Engineering Solid Mechanics 2 (2014) 139-144

Contents lists available at GrowingScience

Engineering Solid Mechanics

homepage: www.GrowingScience.com/esm

Effect of double thermal modulation on heat transfer in a square cavity heated from bellow

Aziz Akhiate[a*], Al alami Semma[a], El Ganaoui[b*] and Abdelah Anouar[a]

[a]Université Hassan I[er] FST de Settat, B.P. 577, Settat, Maroc
[b]University of Limoges, SPCTS, UMRCNRS 66 38, Limoges, France

A R T I C L E I N F O

Article history:
Received September 20, 2013
Received in Revised form
October, 14, 2013
Accepted 12 January 2014
Available online
14 January 2014

Keywords:
Thermo-vibrational convection
Square cavity
Finite volumes method

A B S T R A C T

This paper deals with the investigation of thermo-vibrational convection induced by harmonic vibrations of the temperature boundary conditions in a square cavity heated from bellow and containing a low Prandtl number fluid. The governing equations are solved by using a finite volumes method. Effects of thermal modulation on the all regimes occurring in the cavity when convection intensity increases are analyzed. A characteristic modulation frequency allowing the reduction of the average intensity of the flow and heat transfer at the cold wall has been identified. The effect of phase difference between hot and cold temperature is also studied.

NOMENCLATURE

C	dimensionless concentration
D	mass diffusivity
g	gravitational acceleration
H	height of the enclosure
k	thermal conductivity
Le	Lewis number, $=\alpha/D$
N	buoyancy ratio, $=\beta_C \Delta C'/\beta_T \Delta T'$
Nu	Nusselt number, see Eq (9)
Pr	Prandtl number, $=\nu/\alpha$
Ra	thermal Rayleigh number, $=g\beta_T \Delta T' H^3/\upsilon\alpha$
Sh	Sherwood number
t	time
α	thermal diffusivity
β	expansion coefficient
ν	kinematic viscosity of fluid
ρ	density of fluid
ψ	dimensionless stream function

SUBSCRIPT

c	critic
C	cold
H	hot
max	maximum
S	solutal
T	thermal
0	reference

SUPERSCRIPT

* Corresponding author.
E-mail addresses: a.akhiate@gmail.com (A. Akhiate)

doi: 10.5267/j.esm.2014.1.002

T temperature
x, y coordinate system
x, y velocities u, v

$'$ dimensional variable

1. Introduction

Convective flows generated by the buoyancy forces were the subject of several numerical and experimental studies. The control of these movements and the optimization of the heat transfer in these configurations is a significant challenge in the research works oriented to themo-fluid engineering. In recent years, attention has been given to natural convection in enclosures with time dependent thermal boundary conditions (Antohe & Lage, 1996a, 1996b; de Vahl Davis, 1983; Hyun, 1994). This interest is from the importance of these problems in many engineering applications such as convective heat loss from solar collectors, thermal comfort of buildings, building structure, air conditioning, electronic cooling, the crystal growth from the melt manufacturing, and nuclear engineering (Lin & Violi, 2010).

Numerical simulations and experiments showed that the buoyancy-driven convective activity in the cavity is intensified at certain discrete frequencies of the oscillation of the thermal boundary condition. This has been termed resonance, which is characterized by attaining the maximum amplitude of heat transfer rate through the vertical midplane of the cavity (Larroude et al., 1994). Antohe and his co-worker (Lage & Bejan, 1993; Antohe & Lage, 1996a, 1996b) investigated the effects of heating amplitude and frequency on the heat and flow transfer phenomena considering enclosures filled with a clear fluid and a fully saturated porous medium under time periodic square wave heating in the horizontal direction for a liquid with Pr=0.7. It was underlined that periodic heating is very important since flow resonance appears as the heating frequency matches the natural frequency of the flow inside the enclosure. It was shown that resonance frequency is independent of the heating amplitude for both the clear fluid and porous medium cases.

Kwak and Hyun (1996) studied numerically the effects of the amplitude and frequency of the hot side sinusoidal wall oscillation on the enhancement of heat transfer in a square cavity with fixed thermal Rayleigh number (Ra = 107) and Prandtl number (Pr = 0.7). Once more it was observed that the maximum increase of the time-averaged heat transfer rate occurs at a resonance frequency between natural frequency of the flow and the modulation frequency. Semma et al. (2005) presented results on the effect of thermo-vibrational convection in a vertical Bridgman cavity and studied the frequency dependence of the flow intensity and solid/liquid interface deformation acting on the steady and oscillatory basic states. It was shown that with the stationary basic regime, the solid/liquid interface deformation can be affected at low frequencies. However, for high frequencies, the flow and interface deformation converges toward their free state value. Saravanan and Sivakumar (2010) recently studied the effect of vibration with arbitrary amplitude and frequency in a porous horizontal saturated fluid heated from below. They demonstrated that these vibrations can produce stabilization or destabilization in function of the chosen amplitude and frequency.

