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Free vibration analysis of isotropic plate with stiffeners using finite element method

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A B S T R A C T

This paper presents the free vibration analysis of stiffened isotropic plate by means of finite element method. Stiffeners are used in plates to increase the strength and stiffness. The effect of position of stiffeners on isotropic plate has been studied which involve the possible combination of clamped and free edge condition. The model has been discretized using a 20-node solid element (SOLID186) from the ANSYS element library. The natural frequencies are calculated using Block-Lanczos algorithm. The comparisons of stiffened plate with the available results are found to be in good uniformity. The effect of different boundary conditions, stiffeners location, thickness ratio, stiffener thickness to plate thickness and aspect ratio on the vibration analysis of stiffened plates has been studied.

1. Introduction

Stiffened plate improves the strength to weight ratio and makes the structure cost efficient. Stiffened plates have been widely used in many industrial structures such as aerospace structures, decks and aircraft structures etc. Barton (1951) studied the free vibration problem of skew cantilever plate. Dawe (1966) employed the Parallelogram elements for rhombic cantilever plate problems. Leissa (1973) attempts to present comprehensive and accurate analytical results for the free vibration of rectangular plates. Liu and Chen (1992) investigated the free vibration of a skew cantilever plate with stiffeners by means of a finite element method is described. As the angle of skew is increased, the natural frequency parameters of the modes are generally increased. Mustafa and Ali (1987) studied the application of structural symmetry techniques to the free vibration analysis of cylindrical and conical shells for the prediction of natural frequencies and mode shapes. Gorman (1976) analyzed the first five symmetric and antisymmetric free vibration modes of a cantilever plate for a wide range of aspect ratios. It is shown that it lends itself readily to the entire family of rectangular plates with classical edge conditions: i.e., clamped, and free. Liu and Chang (1990) shows the deflections and natural frequencies of a cantilever plate with stiffeners obtained by a finite element method are presented. Nair and Rao

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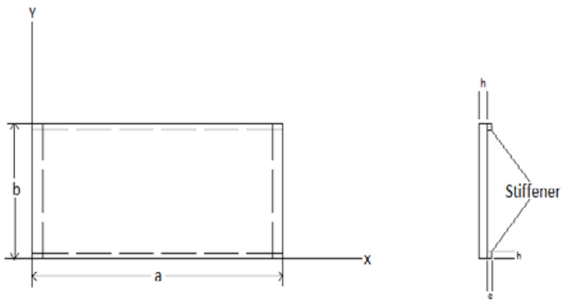
(1984) investigated the effect of stiffener length of a rectangular plate with simply supported or clamped boundary conditions. The effect of this gap between the stiffener tip and the supporting edge on the natural frequencies is investigated hereby using a finite element approach. Liu and Chang (1989) studied to investigate, by a finite element analysis of a cantilever plate, the minimum number of elements in the chord wise direction necessary in order to achieve sufficient accuracy for the first ten Eigen values. The minimum number of elements in the chord wise has been determined, and it has also been found that, with a new, modified element, one chord wise element will provide sufficient accuracy. Mizusawa et al. (1979) presented vibration analysis of isotropic rectangular plates with free edges by the Rayleigh-Ritz method with B-spline functions. Accurate frequencies of rectangular plates are analyzed for different aspect ratios and boundary conditions. The effects of Poisson’s ratio on natural frequencies of square plates with free edges are also investigated. Laura and Gutierrez (1976) deals with the determination of the fundamental frequency of vibration of rectangular plates with edged elastically restrained against rotation. Wu and Liu (1988) analyzed the free vibration of stiffened plates with elastically edges restrained and intermediate stiffeners by using the Rayleigh-Ritz method. Aksu and Ali (1976) studied the free vibration characteristics of rectangular stiffened plates having a single stiffener have been examined by using the finite difference method. Gupta et.al (1986) studied the free vibration characteristics of a damped stiffened panel with applied viscoelastic damping on the flanges of the stiffeners are studied using finite element method. Sivakumaran (1987) concerned with the estimation of natural frequencies of undamaged laminated rectangular plates having completely free edges. The Rayleigh-Ritz energy approach is employed here to obtain the approximate natural frequencies of symmetrically laminated plates. Mukherjee and Mukhopadhyay (1988) studied an isoparametric stiffened plate element is introduced for the free vibration analysis of eccentrically stiffened plates. Bardell and Mead (1989) studied the hierarchical finite element method is used to establish the stiffness and mass matrices of a cylindrically curved rectangular panel. Some natural frequencies and modes of two such panels, each with different boundary conditions, are then determined. The vibration analysis of stiffened plates and the effects of various parameters such as the boundary conditions of the plate, along with orientation, eccentricity, dimensions and number of the stiffeners on free vibration characteristics of stiffened panels have been studied by Hamedani et al. (2012). Samanta and Mukhopadhyay (2004) studied the development of a new stiffened shell element and subsequent application of this element in determining natural frequencies and mode shapes of the different stiffened structures. Qing et al. (2006) studied the free vibration analysis of stiffened laminated plates is developed by separate consideration of plate and stiffeners. The method accounts for the compatibility of displacements and stresses on the interface between the plate and stiffeners, the transverse shear deformation, and naturally the rotary inertia of the plate and stiffeners. Sharma and Mittal (2010, 2011, 2013 and 2014) studied the free vibration analysis of laminated composite plates with elastically restrained edges by applying FEM. In this paper the effects of different geometric parameters i.e. aspect ratio, skew angle, thickness ratio, boundary conditions, stiffener thickness to plate thickness, stiffeners location on the free vibration responses of isotropic plate are studied in detail.



