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Size and crack length effects on fracture toughness of polycrystalline graphite

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ABSTRACT

In this paper, the effects of specimen size and crack length on the fracture toughness of polycrystalline graphite are studied. The experimental results reported in the previous studies showed that the fracture toughness of graphite increase in bigger specimen. It has been also demonstrated that the fracture toughness of graphite is nearly identical in specimens with crack length ratio less than 0.7 but decreases for grater crack length ratios. To justify the size and crack length dependency of fracture toughness, the modified form of maximum tangential stress (MMTS) criterion, which makes the use of higher order terms in calculating the stress field around the crack tip is employed. It is shown that the MMTS criterion can provide good estimates for the fracture toughness of graphite obtained from specimen with different sizes. It is also indicated that the MMTS criterion can predict very good the reported experimental fracture toughness data for samples with the crack length ratios less than 0.7.

1. Introduction

Since graphite has good thermal stability, low permeability, high corrosion resistance, good performance in electrical and thermal conductivity and good thermal shock resistance, it is employed frequently in different engineering components such as sliding current connector, refractory graphite parts, sealing rings, carbon brushes, graphite electrodes, heating elements, etc. However, graphite is prone to mechanical or thermal failure, especially when graphite parts contain stress concentrators such as cracks or notches. In other words, the graphite parts might contain cracks, inherent discontinuities and flaws, which play the role of stress raiser and make the graphite parts very susceptible to suddenly fracture. The cracks are often created during the manufacturing or machining processes, or due to the mechanical or thermal loads applied under service conditions. Therefore, it is urgently required to study the mechanical failure of cracked graphite components. Researchers and

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engineers have used frequently a fundamental parameter so-called fracture toughness K_{If} for determining the strength of cracked components. Fracture toughness presents the resistance of materials against crack growth under mode I or opening mode loading. Fracture toughness of graphite is usually measured from standard laboratory specimens with specific geometry, shape and size limits. It is expected that the fracture toughness obtained from laboratory specimen of different sizes and shapes is nearly identical. However, experimental results have shown that the fracture toughness of graphite is significantly dependent on the size, length of initial crack and geometry of specimen (Chi, 2013; Li et al., 2013; Sakai & Kurita, 1996; Sakai & Nonoyama, 2005; Yamauchi et al., 2000, 2001; Yoon et al., 2011). For example, Li et al. (2013) showed experimentally that the fracture toughness of NBG graphite obtained from the single-edge-notch beam samples increases by increasing the size of specimen. For another example, Yamauchi et al. (2000, 2001) demonstrated that the fracture toughness of graphite measured from two test configurations including edge-cracked semi-circular bend (SCB) specimen subjected to three-point loading and cracked Brazilian disk (CBD) are significantly different. Moreover, Sakai and Nonoyoma (2005) showed that the length of initial crack has the influence on the fracture toughness of graphite when the crack length ratio is more than 0.7. Thus, in order to use the fracture toughness obtained from the laboratory-size specimen for predicting the onset of crack growth in real size and geometry graphite parts, the effects of specimen size, initial crack length and geometry on fracture toughness should be considered.

On the other hand, there are several criteria for investigating the size effects on mode I fracture toughness of quasi-brittle materials such as rocks, concretes, ceramics, etc. For instance, Bazant's size effect law (SEL) proposed by Bazant (1984)is a well-known criterion which has been used frequently for taking into account the specimen size effects on fracture toughness of graphite (Li et al., 2013; Sakai & Kurita, 1996; Sakai & Nonoyama 2005). However, almost all of the size effect criteria (e.g. SEL) are not able to consider simultaneously the geometry and crack length effects on the fracture toughness. The aim of this paper is to investigate the size and crack length effects on the fracture toughness of graphite simultaneously. For this purpose, test data reported by Sakai and Kurita(1995) is used. They showed experimentally that the fracture toughness for a type of polycrystalline graphite (IG-11) depends significantly on the size and crack-length of specimen. In order to justify the size and crack-length dependence of fracture toughness, a modified form of the maximum tangential stress (MMTS) criterion is used. This criterion takes into account the influence of the higher order terms in calculating the stress field around the crack tip in addition to the singular terms. As an important parameter in the MMTS criterion, the critical distance r_c is assumed to be size dependent and a formula proposed recently by Ayatollahi and Akbardoost (2012) is employed for describing the size dependence of r_c. The values of K_{If} for specimens with different crack lengths are also predicted using the MMTS criterion based on the constant value of critical distance. It is shown that the MMTS criterion is able to provide good estimates for the fracture toughness of graphite by taking into account the effects of specimen size and the length of initial crack.