The whole of the work which treated the thermal vibration problems considered only the active wall temperature variable. However, the cold temperature can be also variable or varied to increase the heat control. The objective of this study is the numerical investigation on the heat transfer of natural convection in a square cavity subjected to thermal boundary condition. In addition, the temperatures are time dependent at both hot and cold walls. The study is focused on periodic variations and a special attention is given to the effect of the amplitude, of the period, and of the dephasing of the exciting temperatures on the enhancement of heat transfer and fluid circulation inside the cavity.

2. Model and solution method

2.1. Mathematical model

The studied configuration, depicted in Figure 1, is a square cavity heated from below and cooled from the top as is shown schematically in Figure 1-a. The lateral walls in the hot zone (at T_H), and the cold zone (at T_C) are separated by an adiabatic zone of length $H_{\Delta T}$.

The problem can become dimensionless using H, the height of the cavity, as the scale factor for length; H^2/α and $\rho\alpha^2/H^2$ as the scaling factors for time and pressure respectively. The dimensionless temperature is $T = (\dot{T} - \dot{T}_C)/(\dot{T}_H - \dot{T}_C)$. The cavity is filled with a Newtonian and incompressible fluid at low Prandtl (*Pr=0.01*). The sinusoidal hot and cold temperature are characterised respectively by their amplitudes ε_H and ε_C and their frequencies f_H and f_C. The dephasing between the two temperatures is φ. All properties of the fluid are constant except the density that is assumed to be a linear function of temperature,

$$\rho = \rho_0[1 - \beta_T(T - T_0)] \tag{1}$$

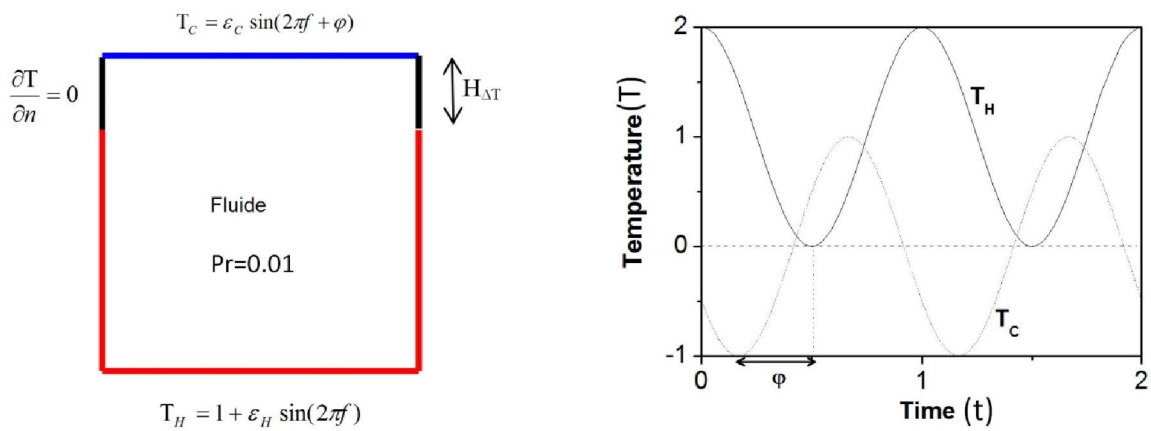


Fig. 1. Sketch of the flow configuration with boundary condition

The governing equations are the continuity, momentum and energy conservation. Corresponding Partial Differential Equations (PDE), written in Cartesian coordinates in non-dimensional form are as following:

Continuity:

$$\nabla.\mathbf{u} = 0 \tag{2}$$

Momentum:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + Pr\Delta\mathbf{u} + RaPr\,\frac{\mathrm{g}}{||\mathrm{g}||}\,T \tag{3}$$