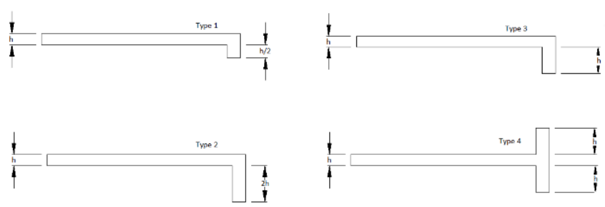


Fig. 1. Sketch of Plate with stiffeners **Fig. 2.** Different types of stiffeners

2. Material Properties

Consider a thin stiffened plate as shown in Fig. 1, here *a* and *b* are geometric dimensions, *h* is the thickness of the plate and α is the skew angle. The stiffeners are located along the upper, lower and all edges of the plate respectively. Fig. 2 shows the different types of stiffeners which are considered for the study of vibration analysis of plate. To show the computational efficiency of FEM, an isotropic stiffened plate has been considered. The default material properties of isotropic plate are

$$E_x = 200e9 \ , \ \nu = 0.3 \ , \ \rho = 7860 kg/m^3 \ , \ \frac{h}{a} = 0.5 , \ \ \frac{a}{b} = 1 , \ \ \frac{e}{h} = 1$$

2.1 Plate Element

The SOLID186 is a higher order 3-D 20-node solid element that exhibits quadratic displacement behavior. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. The element supports plasticity, hyperelasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyperelastic materials. The geometry, node locations, and the element coordinate system for this element are shown in Fig. 3.

