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Free vibration analysis of thin circular and annular plate with general boundary conditions

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ABSTRACT

This paper presents a numerical analysis of free vibration of thin circular and annular plate using finite element method. The first five natural frequencies are presented for uniform annular plates of various inner-to-outer radius ratios, with nine possible combinations of free, clamped and simply supported boundary conditions at the inner and outer edges of plates. The accuracy of the method is established by comparing the results available in the literature. Results show that natural frequency parameter increases as the inner-to-outer radius ratio increases except in case of free boundary condition, for which it decreases with the inner-to-outer radius ratio. This result provides benchmark values that can be used to validate result obtained by other approximate approaches such as finite difference method, differential quadrature method and boundary element method.

1. Introduction

Circular and annular plates are the fundamental structural elements used in various engineering fields. The lateral vibration of such plates has been the subject of numerous studies. Yuan and Dickinson (1996) studied the natural frequency parameters for the free vibration of annular, circular and sectorial plates using a Ritz solution. Vera et al. (1998, 1999) studied free vibration of annular plates with four combinations of boundary conditions Case (i) clamped at both edges, Case (ii) clamped at outer edge and simply supported at inner edge, Case (iii) simply supported at outer edge and clamped at inner edge and Case (iv) simply supported at both edges and also the free-free edge conditions.

Chakraverty and Petyt (1999) studied elliptical and circular plates with seven types of orthotropic material properties for all the classical free, simply supported and clamped boundary conditions using the Rayleigh–Ritz method with two-dimensional boundary characteristic orthogonal polynomials as the admissible functions. They presented an exhaustive graphical result of the first five frequencies for various aspect ratios. Chakraverty et al. (2000, 2001) also studied the orthotropic annular elliptic plates.

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Their study contains results for the first eight frequency parameters for various values of aspect ratios of the outer and inner ellipse. Wu et al. (2002) studied the free vibration of solid circular plates using the generalized differential quadrature rule (GDQR). Kim (2003) analyzed natural frequency parameters for isotropic elliptical and circular plates using the Rayleigh–Ritz method. Farag and Pan (2003) used the assumed method to thoroughly investigate the in-plane modal characteristics of a solid circular disk with clamped outer boundary.

Wang (2003) studied the vibration of an annular membrane attached to a free rigid core. Wang et al. (2005) studied the effect of core on the fundamental frequencies of annular plates with four different types of ideal boundary conditions. Bashmal et al. (2009) used the Rayleigh-Ritz method with boundary characteristic orthogonal polynomials (BCOP) to study the free in-plane vibrations of annular plates for different combinations of inner/outer edge conditions. The latter authors subsequently derived the exact frequency equations, based on the 2D linear plane stress theory, in terms of Bessel functions (2010). Hassani et al. (2010) employed a Rayleigh-Ritz approach with two-dimensional BCOP and linear plane stress theory to investigate the in-plane modal characteristics of annular circular and elliptic plates of non-uniform thickness for all classical boundary conditions.

Komur et al. (2010) carried out a buckling analysis for laminated composite plates with an elliptical/circular hole centered in the plate using finite element method (FEM) using ANSYS finite element software. Chen and Ren (1998) studied the lateral vibration of thin annular and circular plates with variable thickness using finite element analysis. Liang et al. (2007) extended a new method using the limited finite element method (FEM) for the analysis of the natural frequencies of circular/annular plates of polar orthotropy, stepped and variable thickness. Moreover, free vibration analysis of different geometries and materials have also been studied numericaly or theoretically in recent years (Torabi et al. 2013; Nia et al. 2014; Vimal et al. 2014; Yadav et al. 2015; Bhardwaj et al. 2015; Samaei et al. 2015). In this paper, the effects of different radii ratio, with nine possible combinations of free, clamped and simply supported boundary conditions at the inner and outer edges of plates on the free vibration responses are discussed in detail.

2. Material and Methods

A finite element analysis was made for obtaining the first five natural frequencies using ANSYS. The free vibration is computed using Block-Lanczos algorithm. In addition SHELL 181 is suitable for analyzing thin to moderately-thick shell structures. As demonstrated in Fig. 1, the element contains four-node with six degree of freedom at each node. SHELL 181 is well suited for linear, large rotation, and/or large strain nonlinear applications.