Energy:

$$\frac{\partial T}{\partial t} + (\mathbf{u}.\nabla)T = \Delta T \tag{4}$$

Eq. (1) to Eq. (3) are subjected to the following boundary conditions:

$$T_C = \varepsilon_C \sin(2\pi f + \varphi),\ y = 1 \text{ at } u = v = 0 \tag{5}$$

$$T_H = 1 + \varepsilon_H \sin(2\pi f),\ u = v = 0 \text{ at } y = 0;\ x = 0,1 \text{ and } y \in [0\ ,\ 0.75] \tag{6}$$

$$\frac{\partial T}{\partial x} = 0\ ,\ u = v = 0$$

$$\text{at} \quad x = 0,1 \text{ and } y \in [0.75\ ,\ 1.0] \tag{7}$$

In order to study the effect of the oscillations of hot and cold wall temperature on the heat transfer, the following quantities are introduced in the manner described by Kwak and Hyun (1996). Following functions are based on variable ϕ which can be either Nusselt number, Nu or dimensionless stream function, ψ. Therefore, the relative values for periodic regimes can be defined as:

$$G(\phi) = \frac{\bar{\phi}(\varepsilon, f)}{\bar{\phi}(0,0)} \tag{8}$$

$$A(\phi) = \frac{\phi_{\max}(\varepsilon, f) - \phi_{\min}(\varepsilon, f)}{\bar{\phi}(0,0)} \tag{9}$$

In the above equations, ϕ stands for an arbitrary physical variable and $\bar{\phi}$ its average value.

2.2. Numerical Method

For numerical approximations of the considered problem a finite volume method has been used (Patankar, 1980). The conductive terms have been discretized with a central scheme and the convective one by using a third order *QUICK* scheme subjected to a flux limiter developed by Leonard (1979,1991). To resolve the velocity - pressure coupling, the *SIMPLEC* algorithm has been used (Van Doormaal & Raithby, 1984). The temporal discretization is done using a second order Euler scheme. The governing equations are solved on a staggered grid. All the boundary conditions are treated by using second order differencing to maintain the same accuracy in the whole computational domain.

Extensive validation of the performance of the present code with and without phase change has been done elsewhere (Semma et al., 2005). A study of grid was carried out and showed that spatial resolution 64×64 and time step $\Delta t=10^{-4}$ allow an accurate description of the development of thermo convective phenomena within the cavity. Grid convergence tests have been carried out by increasing number of mesh points from 32x32 to 128x128; the computed results obtained for these meshes differed by less than 0.2%. The convergence of the solution was declared at each time step when the maximum relative change between two consecutive iteration levels fell below 10^{-6} for velocity *u,v* and temperature *T*.

Validation of the code to predict the velocity and temperature fields in a square cavity was performed by comparing results against the benchmark solution for the laterally heated/cooled square cavity(de Vahl Davis, 1983). With the computational mesh selected, the vertical velocity v is hardly affected by changes in the grid size. At a relatively high at $Ra_H = 10^6$ the maximum value of the dimensionless vertical velocity V_{max}=220.60 is within five percent of the benchmark results of 217.36 (de Vahl Davis, 1983). Agreements to less than 1% of the mean Nusselt number Nu_H were found for $Ra_H = 10^6$, namely 8.72 in this work and 8.799 in (de Vahl Davis, 1983).

3. Results and discussion

Without modulation, the various flow structures and the transition thresholds from steady to unsteady regime for the same configuration are available in the literature. It has been shown that for the low *Ra*, the solution is stationary and composed of two symmetric counter-rotating cells (called **SS**) with respect to the centerline (*x=1/2*) in which the fluid rises from the vertical walls of the cavity towards the center. The heat transfer is dominated by the diffusive regime. The first transition occurs from symmetrical solution (**SS**) to asymmetrical one (called **SAS**) for Ra=3500 indicating the development of the convective regime. The next transition to the oscillatory flow occurs around *Ra*=17500 and leads to a periodic solution (**P1**) with a dimensionless internal frequency f_i=6.67. The asymmetric solution is retained with the same flow structure dominated by a one-cell flow varying between a quasi-circular to a deformed shape because of the competition with the secondary cells developing near the corners. The frequency of the oscillations increases with *Ra*. Increasing *Ra* to 8.5×10^4, the flow becomes **P2** type characterized by the frequencies f and $f/2$ (f = 33.33). The quasi-periodic regime (**QP**) which starts around $Ra = 1.75\times10^5$ exhibits complex non-periodic oscillation.

The stationary solution corresponding to $Ra=10^4$ is fixed as basic solution and modulation applied to both hot and cold temperatures. This application of a double modulation modifies the evolution of the flow structure that depends on the frequency, the amplitude and the phase-difference. In the first, we keep ε_C constant equal to 1, the phase difference equal to $\pi/2$ and we vary the frequency f for various amplitude values of the hot temperature ε_H. We identify the presence of a critical frequency for which the intensity of flow reaches minimal relative value which passes from 0.87 for ε_H=0 to 0.61 for ε_H=1. This frequency is practically independent of the value of the amplitude ε_H (Fig. 2).