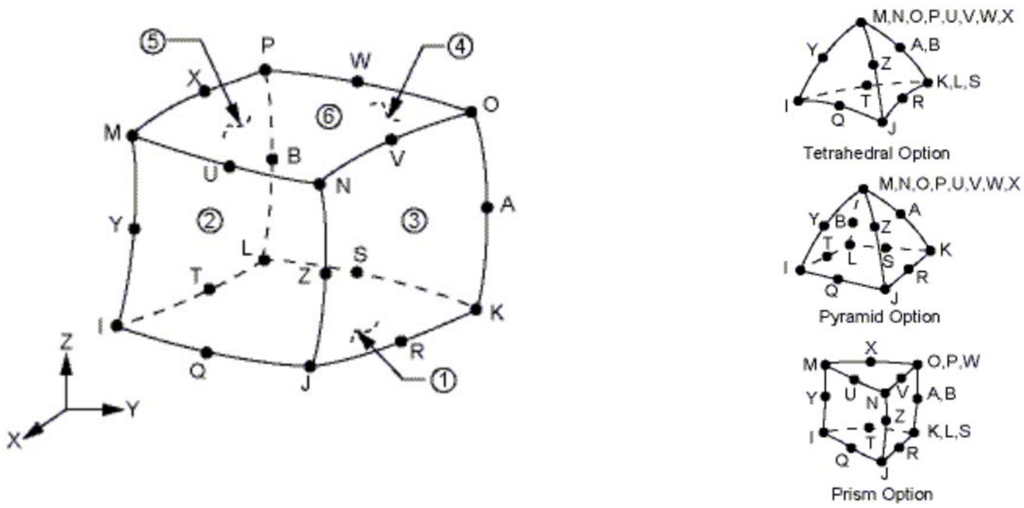


Fig. 3. SOLID186 element

3. Numerical results and discussion

The study shows the vibration analysis of different types of stiffened isotropic plates. Table 1 shows the comparison of non dimensional fundamental frequencies for the isotropic skew plate for the different skew angles. The results are compared with Barton (1951), Dawe (1966), Liessa (1973) and Liu et al. (1992) are found in good conformity.

Table 2 shows the comparison of the first ten non-dimensional frequencies of isotropic skew plate without stiffener for the boundary condition (CFFF- i.e. clamped-free-free-free-free) and the results are compared for the skew angle, α=0, 15, 30, 45 and 60 with the Liu and Chen (1992).

Table 1. Comparison of non-dimensional frequency parameters $\overline{\omega_i} = \omega_i * a^2 \sqrt{\frac{\rho}{D}}$ for skew plates; a/b = 1 and ν = 0.3 (CFFF)

Skew Angle	Mode	Barton (1951)	Dawe (1966)	Leissa (1973)	Liu et al. (1992)	Present results
0	1	3.43	3.47	3.4917	3.4750	3.4692
	2	8.32	8.52	8.5264	8.5176	8.4781
	3	-	21.54	21.429	21.3251	21.2584
	4	-	-	-	-	27.1316
	5	-	-	-	-	30.8420
	6	-	-	-	-	53.9143
15	1	3.44	3.59	-	3.5884	3.5810
	2	8.68	8.71	-	8.7130	8.6678
	3	-	21.59	-	22.2818	22.1968
	4	-	-	-	-	26.2676
	5	-	-	-	-	33.7346
	6	-	-	-	-	51.8408
30	1	3.88	3.95	-	3.9383	3.9249
	2	9.33	9.42	-	9.4618	9.3808
	3	-	25.56	-	25.4436	25.2391
	4	-	-	-	-	25.8722
	5	-	-	-	-	41.1649
	6	-	-	-	-	50.4585
45	1	4.33	4.59	-	4.5469	4.5060
	2	11.21	11.14	-	11.4237	11.2189
	3	-	27.48	-	27.4364	26.9349
	4	-	-	-	-	31..4509
	5	-	-	-	-	50.5120
	6	-	-	-	-	58.8470

Table 2. Comparison of Natural frequency parameters of skew plates without stiffener for different skew angle; a/b = 2 and ν = 0.3 (CFFF).

Skew angle		$\overline{\omega_i} = \omega_i * a^2 \sqrt{\frac{\rho}{D}}$									
α		i=1	2	3	4	5	6	7	8	9	10
0	Liu-Chen	3.4292	14.529	21.340	47.502	60.262	92.116	92.936	119.254	127.096	153.35
	Present	3.4368	14.706	21.408	47.858	60.027	91.895	92.808	118.015	126.103	143.62
15	Liu-Chen	3.4989	14.780	22.218	46.818	63.681	88.986	99.859	123.989	138.168	147.98
	Present	3.5063	14.920	22.292	46.994	63.479	88.468	99.571	122.676	136.602	142.13
30	Liu-Chen	3.7096	15.735	25.041	46.739	73.360	86.541	120.227	138.765	146.644	170.88
	Present	3.7149	15.769	24.994	46.595	72.604	85.283	119.105	135.005	136.803	143.12
45	Liu-Chen	4.0846	17.868	31.004	49.709	88.620	94.427	142.561	164.303	200.859	209.86
	Present	4.0568	17.772	29.944	49.183	84.049	92.104	124.959	133.101	155.345	188.34
60	Liu-Chen	4.7479	22.034	44.834	60.173	107.94	141.330	184.960	244.481	292.234	371.40
	Present	4.5139	21.417	39.798	56.518	96.412	101.990	122.565	149.989	200.530	214.85

Table 3 shows the comparison of the first ten non-dimensional frequencies of isotropic skew plate with stiffener located along upper edge for the boundary condition (CFFF) and the results are compared for the skew angle, α=0, 15, 30 and 45 with the Liu and Chen (1992).