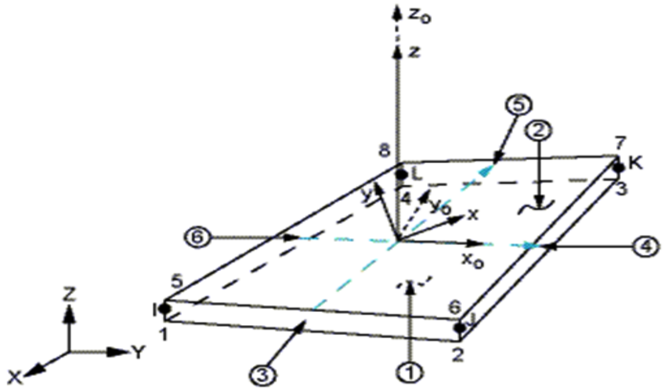


Fig. 1. Four noded SHELL 181 element

Consider an isotropic, homogeneous annular plate with uniform thickness h in cylindrical coordinate (r, θ, z) with the z-axis along the longitudinal direction R_1 and R_2 are the inner radius and outer radius as shown in Fig. 2.

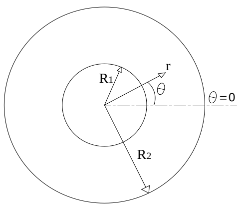


Fig. 2. Geometry and coordinate system of the annular plate

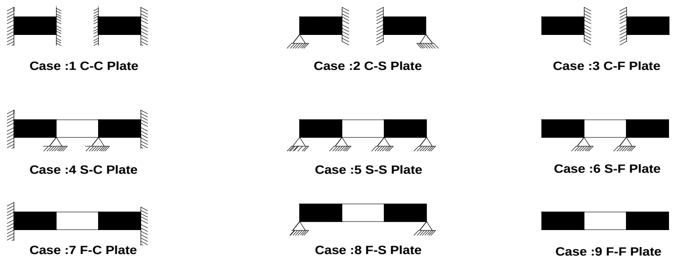


Fig. 3. The boundary conditions of the annular plates analyzed

In this study, isotropic plates made of steel were used. The thickness of plate remains uniform throughout the study. The mechanical properties of steel are listed in Table 1.

Table 1. Mechanical properties of the steel (Reddy 2004)

E_1	E_2	G_{12}	G_{13}	G_{23}	μ
30	30	11.24	11.24	11.24	0.29

Moduli are in msi=million psi ; 1 psi = 6.895 kN/m^2

3. Results and discussion

3.1 Circular plate

The present study is first validated by carrying out convergence study of non-dimensional frequency parameter Ω defined by $\Omega = \omega R_2\sqrt{\rho h/D}$, with respect to mesh dimensions (M × N) and by comparison with results available in the literature. The rates of convergence of the first five frequency parameter for free, clamped and simply supported boundary conditions are presented in Tables 2 to 4. It can be seen that M=12, N=12, is sufficient for converged result. The first five natural modes of flexural vibrations for free, clamped and simply plates are shown in Fig. 4. It can be noted that for free plate mode (1, 0) has higher frequency than mode (1, 2), as well as mode (1, 1) than (1, 3).

Table 2. Value of frequency parameter $\Omega = \omega R_2\sqrt{\rho h/D}$ for circular plate with clamped boundary condition.

NxM	Mode Number				
	1	2	3	4	5
6x6	10.424	22.443	38.613	48.544	58.704
7x7	10.310	22.191	37.554	45.416	57.600
8x8	10.293	21.772	36.900	45.104	55.240
9x9	10.253	21.78	36.318	43.808	54.116
10x10	10.192	21.408	35.504	42.636	52.976
11x11	10.186	21.396	35.469	42.100	52.364
12x12	10.148	21.454	35.694	41.136	52.952
Chakraverty et al. (2001)	10.220	21.260	34.880	39.770	51.030
Zhou et al. (2011)	10.2158	21.260	34.877	39.7711	51.0306

