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Study of free vibration analysis of laminated composite plates with triangular cutouts

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A R T I C L E I N F O

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A B S T R A C T

Cutouts are commonly used as access port for mechanical and electrical structures. Most of the structures generally work under severe dynamic loading and different constrained conditions during their service life. This may lead to vibration of the structure. Therefore, it is necessary to predict the vibration responses of laminated composite plates with cutouts precisely with less computational cost and good accuracy of these complex structures. A suitable finite element model is proposed and developed based on first order shear deformation theory using ANSYS parametric design language (APDL) code. The model has been discretized using an appropriate eight nodded element (SHELL 281) from the ANSYS element library. The free vibrations are computed using Block-Lanczos algorithm. The convergence study has been done of the developed model and compared with those available published literature. Effects of different geometric parameters (aspect ratio, thickness ratio, boundary conditions, number of layers, angle of lamina geometry of cutout, cutout side to plate side ratio and distance between cutouts) and material properties on the free vibration responses are discussed in detail. The frequency increases with increase in the number of layers, modulus ratio of plate and angle of lamina. The frequency decreases with increase in aspect ratio, thickness ratio, size of cutout and distance between cutouts. The boundary conditions of the plate play an important role in the free vibrations of the plate with cutouts. The Non-dimensional frequencies are higher for fully clamped boundary condition in comparison to other boundary conditions.

1. Introduction

Composite laminates are assemblies of layers of fibrous composite materials which can be joined to provide required engineering properties, including in-plane stiffness, bending stiffness, strength and coefficient of thermal expansion. Laminated composite structures are increasingly being used in many engineering fields such as aerospace, marine vehicles, automotive, chemical and nuclear engineering. Cutouts are essential for assembling the components, access ports, damage inspection, fuel lines and electrical lines, opening in a structure to serve as doors and windows, provide ventilation, to reduce weight and for accessibility to other parts of the structure. It is required at the

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bottom plate for some passage of liquid in liquid retaining structures. It is also well recognized that these structures are exposed to the undesirable vibration and many more during their service life and again these plate structures having cutout may change the responses considerably. The plates with the cutouts reduces the total weight, which in turn influence on the vibration response. Hence, there is a need to study the vibration behavior of laminated composite structure with cutouts precisely.

The vibration responses of laminated composite plates have been extensively studied by a number of researchers. Some of the selected studies are discussed in the following lines. Civalek (2008) proposed the discrete singular convolution (DSC) technique for free vibration analysis of moderately thick symmetrically laminated composite plates based on the first order shear deformation theory. The results were also compared with those obtained by the other numerical and analytical method. Ngo-Cong et al. (2011) presented radial basis function (RBF) collocation technique for the free vibration analysis of laminated composite plates applying the first order shear deformation theory (FSDT). Several examples concerning various thicknesses to material properties, span ratios and boundary conditions were considered. Cui et al. (2011) investigated the bending and vibration analysis of laminated composite plate using a novel triangular composite plate element based on an edge based smoothing technique. Xiang et al. (2011) investigated the free vibration properties of laminated composite shells using the first order shear deformation theory and a meshless global collocation technique based on thin plate spline radial basis function. Grover et al. (2013) extended the inverse hyperbolic shear deformation theory to investigate the free vibration response of laminated composite and sandwich plates. Boscolo and Banerjee (2014) developed dynamic stiffness technique by using a sophisticated layer-wise theory which complies with the C_z^0 requirements and delivered high accuracy for the analysis of laminated composite plates. Marjanovic and Vuksanovic (2014) extended the layer wise plate theory of Reddy for the analysis of delamination as a basis for development of enriched finite elements. Effects of plate geometry, lamination scheme, degree of orthotropy and delamination size or position on dynamic characteristics of the plate are presented.