Table 3. Comparison of natural frequency parameters of skew plates with stiffener located along upper edge for different skew angle; a /b = 2 and ν = 0.3 (CFFF)

Skew angle		$\overline{\omega}_i = \omega_i * a^2 \sqrt{\frac{\rho}{D}}$									
α		i=1	2	3	4	5	6	7	8	9	10
0	Liu-Chen	4.3564	14.5808	26.1385	52.9751	67.9598	85.2916	111.8705	120.9210	130.5247	181.087
	Present	3.7741	15.0347	23.4841	50.2775	64.7753	90.5415	100.6826	122.7337	128.3659	145.369
15	Liu-Chen	4.3997	16.0985	25.8962	53.9236	68.5468	91.7526	112.8031	125.2906	136.5247	182.959
	Present	3.9138	15.0861	25.1457	48.6607	69.8191	95.2315	97.5024	127.0699	143.1517	143.945
30	Liu-Chen	4.8835	18.7688	27.7397	54.4217	76.1127	104.707	122.5785	139.2412	162.2579	186.824
	Present	4.2382	15.8747	28.8965	48.0683	79.9602	92.4380	117.9615	137.8693	142.3166	151.840
45	Liu-Chen	6.2117	22.2936	32.8171	58.2128	91.7804	116.0766	159.9638	170.3824	195.4136	246.140
	Present	4.7566	18.0708	35.1617	50.9728	91.3561	102.3323	125.2721	138.7990	172.7521	192.836

Table 4 shows the comparison of the first ten non-dimensional frequencies of isotropic skew plate with stiffener located along upper edge with different type of stiffeners and the results are compared with the Liu and Chen (1992).

Table 4. Comparison of natural frequency parameters of skew plates with different types of stiffener located along upper edge; a/b = 2, ν = 0.3 and α = 30 (CFFF)

Skew angle	$\overline{\omega}_i = \omega_i * a^2 \sqrt{\frac{\rho}{D}}$									
α	i=1	2	3	4	5	6	7	8	9	10
Type1 Liu & Chen (1992)	4.2496	17.9674	25.9397	49.7959	74.8985	93.4623	121.9020	138.3669	154.2560	176.9327
(Present)	3.8709	15.7991	26.2869	46.9698	76.0313	86.6743	118.2289	137.2645	141.0782	143.4808
Type2 Liu & Chen (1992)	4.8835	18.7688	27.7397	54.4217	76.1127	104.7073	122.5785	139.2412	162.2579	186.8242
(Present)	4.2374	15.8715	28.8924	48.0559	79.9314	92.4134	117.9244	137.8651	142.2342	151.7706
Type3 Liu & Chen (1992)	6.7270	19.8983	30.6577	66.2300	81.2645	109.2392	127.0319	143.5953	175.4043	197.9770
(Present)	5.5099	16.3446	34.8157	53.8073	82.4122	108.0014	121.4337	138.6921	142.8308	166.9966
Type4 Liu & Chen (1992)	5.4461	18.4342	29.0272	57.0872	76.2690	105.2199	122.4044	136.9742	164.2620	188.4013
(Present)	4.7241	15.9270	31.8088	49.5534	81.1450	100.5632	117.3979	138.8114	141.0247	157.7153

Table 5 shows the first ten non-dimensional frequencies of isotropic skew plate with stiffener located along all edge for the boundary condition (CFFF), a/b=0.5 and e/h=0.5. It is observed from the results that as the skew angle increases the frequency parameter increases. Table 6 shows the first ten non-dimensional frequencies of isotropic skew plate with stiffener located along upper edge for the boundary condition (CFFF), a/b=1 and a/b=2 and e/h=0.5. It is clear from the results that as the skew angle increases the frequency parameter increases and also observed that as the aspect ratio increases the natural frequency parameter increases.

