

## Application of maximum tangential stress criterion in determination of fracture initiation angles of silicon/ glass anodic bonds

M.M. Mirsayar<sup>a\*</sup> and A. T. Samaei<sup>b</sup>

<sup>a</sup>Zachry Department of Civil Engineering, Texas A&M University, College Station, TX 77843-3136, USA

<sup>b</sup>Young Researchers & Elite Club, Chalous Branch, Islamic Azad University, Chalous, Iran

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### ABSTRACT

Silicon/glass bi-materials are used in micro-assembly and packaging of micro-electromechanical systems (MEMS) and micro-electronics devices. In this paper, maximum tangential stress (MTS) concept is used for determination of the fracture initiation angles of silicon/ glass bi-material notches. First, the MTS criterion is analytically formulated for a bi-material notch problem. Then, the criterion used for prediction of fracture initiation angles of some experimental data given in literature for silicon/ glass bi-material notches. In addition, the modified MTS (MMTS) criterion, which considers the effect of I-stress, was compared with the MTS criterion and the experimental data. It was shown that MMTS criterion provides more accurate results than the MTS criterion for estimation of the fracture initiation angle.

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## 1. Introduction

Nowadays bi-material notches can be seen in different modern industrial structures such as aircrafts, automobiles and microelectronics structures. Among them, the use of micro-electromechanical systems (MEMS) and micro-electronics devices is going to be increased in order to meet the modern technological needs. These devices contain numerous multi-material and bi-material interface corners (Chen et al., 1997; Dunn et al., 2000; Labossiere et al., 2002; Lu et al., 2002). At the corner of these interfaces, fracture may occur due to the high stress/ strain gradient as a result of material discontinuity and geometrical configuration. In order to understand the effect of each fracture mode (mode I and II) as well as the higher order terms, the analysis of stress field parameters is one of the first steps. Heretofore, elastic stress field of bi-material/ simple notches and cracks has been studied by many researchers (Ayatollahi et al., 2010; Ayatollahi et al., 2011; Mirsayar & Samaei 2013; Mirsayar 2013; Ayatollahi et al., 2013; Ayatollahi et al., 2010; Arabi et al., 2013; Mirsayar

\* Corresponding author. Tel.: +1 (979) 777-6096

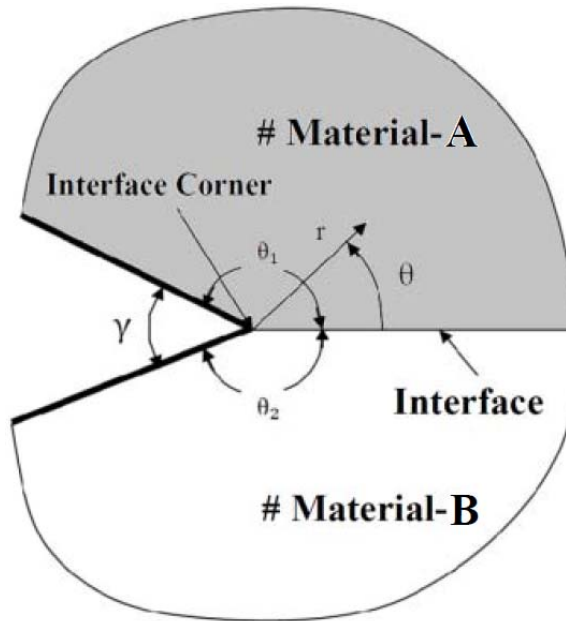
E-mail addresses: [mirmilad@tamu.edu](mailto:mirmilad@tamu.edu) (M.M. Mirsayar)