Aydogdu and Timarchi (2003) investigated the vibration analysis of cross-ply laminated square plates based on PSDPT with general boundary conditions. Sharma and Mittal (2010, 2011, 2013, 2014) studied the free vibration analysis of laminated composite plates with elastically restrained edges by applying FEM. Karami et al. (2006) developed the free vibration analysis of moderately thick symmetric laminated plates using elastically restrained edges. Liew et al (2003) investigated the free vibration analysis of symmetrically laminated plates based on FSDT using the moving least squares differential quadrature technique. Free vibration analysis of laminated composite rectangular plate was presented in the work of Pandit et al. (2007) by applying FEM. Numerical instances of isotropic and composite rectangular composite plates having various fiber orientation angle, thickness ratio and aspect ratio were also solved. Brethee (2009) studied the free vibration analysis of a symmetric and anti-symmetric laminated composite plate with a cut-out at the center. Lahouel and Guenfoud (2013) investigated the comparative analysis of vibration between laminated composite plates with and without holes under compressive loads. Luccioni and Dong (1998) presented Levi-type (semi-analytical) finite element analysis of free vibration and stability of laminated composite rectangular plates based on both classical and first order shear deformation theories. Xiang et al. (2010) presented a meshless method based on thin plate spline radial basis functions and higher order shear deformation theory to study the free vibration of clamped laminated composite plates. Chen and Lue (2005) studied the free vibration analysis of cross-ply laminated plates with one pair of opposite edges simply supported. Suresh Kumar et al. (2011) developed an analytical procedure to investigate the free vibration characteristics of various laminated composite plates based on higher order shear displacement model with zig-zag function. In aerospace structures, panels with triangular cutouts are often used. Therefor it is important to study the vibration behavior of laminated composite structure with triangular cutout precisely. In this paper the effects of different geometric parameters (aspect ratio, thickness ratio, boundary conditions, number of layers, angle of lamina, geometry of cutout,

cutout side to plate side ratio and distance between cutouts) and material properties on the free vibration responses are discussed in detail. It is shown that the frequency increases with increase in the number of layers, modulus ratio of plate and cutout size.

2. Material and Methods

The finite element software (ANSYS) is implemented with the aim of analyzing. In addition SHELL 281 is suitable for analyzing thin to moderately-thick shell structures. As demonstrated in Fig. 1, the element contains eight nodes with six degree of freedom at each node. SHELL 281 is well-suited for linear, large rotation and/or large strain non-linear applications. SHEEL 281 may be used for layered applications for modeling composite shells or sandwich construction.

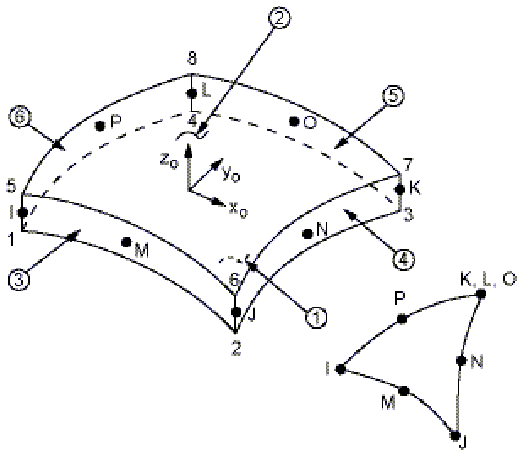

Fig. 1. SHELL 281 element

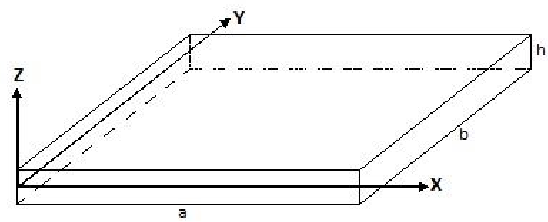

Fig. 2. Geometry of the composite laminated plate

It is mentioned that for free vibration analysis, subspace method is applied. The subspace iteration method was described in detail by Bathe (1996). Enhancements as suggested by Wilson and Itoh (1983) were also included as outlined subsequently.

3. Numerical results and discussion

Different cases of cross-ply composite laminates with triangular cut-out are examined here. Without loss of generality equal number of grid points in x- and y- direction is assumed. To define the boundary conditions along the edges the alphabet symbolism will be used, so that S-C-S-C indicates a plate with edge x=0 simply supported, edge y=0 clamped, edge x=a simply supported and edge y=b clamped, in which the x,y and z axes have been shown in Fig. 2 for the investigated laminate.

3.1 Convergence study

Cross-ply laminated plate

To show the computational efficiency of FEM, a thin square plate composed with three orthotropic layers $(0^0/90^0/0^0)$ is considered. The material properties of each layer are

$$\frac{E_{11}}{E_{22}} = 25, \qquad G_{12} = G_{13} = 0.5E_{22}, \ G_{23} = 0.2E_{22}, \nu_{12} = 0.25, \qquad \rho = 2700\, kg/m^3$$

The accuracy and convergence behaviors of the solutions for a square laminate plate with two orthotropic layers $(0^0/90^0)$ with cut-out at the center (shown in Fig. 3) were investigated, the results of which are shown in Table 1. To compare the solutions the results of Pandit et al. (2007) and

Brethee (2009) are also cited. A good agreement of results is obtained between the present results and the results given by Pandit et al. (2007) and Brethee (2009).