Table 5. Natural frequency parameters of skew plates with stiffener located along all edges for different skew angle; a /b = 0.5 and ν = 0.3 and e /h = 0.5 (CFFF)

Skew angle	$\overline{\omega_i} = \omega_i * a^2 \sqrt{\frac{\rho}{D}}$									
α	i=1	2	3	4	5	6	7	8	9	10
0	5.1746	8.0972	15.6309	29.7562	32.3681	37.7260	48.9017	51.5120	65.8319	80.3270
15	5.3922	8.3285	15.9372	30.0636	34.2106	39.5215	50.8480	51.5701	68.9170	78.2285
30	6.1213	9.3133	17.1491	31.4403	40.2580	46.0615	52.6389	58.9281	77.6218	79.0033
45	7.3820	12.1758	20.3657	35.3111	49.4434	55.8973	68.2845	75.0380	87.8922	98.1697
60	8.8639	20.9194	29.3528	45.1321	60.4522	74.0790	94.5125	118.297	131.855	143.856

Table 6. Natural frequency parameters of skew plates with stiffener located along upper edge for different skew angle, ν = 0.3 and e /h = 0.5 (CFFF)

Aspect ratio	Skew Angle,	$\overline{\omega_i} = \omega_i * a^2 \sqrt{\frac{\rho}{D}}$									
	α	i=1	2	3	4	5	6	7	8	9	10
	0	5.1473	12.4286	31.5581	38.2464	46.3394	78.8097	88.4056	90.6415	107.397	132.445
	15	5.4007	12.6988	33.2274	38.0961	49.8271	74.7739	93.4980	97.7974	117.240	123.861
a /b = 1	30	6.0625	13.6666	36.2233	41.1216	59.1433	72.1217	103.070	116.673	124.365	141.691
	45	7.1852	16.2194	40.1536	50.2651	72.1217	86.2559	111.377	139.404	156.881	169.893
	60	8.6496	23.5242	48.2646	68.6308	89.3445	122.666	151.601	175.321	198.558	233.787
	0	5.4545	23.152	38.012	77.987	108.866	152.683	177.411	235.001	255.472	334.537
	15	5.7797	23.939	42.998	84.176	135.961	183.171	197.879	320.979	352.984	365.245
a /b = 2	30	6.4510	24.627	52.426	84.713	152.085	200.599	239.523	344.273	362.047	390.922
	45	7.5975	28.841	64.785	96.077	189.415	219.127	300.154	355.934	385.283	442.092
	60	9.2349	38.465	87.041	124.043	222.369	290.149	329.169	400.674	559.364	612.949

Table 7 shows the first ten non-dimensional frequencies of isotropic skew plate with stiffener located along all edge for the boundary condition (CFFF), a/b=1 and e/h=0.5. It is examined from the results that as the skew angle increases the frequency parameter increases.

Table 7. Natural frequency parameters of skew Plates with stiffener located along all edges for different skew angle; a /b = 1 and ν = 0.3 and e /h = 0.5(CFFF)

Skew angle	$\overline{\omega_i} = \omega_i * a^2 \sqrt{\frac{\rho}{D}}$									
α	i=1	2	3	4	5	6	7	8	9	10
0	5.3833	12.921 5	33.520	40.9275	49.4284	83.8429	93.486	96.8877	114.283	141.729
15	5.5525	13.1536	35.0240	40.5448	52.6218	80.4211	99.053	103.715	122.469	132.138
30	6.1591	14.1906	39.9841	41.4024	62.8760	78.4068	116.483	120.530	127.071	150.705
45	7.2830	16.9798	44.5045	50.3877	82.3591	87.2957	127.613	143.356	170.988	181.923
60	8.8127	24.7879	52.6220	70.3529	100.8928	136.1817	160.930	193.262	216.123	269.285

Table 8 shows the first ten non-dimensional frequencies of isotropic skew plate with stiffener located along upper edge for the boundary condition (CCFF- i.e. clamped-clamped-free-free), a/b=0.5 and e/h=0.5. It is clear from the results that as the skew angle increases the frequency parameter increases.