Table 3. Value of frequency parameter $\Omega = \omega R_2\sqrt{\rho h/D}$ for circular plate with simply supported boundary condition

NxM	Mode Number				
	1	2	3	4	5
6×6	4.9228	14.31	27.297	34.354	44.16
7×7	4.9028	14.216	26.823	32.656	43.364
8×8	4.8936	14.002	26.411	32.51	42.176
9×9	4.8852	13.984	26.138	31.807	41.38
10×10	4.8696	13.842	25.718	31.127	40.812
11×11	4.8708	13.83	25.66	30.846	40.392
12×12	4.8664	13.864	25.792	30.285	40.7
Chakraverty et al. (2001)	4.984	13.94	25.65	29.76	34.00*
Zhou et al. (2011)	4.9352	13.8983	25.6148	29.7193	39.9574

Table 4. Value of frequency parameter $\Omega = \omega R_2\sqrt{\rho h/D}$ for circular plate with free boundary condition

NxM	Mode Number				
	1	2	3	4	5
6×6	5.4088	9.2032	12.911	21.484	23.684
7×7	5.3628	9.0912	12.691	21.242	23.009
8×8	5.3408	9.0768	12.592	20.867	22.643
9×9	5.32	9.0404	12.515	20.833	22.358
10×10	5.3132	8.9792	12.467	20.547	22.232
11×11	5.298	8.9688	12.408	20.556	22.092
12×12	5.2936	8.9288	12.38	20.587	21.985
Chakraverty et al. (2001)	5.251	9.076	12.22	20.52	21.49
Zhou et al. (2011)	5.3583	9.0031	12.439	20.4745	21.8352