2014). Several fracture criteria are suggested for prediction of fracture initiation angle at the interface corners. These criteria are mostly energy based (like strain energy density and energy release rate) and stress based criteria have not been extended well for bi-material notches (Spyropoulos 2003; Klusak & Knesl 2007). Recently, Mirsayar et al. (2014) suggested a stress based fracture criterion considering the effect of first non-singular stress term of elastic stress field, called I-stress, to estimate the direction of fracture initiation at the interface corner. They showed that I-stress can play an important role on estimation of fracture initiation angle at the interface and proposed a modified MTS (MMTS) criterion. In this paper, the maximum tangential stress (MTS) criterion is used for prediction of direction of fracture initiation at the corner of the interfaces. First, MTS was formulated for bi-material notch problems and the direction of fracture initiation was calculated and plotted as a function of mode mixity. Then, the MTS criterion is used for predicting the fracture initiation angles of experimental data presented by Labossiere et al. (2002) for silicon/glass interface corners which are used in MEMS and microelectronics devices (Labossiere et al., 2002; Dunn et al., 2000). Finally, the experimental data and analytical predictions from MTS and MMTS criteria are compared with each other. It is shown that the MMTS provides more accurate estimation of experimental results than the MTS criterion however the difference between the MTS and MMTS predictions is not significant.

## 2. MTS criterion

### 2.1. Stress and displacement field at the interface corner

Let us consider a bimaterial interface corner as shown in Fig. 1.



**Fig. 1.** General configuration of a bi-material notch

The in-plane singular stress and displacement fields near the interface corner can be expressed as (Labossiere et al., 2002):

$$\begin{aligned}\sigma_{ij}^M &= K_1 r^{\lambda_1-1} f_{ij}^{1M}(\theta) + K_2 r^{\lambda_2-1} f_{ij}^{2M}(\theta) + (I - stress) + \text{Higher order terms } (\lambda_m > 1) \\ u_i^M &= K_1 r^{\lambda_1-1} g_i^{1M}(\theta) + K_2 r^{\lambda_2-1} g_i^{2M}(\theta) + (I - stress) + \text{Higher order terms } (\lambda_m > 1)\end{aligned}\quad (1)$$

where  $(i, j) \equiv (r, \theta)$  are the polar coordinates with the origin at the interface corner,  $K_m$  ( $m=1, 2$ ) are the stress intensity factors,  $\lambda_m$  corresponds to the  $m$  th eigenvalue of the problem. Also in Eq. (1),  $f_{ij}^{mM}$  and  $g_i^{mM}$  are known functions of  $\theta$  and differ in each material ( $M = A, B$ ) (see refs (Ayatollahi

et al., 2010; Ayatollahi et al., 2011; Mirsayar & Samaei 2013; Mirsayar 2013; Ayatollahi et al., 2013; Ayatollahi et al., 2010; Arabi et al., 2013; Mirsayar 2014)) for more details).

It is worth mentioning that the full field stress and displacement fields contain infinite terms of singular and non-singular terms. For bi-material notch problems, there are one or two singular stress terms ( $\lambda_m - 1 < 0$ ) based on the geometrical and material configuration at the interface corner and the higher order ones represent the non-singular terms. Although the non-singular terms, especially the first non-singular one, may sometimes significantly affect the stress distribution adjacent the interface corner (Ayatollahi et al., 2010; Ayatollahi et al., 2011; Ayatollahi & Mirsayar 2011, Mirsayar et al., 2014; Mirsayar 2014), the singular stress terms mainly cover the stress fields in a wide range of materials, geometrical configurations and loading conditions.

## 2.2. MTS criterion for determination of fracture initiation angle

The MTS criterion for brittle materials states that fracture occurs when the tangential stress component  $\sigma_{\theta\theta}^M$  reaches to its maximum possible value ( $\sigma_{\theta\theta,C}^M$ ) at a critical distance of  $r_c$  around the notch tip. This criterion has been used earlier for predicting the angle of fracture in cracked brittle or quasi brittle materials (Awaji and Sato 1978; Shetty et al. 1987; Singh and Shetty 1989; Tikare and Choi 1997; Aliha and Ayatollahi 2008, 2009, 2012; Ayatollahi and Aliha 2011, Aliha et al. 2008, 2012; Yamauchi et al. 20011, Marsavina et al. 2013). The fracture initiation angle ( $\theta_0$ ) could easily be obtained by satisfying the following equation:

$$\begin{cases} \left( \frac{\partial \sigma_{\theta\theta}^M(\theta)}{\partial \theta} \right)_{\theta_0} = 0 \\ \left( \frac{\partial^2 \sigma_{\theta\theta}^M(\theta)}{\partial \theta^2} \right)_{\theta_0} < 0 \end{cases} \quad (2)$$