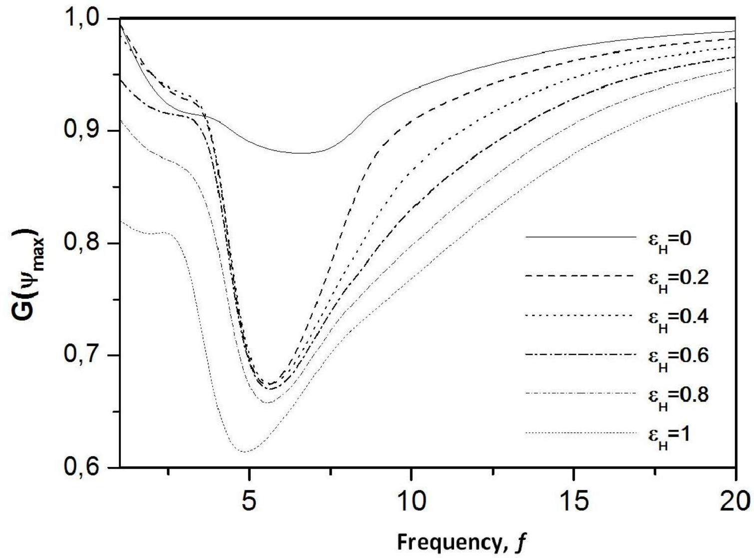
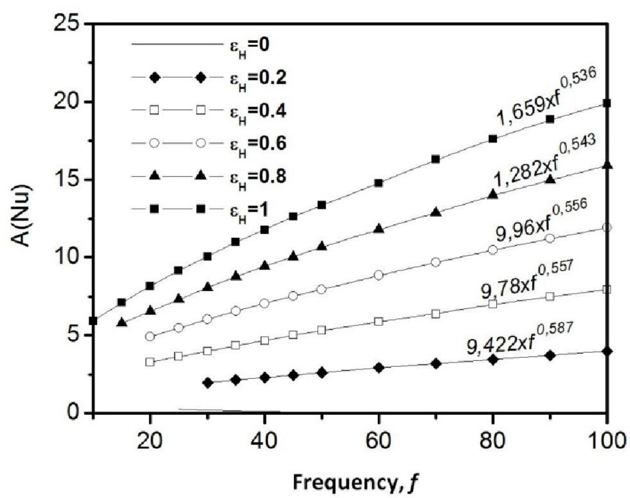


Fig. 2. Variation of relative value of maximal intensity G(ψmax) according to the frequency f (Ra=10^4)

For high frequency values, the relative value of maximal flow intensity tends to 1. This change recorded in the flow intensity for low frequencies strongly influences the behavior of the heat transfer who falls for the critical frequency.

For high frequencies, the value of the relative Nusselt number tends towards 0 but the value of its amplitude increases according to the following correlations for $\varphi=\pi/2$:

$A(Nu)=a(\varepsilon)\text{x}f^{\,b(\varepsilon)}$



ε=0.2 *a=1.659, b=0.536*

ε=0.4 *a=1.282, b=0.543*

ε=0.6 *a=9.96, b=0.556*

ε=0.8 *a=9.78, b=0.557*

ε=1.0 *a=9.422, b=0.587*

Fig. 3. Variation of relative value of Nusselt number amplitude A(Nu) according to the frequency f for different values of ε_H (Ra=10^4)

The effect of the phase difference between the hot and cold temperature is investigated. This parameter influences the instantaneous variation between cold and hot temperature walls.

For low frequencies, the heat transfer is very sensitive to the dephasing between the two boundary temperatures (Fig. 4). For high frequencies, the asymptotic behavior of heat transfer does not depend on the dephasing value. We notice that the critical frequency increases linearly with the difference phase according to the following relation:

$$f^C=0.4\text{x}\varphi/\pi+3.22 \qquad \text{for} 0\leq\varphi\leq\pi \tag{10}$$