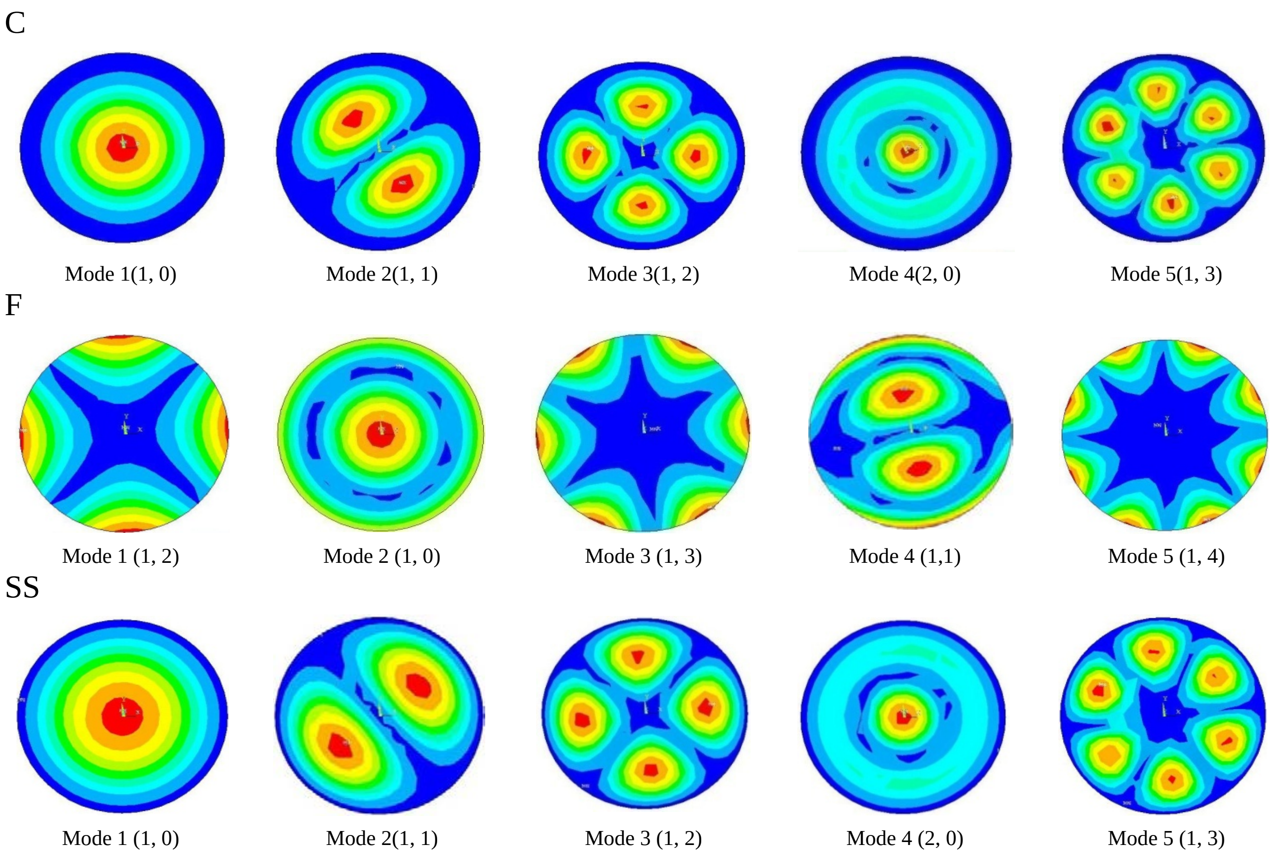


Fig. 4. The first five natural modes of clamped (C), free (F) and simply supported (SS) circular plate

3.2 Annular Plate

The first five non-dimensional frequency parameters, for uniform annular plates of various inner-to-outer radius ratios varying from 0.1 to 0.8 at interval of 0.1 are computed and presented in Tables 5-13. Result are provided for nine cases of boundary conditions (C-S, C-F; S-C, S-S, S-F; F-C, F-S, F-F) at both the inner and outer edges of plates (Fig. 3). Here, the designation C-S identifies a plate with the outer edge clamped and the inner edge simply supported and F-C corresponds to a free outer edge.

Table 5. Value of frequency parameter Ω for C-C plate

	Mode number				
R_1/R_2	1	2	3	4	5
0.1	27.252	28.882	36.607	51.328	70.044
0.2	34.613	36.109	41.828	53.448	70.468
0.3	45.48	46.776	51.26	60.196	74.236
0.4	61.864	62.948	66.552	73.42	84.372
0.5	89.62	90.592	93.66	99.268	107.9
0.6	139.596	140.452	143.088	147.736	154.664
0.7	248.528	249.28	251.576	255.536	261.268
0.8	559.16	559.8	561.84	565.32	570.16

Table 6. Value of frequency parameter Ω for C-S plate

	Mode number				
R_1/R_2	1	2	3	4	5
0.1	10.106	21.154	34.549	39.387	51.104
0.2	10.319	20.46	33.751	42.816	50.576
0.3	11.305	19.364	32.556	49.18	51.648
0.4	13.24	19.612	30.924	46.408	65.632
0.5	17.546	21.73	31.7224	45.456	62.8
0.6	25.468	28.42	36.326	47.928	62.812
0.7	42.86	44.952	50.972	60.416	72.92
0.8	92.556	94.052	98.504	105.816	115.864

Table 7. Value of frequency parameter Ω for C-S plate

	Mode number				
R_1/R_2	1	2	3	4	5
0.1	22.551	25.098	35.330	51.168	65.696
0.2	26.595	29.138	37.567	51.736	69.996
0.3	33.683	35.840	42.708	54.676	71.312
0.4	44.628	46.400	51.956	61.700	75.836
0.5	63.964	65.476	70.128	78.200	89.912
0.6	98.640	99.972	103.884	110.588	120.248
0.7	174.144	175.248	178.588	184.248	192.316
0.8	389.128	390.08	392.968	397.824	404.640

Table 8. Value of frequency parameter Ω for F-C plate

	Mode number				
R_1/R_2	1	2	3	4	5
0.1	3.47096	4.2524	5.506	12.2316	21.5396
0.2	4.808	5.2016	6.3324	12.3832	21.5248
0.3	6.550	6.6884	7.8444	13.0412	21.7496
0.4	8.926	8.9864	10.2048	14.5104	22.366
0.5	13.0636	13.2884	14.582	18.2792	25.1664
0.6	20.5544	20.9228	22.3584	25.652	31.540
0.7	36.9756	37.472	39.110	42.276	47.448
0.8	84.456	85.064	86.944	90.232	95.128