Table 8. Natural frequency parameters of skew plates with stiffener located along upper edge for different skew angle; a /b = 0.5 and ν = 0.3 and e /h = 0.5 (CCFF)

Skew angle	$\overline{\omega_i} = \omega_i * a^2 \sqrt{\frac{\rho}{D}}$									
α	i=1	2	3	4	5	6	7	8	9	10
0	31.8571	34.5943	40.8244	52.0429	69.8811	87.5091	91.8131	95.1477	100.512	114.016
15	33.8159	36.5532	42.6660	53.8617	71.6858	92.8109	96.5697	98.1551	106.673	119.395
30	40.0936	43.6528	49.3005	60.5367	78.4031	101.689	111.775	119.322	127.516	132.380
45	52.6271	59.8236	67.6989	78.7949	96.4687	118.710	139.392	158.372	171.355	178.091
60	81.3049	92.7783	129.752	137.662	151.302	169.694	192.515	220.491	255.070	279.339

Table 9 shows the first ten non-dimensional frequencies of isotropic skew plate with stiffener located along upper edge for the boundary condition (CCCC), a/b=2 and e/h=0.5. It is observed from the results that as the skew angle increases the frequency parameter increases.

Table 9. Natural frequency parameters of skew plates with stiffener located along upper edge for different skew angle; a /b = 2 and ν = 0.3 and e /h = 0.5 (CCCC)

Skew angle	$\overline{\omega_i} = \omega_i * a^2 \sqrt{\frac{\rho}{D}}$									
α	i=1	2	3	4	5	6	7	8	9	10
0	144.54	185.28	258.41	363.78	378.41	418.39	487.39	500.12	587.23	666.61
15	154.29	195.85	269.81	374.49	405.98	445.47	510.53	517.99	623.78	666.71
30	189.35	232.43	308.48	413.80	501.31	524.18	567.56	613.23	706.76	730.16
45	282.36	328.88	409.83	522.05	652.43	753.32	792.11	816.43	926.40	935.68
60	569.67	620.50	710.68	838.71	994.87	1165.66	1345.14	1540.26	1563.31	1629.59

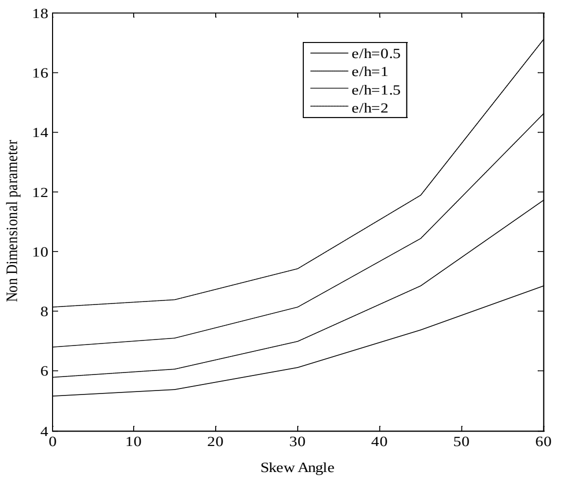


Fig. 4. Variation of frequency parameter with different skew angles for isotropic skew plates with stiffener located at all edges and a/b = 0.5 for CFFF boundary conditions

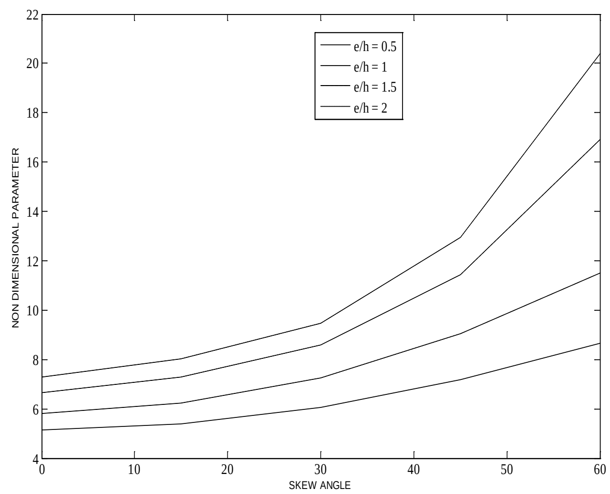


Fig. 5. Variation of frequency parameter with different skew angles for isotropic skew plates with stiffener located at upper edge and a/b =1 for CFFF boundary conditions