By putting the extended form of tangential stress component from Eq. 1 into Eq. 2, the Eq. 2 can be rewritten as:

$$\begin{cases} \left( \frac{\partial \sigma_{\theta\theta}^M(\theta)}{\partial \theta} \right)_{\theta_0} = 0 \rightarrow K_1 r^{\lambda_1 - 1} \frac{\partial f_{\theta\theta}^{1M}}{\partial \theta} + K_2 r^{\lambda_2 - 1} \frac{\partial f_{\theta\theta}^{2M}}{\partial \theta} = 0 \rightarrow \theta_0 \\ \left( \frac{\partial^2 \sigma_{\theta\theta}^M(\theta)}{\partial \theta^2} \right)_{\theta_0} < 0 \end{cases} \quad (3)$$

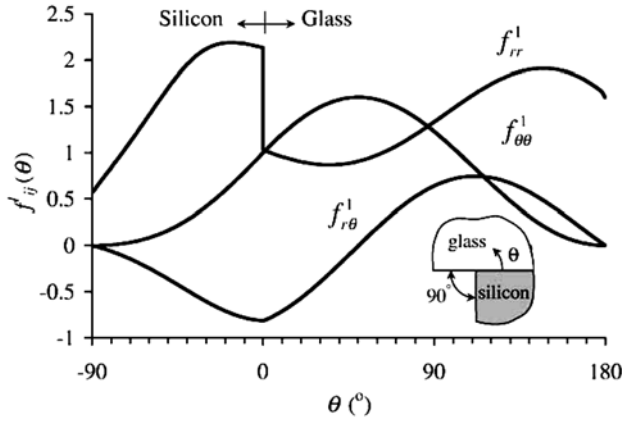
The Eq. 3 represents the MTS criterion for determining the direction of fracture initiation at the interface corner. It should be mentioned that the Eq. 3 considers only singular stress terms and the term corresponding to  $\lambda_2$  is supposed to be a singular term. Therefore, in cases with only one singular eigenvalue, the term corresponding to  $\lambda_2$  should not be considered based on the MTS criterion.

The modified version of the MTS criterion (called MMTS criterion), is recently proposed by Mirsayar et al (2014) which considers the effect of I-stress as well as the singular stress field. According to their criterion, the I-stress term can play an important role on estimation of the fracture initiation angle depending on the geometry and combination of materials at the interface. More details about modified version of the MTS criterion can be found in (Mirsayar et al., 2014).

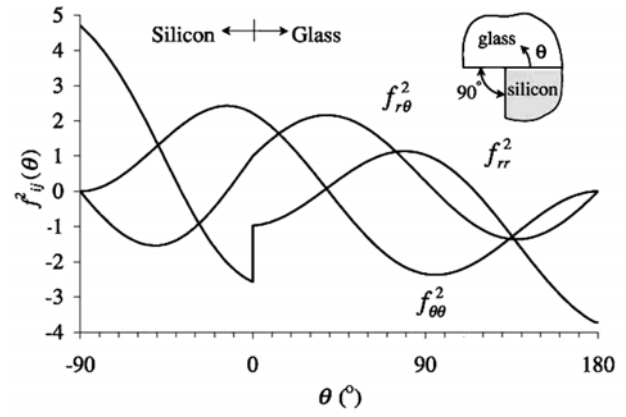
## 3. Experimental data and discussion

In this section, some experimental data for fracture initiation angle of silicon/ glass bi-material notches given by Labossiere et.al (2002) are used for validating the MTS and MMTS criteria. For a silicon/ glass bond with opening angle of  $90^\circ$ , the functions  $f_{ij}^{1,M}$  and  $f_{ij}^{2,M}$  are given in Figs. 2 and

3. The first and the second eigenvalues ( $\lambda_1, \lambda_2$ ) associated with this combination of materials and geometrical configuration are 0.505 and 0.822 respectively (Labossiere et al., 2002).