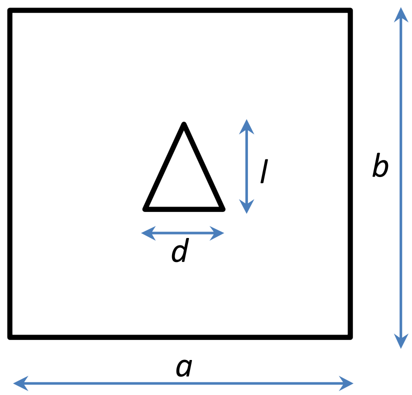

Fig. 3. Square plates with triangular cutout

Angle-ply laminated plate

The material properties of each layer for an angle-ply $(\theta^0/-\theta^0/\theta^0)$ square laminated plate are

$$\frac{E_{11}}{E_{22}} = 40, \qquad G_{12} = G_{13} = 0.6E_{22},\ G_{23} = 0.5E_{22}, \nu_{12} = 0.25, \quad \rho = 2700\ kg/m^3$$

Table 1. Convergence study of non-dimensional frequencies ($\bar{\omega} = \frac{\omega a^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) for a cross-ply laminate $(0^0/90^0)$ having a square cutout at the centre, (K_s(shear correction coefficient)=5/6, b/a=1, h/a=0.01) for boundary condition i.e. CCCC with respect to the results given by Pandit et al. (2007) and Brethee (2009)

	Mode					
Cut-out size	1	2	3	4	5	6
0.2a*0.2a (Present)	9.827	25.833	26.277	40.301	54.818	63.052
Pandit et al. (2007)	9.11	25.63	25.80	38.11	54.23	60.64
Brethee (2009)	9.071	25.057	25.057	37.643	53.154	59.856
0.4a*0.4a (Present)	10.050	21.382	21.984	37.752	46.409	65.219
Pandit et al. (2007)	9.12	20.25	20.34	35.67	44.76	61.81
Brethee (2009)	9.061	19.930	19.930	35.070	42.866	60.270
0.6a*0.6a (Present)	12.402	20.022	20.525	34.749	37.438	56.576
Pandit et al. (2007)	11.31	18.69	18.71	32.81	34.34	53.11
Brethee (2009)	11.085	18.173	18.173	31.560	33.920	51.712
0.4a*0.2a (Present)	9.63	22.35	25.74	39.38	53.40	64.98
Pandit et al. (2007)	8.85	21.31	27.82	39.12	51.22	62.14
Brethee (2009)	8.76	20.57	54.35	36.57	50.83	60.81
0.8a*0.4a (Present)	10.83	13.21	30.58	34.90	55.77	66.97
Pandit et al. (2007)	9.72	11.81	27.36	31.15	50.63	60.37
Brethee (2009)	9.60	11.54	27.03	30.63	49.42	59.27
0.6a*0.2a (Present)	9.57	17.18	25.90	36.97	52.70	62.58
Pandit et al. (2007)	8.54	15.87	25.45	34.87	51.26	60.13
Brethee (2009)	8.49	15.27	25.08	34.02	50.24	59.21

3.2. Vibration of composite laminated plate with triangular cutout

Table 2 shows the variation of first ten non-dimensional frequencies with size ratio (d/b=0.2, 0.3, 0.4) of triangular cut-out for a cross-ply square laminate $(0^0/90^0/0^0)$ of aspect ratio a/b=1 and thickness ratio h/b=0.01 for fully clamped boundary condition. The frequency for all mode decreases as the size ratio of cut-outs increases. Table 3 shows the variation of first ten non-dimensional frequencies with distance between cut-outs (e/b=0.3, 0.4, 0.5) for a thin cross-ply square laminate$(0^0/90^0/0^0)$ of aspect ratio a/b=1, thickness ratio h/b=0.01, having triangular cut-out of size ratio d/b=0.2 at the center for fully clamped boundary condition. The frequency for first four modes decreases as the distance between cut-outs increases. Table 4 shows the variation of first ten non-dimensional frequencies with different number of layers for a thin cross-ply square laminate $(0^0/90^0/0^0 \dots\dots)$ of aspect ratio a/b=1 and thickness ratio h/b=0.01, having triangular cut-out of size ratio d/b=0.2 at the center for fully clamped boundary condition. The frequency in all ten modes increases as the number of layers increases. Table 5 shows the variation of first ten non-dimensional frequencies with different angles of lamina for an angle-ply square laminate $(\theta^0/-\theta^0/\theta^0)$ of aspect ratio a/b=1 and thickness ratio h/b=0.01, having triangular cut-out of size ratio d/b=0.2 at the center for fully clamped boundary condition. The frequency in all ten modes increases as the angle of lamina increases. Table 6 shows the variation of first ten non-dimensional frequencies with different boundary conditions (SSSS, SCSC and CCCC) for a thin cross-ply square laminate $(0^0/90^0/0^0)$ of aspect ratio a/b=1 and thickness ratio h/b=0.01, having triangular cut-out of size ratio d/b=0.2 at the center. The higher constraints at the edges results in higher frequencies.