Table 9. Value of frequency parameter Ω for F-F plate

	Mode number				
R_1/R_2	1	2	3	4	5
0.1	5.1904	8.7956	12.2184	20.4072	21.5392
0.2	5.0408	8.4216	12.1760	19.6464	21.4972
0.3	4.8132	8.2964	12.0552	18.1508	21.4724
0.4	4.4596	8.3960	11.6068	16.7792	21.0590
0.5	4.1980	9.2044	11.2432	16.9104	20.7572
0.6	3.8546	10.5164	10.5340	18.1472	19.7876
0.7	3.5139	9.7088	12.9820	18.4272	21.4624
0.8	3.1883	8.8480	16.8488	18.2100	27.1744

Table 10. Value of frequency parameter Ω for F-S plate

	Mode number				
R_1/R_2	1	2	3	4	5
0.1	2.3690	3.4316	5.2996	12.222	20.820
0.2	2.8385	3.3052	5.5272	12.240	21.5048
0.3	3.3130	3.379	5.9456	12.3812	21.5576
0.4	3.5664	3.8638	6.5568	12.618	21.5056
0.5	4.060	4.760	7.7936	13.7384	22.3832
0.6	4.795	6.030	9.6940	15.755	24.2040
0.7	6.0924	8.1648	13.080	19.9432	28.7308
0.8	8.758	12.448	20.098	29.4356	40.1720

Table 11. Value of frequency parameter Ω for S-C plate

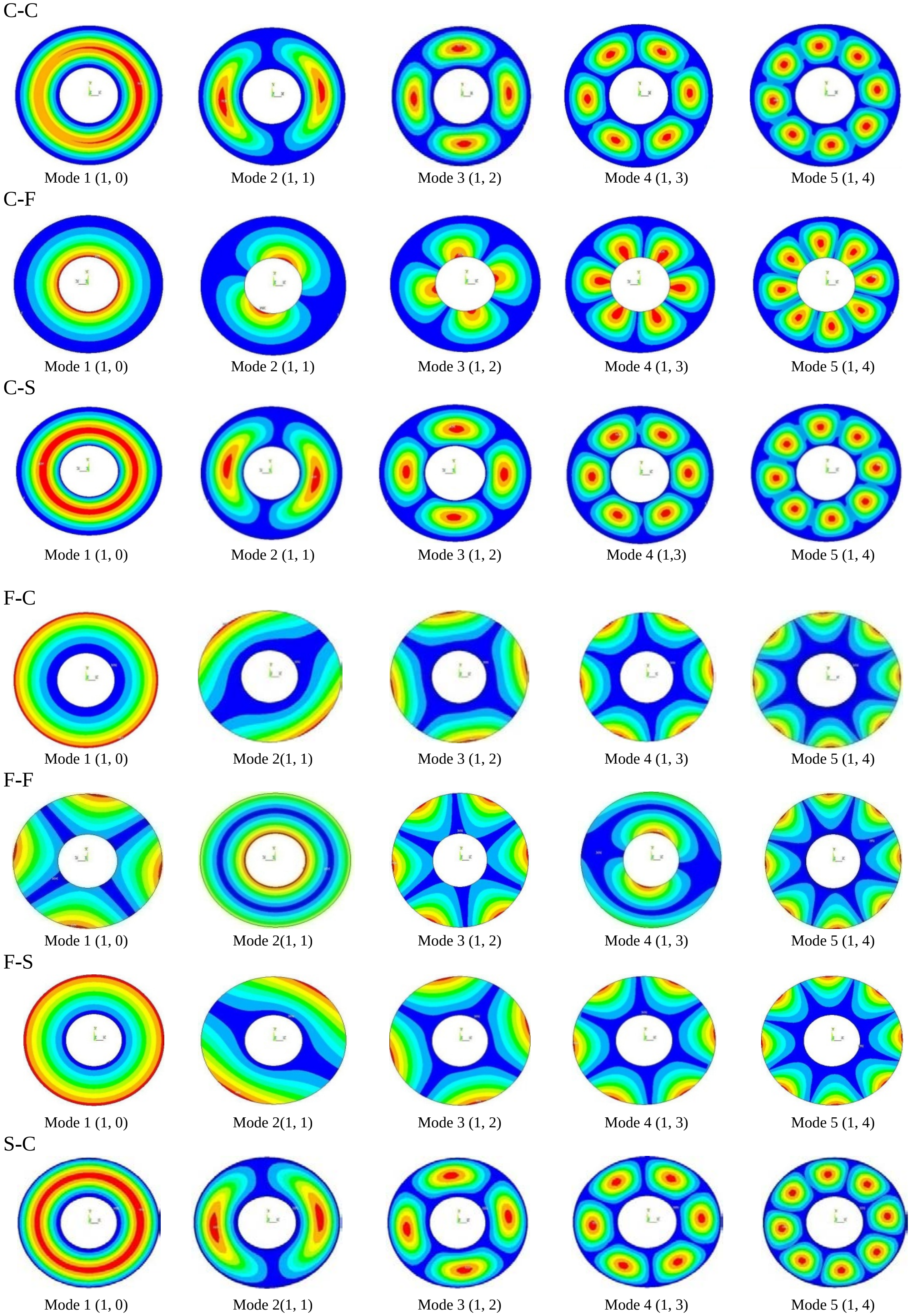
	Mode number				
R_1/R_2	1	2	3	4	5
0.1	17.814	19.4124	26.7436	40.180	57.164
0.2	22.7524	24.3084	30.1192	41.392	57.320
0.3	30.076	31.498	36.330	45.572	59.492
0.4	41.028	42.284	46.372	53.928	65.516
0.5	60.020	61.180	64.808	71.276	80.968
0.6	94.168	95.224	98.452	104.056	112.256
0.7	168.560	169.516	172.424	177.224	184.488
0.8	381.288	382.156	384.804	389.256	395.54