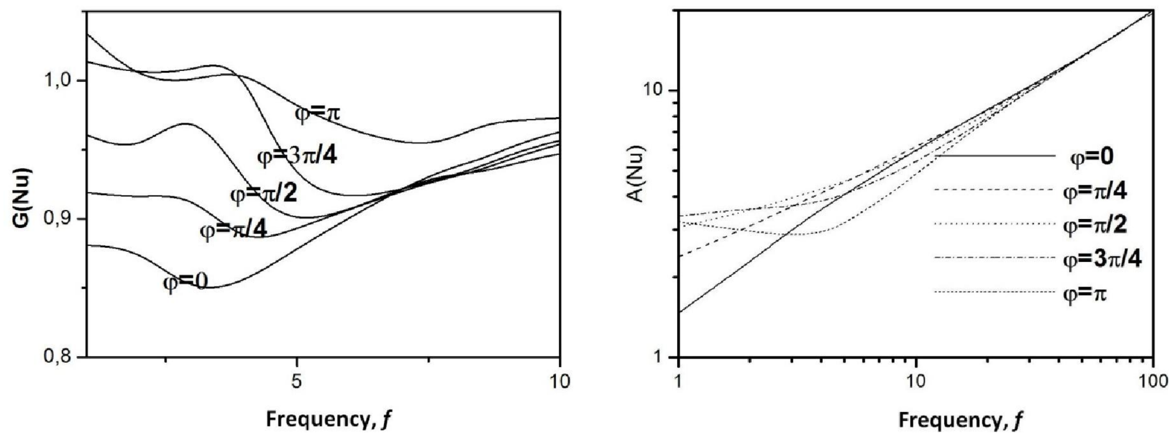


Fig. 4. Variation of the relative value of average and amplitude of the Nusselt number according to the frequency for different values of φ (Ra=10^4).

4. Conclusion

The effect of double thermal modulation on heat transfer and flow structure is studied; the existence of a characteristic modulation frequency allowing the reduction of the average intensity of the flow and heat transfer at the cold wall is noticed. The effect of the dephasing between the hot and cold temperature on heat transfer is analysed; correlation between critical frequency and phase difference is identified. The present work can be considered as a step to quantify dephasing modulation effects between the hot and the cold walls. For the future work, the different shape and geometry, and the effect of oscillatory convection can be considered.

References

Antohe, B. V., & Lage, J. L. (1996a). Amplitude effect on convection induced by time-periodic horizontal heating. *International Journal of Heat and Mass Transfer*, 39(6), 1121-1133.

Antohe, B. V., & Lage, J. L. (1996b). Experimental investigation on pulsating horizontal heating of a water-filled enclosure. *Journal of Heat Transfer*, 118(4), 889-896.

de Vahl Davis, G. (1983). Natural convection of air in a square cavity: a bench mark numerical solution. *International Journal for Numerical Methods in Fluids*, 3(3), 249-264.

Hyun, J. M. (1994). Unsteady buoyant convection in an enclosure. *Advances in heat transfer*, 24, 277.

Kwak, H. S., & Hyun, J. M. (1996). Natural convection in an enclosure having a vertical sidewall with time-varying temperature. *Journal of Fluid Mechanics*, 329, 65-88.

Lage, J. L., & Bejan, A. (1993). The resonance of natural convection in an enclosure heated periodically from the side. *International Journal of Heat and Mass Transfer*, 36(8), 2027-2038.

Larroude, P. H., Ouazzani, J. L. I. D., Alexander, J. I. D., & Bontoux, P. (1994). Symmetry breaking flow transitions and oscillatory flows in a 2D directional solidification model. *European Journal of Mechanics. B, Fluids,* 13(3), 353-381.

Leonard, B. P. (1991). The ULTIMATE conservative difference scheme applied to unsteady one-dimensional advection. *Computer Methods in Applied Mechanics and Engineering*, 88(1), 17-74.

Leonard, B. P. (1979). A stable and accurate convective modelling procedure based on quadratic upstream interpolation. *Computer Methods in Applied Mechanics and Engineering*, 19(1), 59-98.

Lin, K. C., & Violi, A. (2010). Natural convection heat transfer of nanofluids in a vertical cavity: Effects of non-uniform particle diameter and temperature on thermal conductivity. *International Journal of Heat and Fluid Flow*, 31(2), 236-245.

Patankar, S. V. (1980). *Numerical Heat Transfer and Fluid Flow*. Taylor & Francis.

Saravanan, S., & Sivakumar, T. (2010). Onset of filtration convection in a vibrating medium: The Brinkman model. *Physics of Fluids*, 22, 034104.

Semma A., Cheddadi A., Farchi A. (2005). High order finite volume scheme for phase change problems. Finite Volumes for Complex Applications IV. *Hermes Science Publishing*, 493-503.

Van Doormaal, J. P., & Raithby, G. D. (1984). Enhancements of the SIMPLE method for predicting incompressible fluid flows. *Numerical heat transfer*, 7(2), 147-163.