Fig. 4 shows the variation of first ten non-dimensional frequencies with different skew angle (α=0, 15, 30, 45 and 60) for a stiffened isotropic plate of aspect ratio a/b=0.5 and stiffener located at all edges and ratio of stiffener to plate thickness (e/h= 0.5, 1, 1.5, 2) for CFFF boundary condition. The frequency in all ten modes increases as the skew angle increases. It also shows that as the ratio of stiffener to plate thickness increases, the fundamental frequencies increases. Fig. 5 shows the variation of first ten non-dimensional frequencies with different skew angle (α=0, 15, 30, 45 and 60) for a stiffened isotropic plate of aspect ratio a/b=1 and stiffener located at upper edges and ratio of stiffener to plate thickness (e/h= 0.5, 1, 1.5, 2) for CFFF boundary condition. The frequency in all ten modes increases as the skew angle increases. It also shows that as the ratio of stiffener to plate thickness increases, the fundamental frequencies increases. Fig. 6 shows the variation of first ten non-dimensional frequencies with different skew angle (α=0, 15, 30, 45 and 60) for a stiffened isotropic plate of aspect ratio a/b=1 and stiffener located at all edges and ratio of stiffener to plate thickness (e/h= 0.5, 1, 1.5, 2) for CFFF boundary condition. The frequency in all ten modes increases as the skew angle increases. It also shows that as the ratio of stiffener to plate thickness increases, the fundamental frequencies increases. Fig. 7 shows the variation of first ten non-dimensional frequencies with different skew angle (α=0, 15, 30, 45 and 60) for a stiffened isotropic plate of aspect ratio a/b=2 and stiffener located at upper edges and ratio of stiffener to plate thickness (e/h= 0.5, 1, 1.5, 2) for CFFF boundary condition. The frequency in all ten modes increases as the skew angle increases. It also shows that as the ratio of stiffener to plate thickness increases, the fundamental frequencies increases.

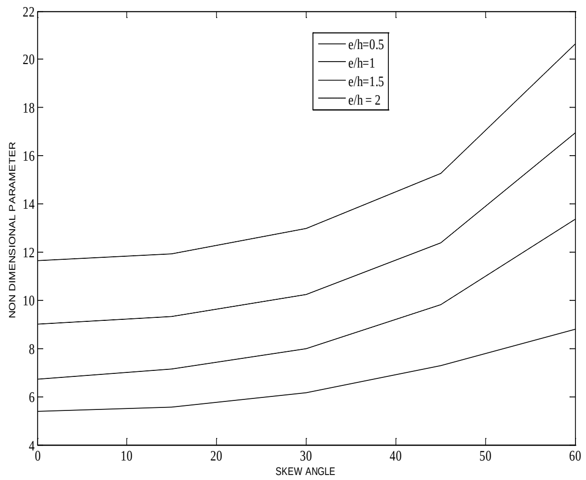


Fig. 6. Variation of frequency parameter with different skew angles for isotropic skew plates with stiffener located at all edges and a/b = 1 for CFFF boundary conditions

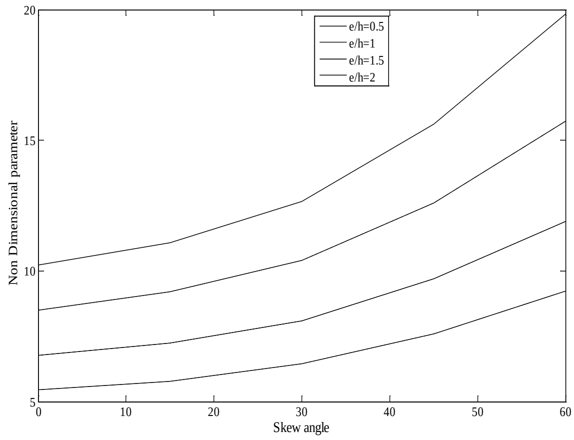


Fig. 7. Variation of frequency parameter with different skew angles for isotropic skew plates with stiffener located at upper edge and a/b = 2 for CFFF boundary conditions