Table 12. Value of frequency parameter Ω for S-F plate

	Mode number				
R_1/R_2	1	2	3	4	5
0.1	4.8764	13.8896	25.4364	29.3612	40.060
0.2	4.7196	13.5604	24.9116	31.2440	39.7288
0.3	4.6460	12.7552	24.1504	36.9076	38.8872
0.4	4.6536	11.7168	22.7908	36.8832	36.8884
0.5	5.0284	11.4416	22.1488	35.4844	51.988
0.6	5.6472	11.6984	22.0988	34.6840	49.832
0.7	6.8448	13.0548	23.8608	36.4640	50.968
0.8	9.4284	16.7356	29.5732	43.9920	59.736

Table 13. Value of frequency parameter Ω for S-S plate

	Mode number				
R_1/R_2	1	2	3	4	5
0.1	14.4144	16.6792	25.9192	40.092	51.78
0.2	16.7048	19.1732	27.2332	40.36	57.068
0.3	21.0264	23.2804	30.2704	41.976	57.748
0.4	27.7664	29.7328	35.734	45.852	59.988
0.5	40.008	41.764	47.056	55.944	68.400
0.6	62.016	63.568	68.224	76.012	86.932
0.7	109.872	111.248	115.38	122.28	131.94
0.8	247.0716	248.28	251.996	258.158	266.790


