular orthotropic plates of non-linearly varying thickness resting on two parameter foundation (Pasternak foundation) have been studied on the basis of classical plate theory. It is observed that frequency parameter Ω increases with the increase in non-homogeneity parameter μ and aspect ratio a/b keeping other plate parameters fixed. Further Ω is found to decrease with the increasing value of density parameter β keeping all other plate parameters fixed for all the three boundary conditions. However, its behavior with taper parameter α is not monotonous. It is appeared that the parameter K and G of the Winkler and Pastenak foundation has been found to have a significant influence on the displacements of the plates. In fact, similar results were previously found. Consequently, by comparing the computed results with those available in published works, the present analysis by the DQM is examined and a very good agreement is observed.

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