Fig. 8 shows the variation of first ten non-dimensional frequencies with different skew angle (α=0, 15, 30, 45 and 60) for a stiffened isotropic plate of aspect ratio a/b=0.5 and stiffener located at upper edges and ratio of stiffener to plate thickness (e/h= 0.5, 1, 1.5, 2) for CCFF boundary condition. The frequency in all ten modes increases as the skew angle increases. It also shows that as the ratio of stiffener to plate thickness increases, the fundamental frequencies increases. Fig. 9 shows the variation of first ten non-dimensional frequencies with different skew angle (α=0, 15, 30, 45 and 60) for a stiffened isotropic plate of aspect ratio a/b=2 and stiffener located at upper edges and ratio of stiffener to plate thickness (e/h= 0.5, 1, 1.5, 2) for fully clamped boundary condition. The frequency in all ten modes increases as the skew angle increases. It also shows that as the ratio of stiffener to plate thickness increases, the fundamental frequencies increases.

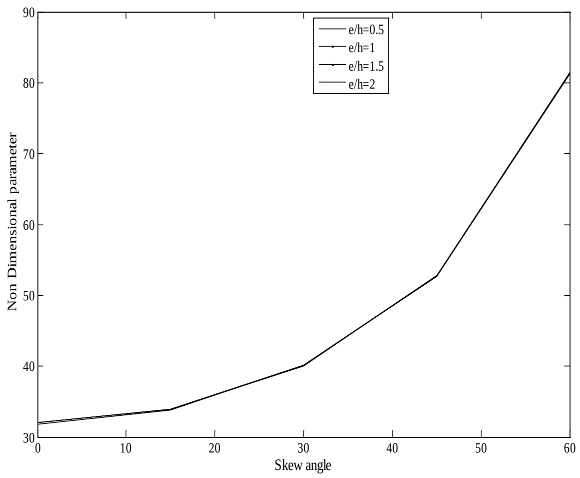


Fig. 8. Variation of frequency parameter with different skew angles for isotropic skew plates with stiffener located at upper edge and a/b = 0.5 for CCFF boundary conditions

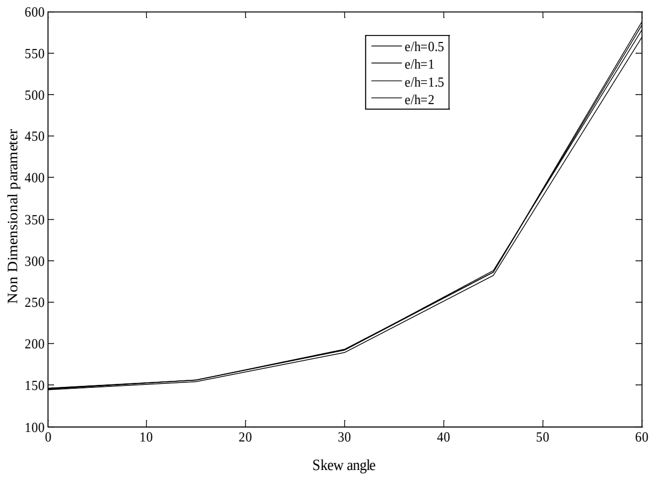


Fig. 9. Variation of frequency parameter with different skew angles for isotropic skew plates with stiffener located at upper edge and a/b = 2 for CCCC boundary conditions

4. Conclusions

The finite element methodology has been used to investigate the free vibration analysis of stiffened plates with stiffeners with adequate results. The all edges clamped boundary conditions gives the higher natural frequencies than the other boundary conditions. From the results it is observed that by increasing the skew angle the natural frequency increases and by increasing the stiffener to the plate thickness the natural frequencies also increases for all the boundary conditions. By providing the stiffener to the plate the natural frequencies of the plate shifted towards the higher side. The effect of various types of stiffeners (Types 1, 2, 3 and 4) on the natural frequencies has been studied.

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