C-C
Mode 1 (1, 0)
Mode 2 (1, 1)
Mode 3 (1, 2)
Mode 4 (1, 3)
Mode 5 (1, 4)
C-F
Mode 1 (1, 0)
Mode 2 (1, 1)
Mode 3 (1, 2)
Mode 4 (1, 3)
Mode 5 (1, 4)
C-S
Mode 1 (1, 0)
Mode 2 (1, 1)
Mode 3 (1, 2)
Mode 4 (1,3)
Mode 5 (1, 4)
F-C
Mode 1 (1, 0)
Mode 2(1, 1)
Mode 3 (1, 2)
Mode 4 (1, 3)
Mode 5 (1, 4)
F-F
Mode 1 (1, 0)
Mode 2(1, 1)
Mode 3 (1, 2)
Mode 4 (1, 3)
Mode 5 (1, 4)
F-S
Mode 1 (1, 0)
Mode 2(1, 1)
Mode 3 (1, 2)
Mode 4 (1, 3)
Mode 5 (1, 4)
S-C
Mode 1 (1, 0)
Mode 2(1, 1)
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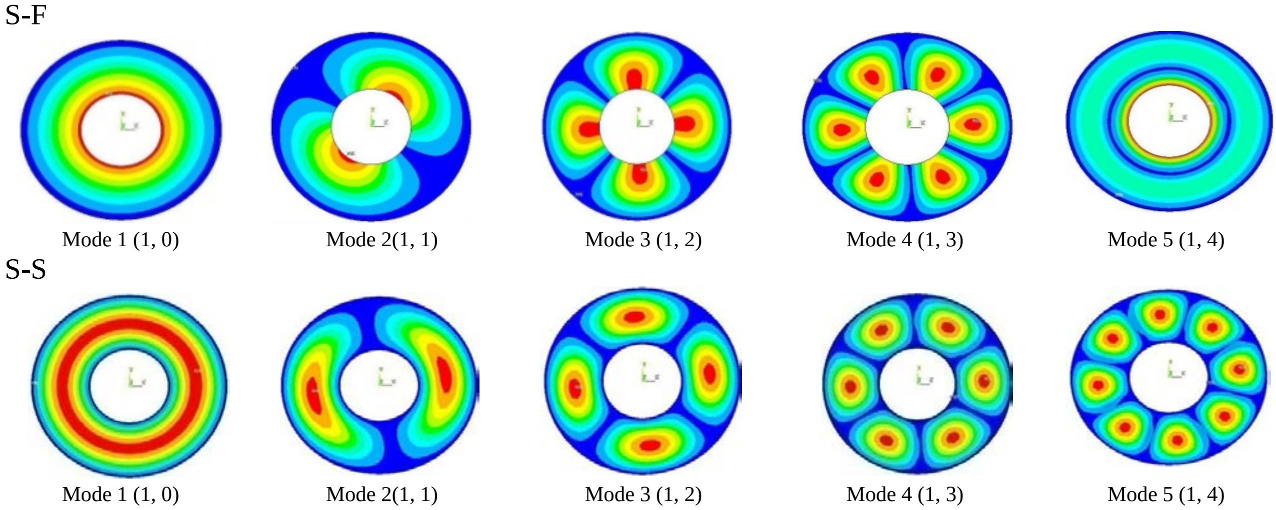

Fig. 5. The first five natural modes, for uniform annular plates with nine possible combinations of free, clamped and simply supported boundary conditions at the inner and outer edges of plates

4. Conclusion

In this paper, finite element method has been employed to solve free vibration of thin, isotropic circular and annular plates. The effects of boundary conditions and hole size on different modes of vibration has been fully investigated and it is found that the natural frequency increases as the hole size increases except when the inner and outer boundaries of the annular plates are free, for which they decrease with the hole size. It is also found that for free plate mode (1, 0) has higher frequency than mode (1, 2), as well as mode (1, 1) than (1, 3) in both circular and annular plates. The numerical results revel that the method is very accurate and can be extended to vibration problems of composite laminated plates which are subjects of investigation nowadays.

References

Bashmal, S., Bhat, R., & Rakheja, S. (2009). In-plane free vibration of circular annular disks. *Journal of Sound and Vibration*, *322*(1), 216-226.

Bashmal, S., Bhat, R., & Rakheja, S. (2010). Frequency equations for the in-plane vibration of circular annular disks. *Advances in Acoustics and Vibration*, *2010*.

Bhardwaj, H., Vimal, J., & Sharma, A. (2015). Study of free vibration analysis of laminated composite plates with triangular cutouts. *Engineering Solid Mechanics*, *3*(1), 43-50.

Chakraverty, S., & Petyt, M. (1999). Free vibration analysis of elliptic and circular plates having rectangular orthotropy. *Structural Engineering and Mechanics*, *7*(1), 53-67.

Chakraverty, S., Bhat, R. B., & Stiharu, I. (2000). Vibration of annular elliptic orthotropic plates using two dimensional orthogonal polynomials. *Applied Mechanics and Engineering*, *5*(4), 843-866.