Table 2. Variation of first ten natural frequency parameters ($\bar{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with different values of size of cutout (d/b=0.2, 0.3, 0.4) for a cross-ply laminate ($0^0/90^0/0^0$) having triangular cut-out at the centre, (K_s=5/6, a/b=1, h/b=0.01) for fully clamped boundary condition

	Mode									
d/b	1	2	3	4	5	6	7	8	9	10
0.2	26.127	53.175	56.265	84.937	92.959	102.664	111.871	121.441	133.985	144.825
0.3	25.845	41.467	44.886	72.680	78.730	94.092	98.782	112.965	131.683	139.632
0.4	25.542	39.402	40.584	66.684	68.466	92.413	102.781	106.967	120.667	139.085

Table 3. Variation of first ten natural frequency parameters ($\bar{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with different values of distance between cutouts (e/b=0.3, 0.4, 0.5) for a cross-ply laminate ($0^0/90^0/0^0$) having triangular cut-out at the center, (K_s=5/6, a/b=1, l/b=d/b=0.2, h/b=0.01) for fully clamped boundary condition

	Mode									
e/b	1	2	3	4	5	6	7	8	9	10
0.3	27.401	50.955	53.234	73.363	76.742	98.858	111.117	116.352	129.649	133.014
0.4	26.734	44.743	51.141	71.898	83.400	93.864	106.668	119.438	122.36	133.851
0.5	24.518	43.5673	48.889	67.650	80.144	90.491	104.985	114.370	122.876	163.287

Table 4. Variation of first ten natural frequency parameters ($\bar{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with different no. of layers (n=3, 5, 7, 9) for a cross-ply laminate ($0^0/90^0/0^0$……….) having triangular cutout at the center, (K_s=5/6, a/b=1, l/b=d/b=0.2, h/b=0.01) for fully clamped boundary condition

	Mode									
No. of layers	1	2	3	4	5	6	7	8	9	10
3	26.127	53.175	56.265	84.937	92.959	102.664	111.871	121.441	133.985	144.82
5	30.448	62.985	64.627	94.018	106.04	113.791	130.485	136.287	153.539	164.61
7	31.270	61.568	63.603	92.695	112.84	113.554	133.427	135.699	155.531	165.27
9	31.516	64.844	65.914	95.713	109.52	116.155	136.786	140.221	160.177	170.89

Table 5. Variation of first ten natural frequency parameters ($\overline{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with different angle of layers ($\theta = 15^0, 30^0, 45^0$) for an angle-ply laminate ($\theta/-\theta/\theta$) having triangular cutout at the center, (K_s=5/6, a/b=1, l/b=d/b=0.2, h/b=0.01) for fully clamped boundary condition

	Mode									
	1	2	3	4	5	6	7	8	9	10
θ										
15^0	2.948	5.731	6.169	9.646	10.596	11.278	12.600	14.107	14.748	16.742
30^0	3.223	6.070	6.261	9.125	10.844	11.564	13.401	14.089	15.087	16.799
45^0	3.226	5.909	6.142	8.829	9.848	10.964	13.110	13.792	15.983	16.809

Table 6. Variation of first ten natural frequency parameters ($\overline{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with different boundary conditions (CCCC, SSSS, SCSC) for a cross-ply laminate ($0^0/90^0/0^0$) having triangular cutout at the center, (K_s=5/6, a/b=1, l/b=d/b=0.2, h/b=0.01)

	Mode									
	1	2	3	4	5	6	7	8	9	10
B. C.										
SSSS	9.453	29.385	33.903	48.274	58.379	66.761	76.921	78.191	97.493	103.416
SCSC	20.401	39.579	49.978	66.773	68.092	86.765	93.509	96.967	111.396	135.492
CCCC	26.127	53.175	56.265	84.937	92.959	102.664	111.871	121.441	133.985	144.825