**Fig. 2.**  $f_{rr}^1, f_{r\theta}^1$  and  $f_{\theta\theta}^1$  as a function of  $\theta$  for the silicon/ glass bi-material interface corner  $\theta_A=180^\circ$ ,  $\theta_B=90^\circ$  ( $\gamma=90^\circ$ ) (Labossiere et al., 2002)



**Fig. 3.**  $f_{rr}^2, f_{r\theta}^2$  and  $f_{\theta\theta}^2$  as a function of  $\theta$  for the silicon/ glass bi-material interface corner  $\theta_A=180^\circ$ ,  $\theta_B=90^\circ$  ( $\gamma=90^\circ$ ) (Labossiere et al., 2002)

As the fracture always occurred in the glass in all the tested specimens, the critical distance assumed to be 0.05 for glass part. The parameters  $\frac{\partial f_{\theta\theta}^{1M}}{\partial \theta}$  and  $\frac{\partial f_{\theta\theta}^{2M}}{\partial \theta}$  could easily be calculated by numerically derivation of  $f_{ij}^{1,M}$  and  $f_{ij}^{2,M}$  given in Figs. 2 and 3 as follow:

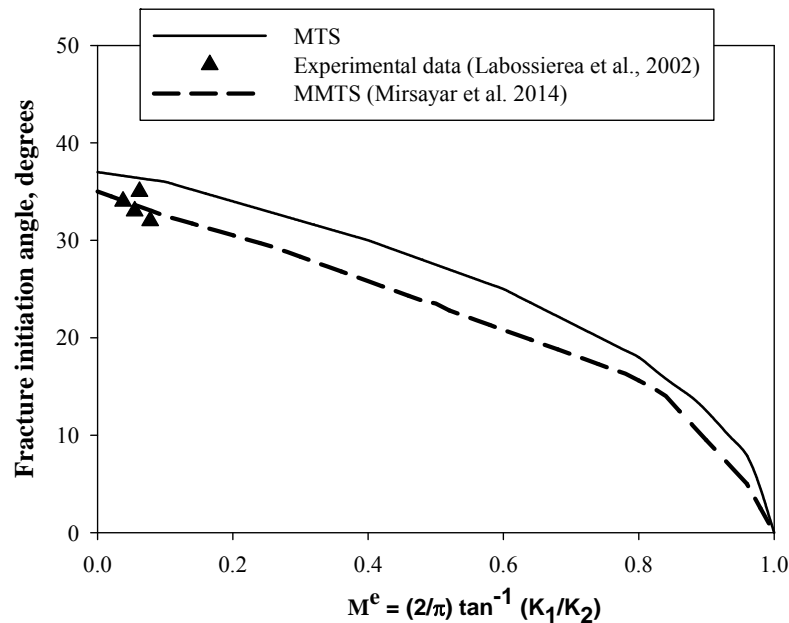
$$\frac{f_{ij}^{mM}(\theta + \Delta\theta) - f_{ij}^{mM}(\theta)}{\Delta\theta}, (\Delta\theta = 0.5^\circ, -90^\circ < \theta < 180^\circ). \quad (4)$$

Table 1 represents the experimental data given in (Labossiere et al., 2002) for the direction of fracture initiation. Based on their results, the fracture was occurred in glass for all of these specimens. The parameter  $M^e$  ( $M^e = \frac{2}{\pi} \tan^{-1}(\frac{K_1}{K_2})$ ), called mode mixity, represents the participation of each fracture mode and varies between zero and one.

**Table 1.** Experimental data for fracture initiation angles in the silicon/glass bonds with opening angle of  $90^\circ$  given in (Labossiere et al., 2002)

$\theta$ (Degrees)	$K_{1cr}$	$K_{2cr}$	$M^e$
35	0.19	1.86	0.065
38	0.19	1.94	0.062
35	0.19	2.18	0.055
36	0.19	2.58	0.047

The experimental data, given in Table 1 and the analytical predictions by the MTS and MMTS criteria are shown in Fig. 4. It can be seen from Fig. 4 that the MMTS criterion provides more accurate results than the MTS criterion; however the difference between the MTS and MMTS predictions is not significant. The details of MMTS criterion can be found in (Mirsayar et al., 2014).



**Fig. 4.** MTS criterion in comparison with MMTS and experimental data

#### 4. Conclusions

The MTS criterion for bi-material notch problems is formulated and applied for some experimental data for silicon/glass interfaces. The direction of fracture initiation was estimated using both MTS and MMTS criterion. For the experimental data discussed in this paper, it was observed that no significant difference exist between the MTS and MMTS predictions. However, it was shown that the MMTS is capable for predicting more accurate results than the MTS criterion in general.

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