Chakraverty, S., Bhat, R. B., & Stiharu, I. (2001). Free vibration of annular elliptic plates using boundary characteristic orthogonal polynomials as shape functions in the Rayleigh–Ritz method. *Journal of sound and vibration*, *241*(3), 524-539.

Chen, D. Y., & Ren, B. S. (1998). Finite element analysis of the lateral vibration of thin annular and circular plates with variable thickness. *Journal of vibration and acoustics*, *120*(3), 747-752.

Farag, N. H., & Pan, J. (2003). Modal characteristics of in-plane vibration of circular plates clamped at the outer edge. *The Journal of the Acoustical Society of America*, *113*(4), 1935-1946.

Kim, C. S. (2003). Natural frequencies of orthotropic, elliptical and circular plates. *Journal of sound and vibration*, *259*(3), 733-745.

Komur, M. A., Sen, F., Ataş, A., & Arslan, N. (2010). Buckling analysis of laminated composite plates with an elliptical/circular cutout using FEM. *Advances in Engineering Software*, *41*(2), 161-164.

Hassani, A., Hojjati, M. H., & Fathi, A. R. (2010). In-plane free vibrations of annular elliptic and circular elastic plates of non-uniform thickness under classical boundary conditions. *International Review of Mechanical Engineering*, *4*(1), 112-119.

Liang, B., Zhang, S. F., & Chen, D. Y. (2007). Natural frequencies of circular annular plates with variable thickness by a new method. *International journal of pressure vessels and piping*, *84*(5), 293-297.

Nia, M., Torabi, K., & Heidari-Rarani, M. (2014). Free vibration analysis of a composite beam with single delamination-An improved free and constrained model. *Engineering Solid Mechanics*, *2*(4), 313-320.

Reddy, J. N. (2004). *Mechanics of laminated composite plates and shells: theory and analysis*. CRC press.

Samaei, A. T., Aliha, M. R. M., & Mirsayar, M. M. (2015). Frequency analysis of graphene sheet embedded in an elastic medium with consideration of small scale. *Materials Physics and Mechanics*, *22*, 125-135.

Torabi, K., Afshari, H., & Heidari-Rarani, M. (2013). Free vibration analysis of a non-uniform cantilever Timoshenko beam with multiple concentrated masses using DQEM. *Engineering Solid Mechanics*, *1*(1), 9-20.

Vera, S. A., Sanchez, M. D., Laura, P. A. A., & Vega, D. A. (1998). Transverse vibrations of circular, annular plates with several combinations of boundary conditions. *Journal of Sound and Vibration*, *213*(4), 757-762.

Vera, S. A., Laura, P. A. A., & Vega, D. A. (1999). Transverse vibrations of a free-free circular annular plate. *Journal of Sound and Vibration*, *224*(2), 379-383.

Vimal, J., Srivastava, R., Bhatt, A., & Sharma, A. (2014). Free vibration analysis of moderately thick functionally graded skew plates. *Engineering Solid Mechanics*, *2*(3), 229-238.

Wang, C. Y. (2003). Vibration of an annular membrane attached to a free, rigid core. *Journal of sound and vibration*, *260*(4), 776-782.

Wang, C. Y., & Wang, C. M. (2005). Examination of the fundamental frequencies of annular plates with small core. *Journal of sound and vibration*, *280*(3), 1116-1124.

Wu, T. Y., Wang, Y. Y., & Liu, G. R. (2002). Free vibration analysis of circular plates using generalized differential quadrature rule. *Computer Methods in Applied Mechanics and Engineering*, *191*(46), 5365-5380.

Yuan, J., & Dickinson, S. M. (1996). On the vibration of annular, circular and sectorial plates with cut-outs or on partial supports. *Computers & structures*, *58*(6), 1261-1264.

Yadav, D., Sharma, A., & Shivhare, V. (2015). Free vibration analysis of isotropic plate with stiffeners using finite element method. *Engineering Solid Mechanics*, *3*(3), 167-176.

Zhou, Z. H., Wong, K. W., Xu, X. S., & Leung, A. Y. T. (2011). Natural vibration of circular and annular thin plates by Hamiltonian approach. *Journal of Sound and Vibration*, *330*(5), 1005-1017.