Fig. 4 shows the variation of first ten non-dimensional frequencies with aspect ratio (a/b=1, 1.5, 2) for a cross-ply square laminate ($0^0/90^0/0^0$) of thickness ratio h/b=0.01, having triangular cut-out of size ratio d/b=0.2 at the center for fully clamped boundary condition. The frequency in all modes decreases as the size of plate increases. Fig. 5 shows the variation of first ten non-dimensional frequencies with thickness ratio (h/b=0.01, 0.02, 0.1, 0.2) for a cross-ply square laminate ($0^0/90^0/0^0$) of aspect ratio a/b=1, having triangular cut-out of size ratio d/b=0.2 at the center for fully clamped boundary condition. The frequency in all ten modes decreases as the thickness ratio increases. Fig. 6 shows the variation of first ten non-dimensional frequencies with different material properties ($\frac{E_{11}}{E_{22}} = 10, 20, 30, 40$) for a thin cross-ply square laminate ($0^0/90^0/0^0$) of aspect ratio a/b=1 and thickness ratio h/b=0.01, having triangular cut-out of size ratio d/b=0.2 at the center for fully clamped boundary condition. The frequency in all ten modes increases as the modulus ratio increases.

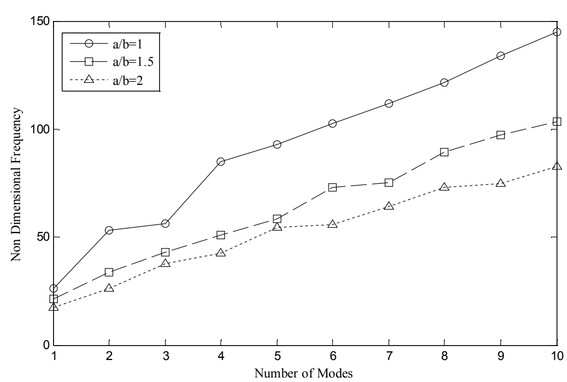

Fig. 4. Variation of first ten natural frequency parameters ($\overline{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with size ratio (a/b=1, 1.5, 2) for a cross-ply laminate ($0^0/90^0/0^0$) having triangular cut-out at the center, (K_s=5/6, l/b=d/b=0.2, h/b=0.01) for fully clamped boundary condition

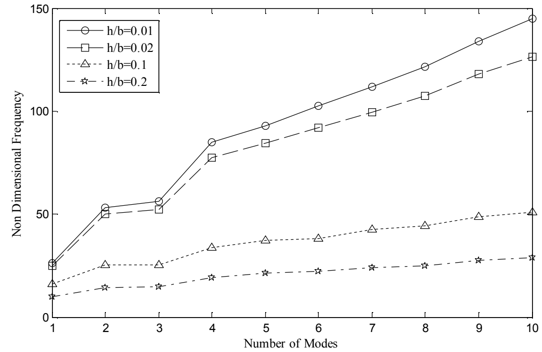

Fig. 5. Variation of first ten natural frequency parameters ($\overline{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with thickness ratio (h/b=0.01, 0.02, 0.1, 0.2) for a cross-ply laminate ($0^0/90^0/0^0$) having triangular cut-out at the centre, (K_s=5/6, a/b=1, l/b=d/b=0.2) for fully clamped boundary condition

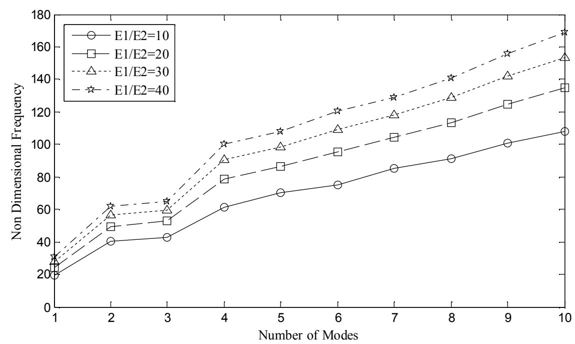

Fig. 6. Variation of first ten natural frequency parameters ($\overline{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with different material properties ($\frac{E_{11}}{E_{22}} = 10, 20, 30, 40$) for a cross-ply laminate ($0^0/90^0/0^0$) having triangular cut-out at the centre, (K_s=5/6, a/b=1, l/b=d/b=0.2, h/b=0.01) for fully clamped boundary condition

4. Conclusions

In this paper, free vibration behavior of laminated composite plate with cutouts has been carried out using APDL code in ANSYS and validated with the available published results. A convergence and validation survey of the free vibration analysis of laminated composite plate has been obtained. It has been observed that the present model converged well with mesh refinement and the differences were within acceptable range. From the present parametric analysis following conclusions are made: The frequency increases with an increase in the number of layers, modulus ratio of plate and angle of lamina. The frequency decreases with an increase in aspect ratio, thickness ratio, size of cutout and distance between cutouts. The boundary conditions of the plate has played an important role in the free vibrations of the plate with cutouts. The Non-dimensional frequencies were higher for fully clamped boundary condition in comparison to other boundary conditions.

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