2. Modified MTS criterion

Based on the classical MTS criterion proposed by Erdogan and Sih (1963), fracture occurs radially from the crack tip and perpendicular to the direction of the maximum tangential stress θ_m. Moreover, the crack will be extended when the tangential stress component $\sigma_{\theta\theta}$ along θ_m and at a critical distance r_c from the crack tip reaches a critical value $\sigma_{\theta\theta c}$ (Erdogan & Sih 1963). The tangential stress around the crack tip under pure mode I loading can be written from the William's series expansion (Williams, 1957):

$$\sigma_{\theta\theta}(r,\theta)=\sum_{n=1}^{\infty}\frac{n}{2}A_n r^{\frac{n}{2}-1}\left[\left(\frac{n}{2}+1\right)\cos\left(\frac{n}{2}-1\right)\theta-\left(\frac{n}{2}+(-1)^n\right)\cos\left(\frac{n}{2}+1\right)\theta\right] \tag{1}$$

where r and θ are the conventional crack tip co-ordinates, n is the order of term in the series expansion and the constant coefficients A_n are dependent on the specimen geometry and loading conditions. These coefficients can be generally written in terms of dimensionless parameters A_n^* as:

$$A_n = \sigma_N \, w^{(1-n/2)} \; A_n^* \tag{2}$$

in which σ_N is the nominal stress and w is a characteristic dimension like the width of single-edge notched beam (SENB) specimens. Moreover, the dimensionless coefficients A_n^* depend only on the configuration parameters like the crack length ratio (a/w) and the loading span to width ratio (S/w). These parameters are independent of the load and the dimensions of samples. Due to symmetry in mode I loading, the crack growth takes places along the crack direction, i.e. θ_m=0. Therefore, the tangential stress $\sigma_{\theta\theta}$ along the fracture direction is obtained from Eq. (1) by setting θ=0:

$$\sigma_{\theta\theta}(r,\theta)\Big|_{\theta=0} = \frac{A_1}{\sqrt{r}} + 3A_3\sqrt{r} + 5A_5 r^{3/2} + \ldots \tag{3}$$

The parameter A_1 is related to the mode I stress intensity factor K_I as $A_1 = K_I/\sqrt{2\pi}$.The conventional MTS criterion (Erdogan & Sih, 1963) considers only the first stress term and ignores the higher order terms. It has been recently demonstrated that the higher order terms of crack tip asymptotic field are no longer negligible and they should be taken into account to characterize the tangential stress more accurately (Awaji & Sato, 1978; Ayatollahi & Akbardoost, 2012; Ayatollahi & Aliha, 2011; Aliha & Ayatollahi, 2009, 2013; Mirsayar et al., 2014; Aliha et al., 2010, 2013). By taking the first two terms in Eq. (3) into consideration, the maximum tangential stress component at the critical distance r_c can be obtained from:

$$\sigma_{\theta\theta c} = \frac{A_{1c}}{\sqrt{r_c}} + 3A_{3c}\sqrt{r_c} = \frac{K_{If}}{\sqrt{2\pi r_c}} + 3A_{3c}\sqrt{r_c} \tag{4}$$

where A_{3c} is the critical value of A_3 and K_{If} is the critical stress intensity factor or apparent fracture toughness which is considered to be dependent on the size and geometry of cracked specimen. By substituting Eq. (2) into Eq. (4) for n= 1 and n=3, Eq. (4) simplifies to:

$$\sigma_{\theta\theta c} = \frac{K_{If}}{\sqrt{2\pi r_c}}\left(1 + 3\frac{A_3^*}{A_1^*}\frac{r_c}{w}\right) \tag{5}$$

According to previous studies and considering the intrinsic features of graphite (Ayatollahi & Aliha, 2011; Li et al., 2013; Sakai & Kurita 1996), the critical value of $\sigma_{\theta\theta c}$ can be assumed to be the tensile strength of materials, f_t. Thus, the mode I brittle fracture occurs when:

$$f_t = \frac{K_{If}}{\sqrt{2\pi r_c}}\left(1 + 3\frac{A_3^*}{A_1^*}\frac{r_c}{w}\right) \tag{6}$$

In order to use Eq. (6) for predicting the onset of fracture, one should first determine the critical distance r_c. There are several formulations in the literature for calculating the value of r_c (Bazant et al., 1991; Bazant & Planas 1998; Karihaloo, 1999; Schmidt, 1980). Recently, Ayatollahi and Akbardoost (2012) proposed a modified form of Schmidt's formula (Schmidt, 1980) for determining the value of r_c. The proposed formula can be written as:

$$r_c = \left[\frac{f_t\sqrt{2\pi} \pm \sqrt{2\pi f_t^2 - 12\dfrac{A_3^*}{A_1^*}\dfrac{K_{If}^{\,2}}{w}}}{6\dfrac{A_3^*}{A_1^*}\dfrac{K_{If}}{w}}\right]^2 \tag{7}$$

In this formula, K_{If} is the mode I fracture resistance, A_1^* and A_3^* are the dimensionless parameters for the coefficients of first and third stress terms in pure mode I loading. It has been also shown (Bazant & Planas, 1998; Karihaloo, 1995) that the value of r_c depends on the size of specimen and increases by increasing the specimen size. Therefore, the size dependent value of r_c should be considered in the proposed method. Here, a simplified formula proposed recently by Ayatollahi and Akbardoost (2012) is used for describing the size dependency of r_c:

$$r_c = \frac{A}{1 + \frac{B}{w}} \quad \textbf{(8)}$$

where the constant coefficients A and B are calculated by a linear regression on fracture resistance obtained from mode I tests conducted on specimens of different sizes. For calculating A and B, the value of r_c corresponding to each specimen size is first determined by replacing the fracture toughness (K_{If}) obtained from the experiment into Eq. (7). Then a linear regression based on $Y=M X + Q$, in which:

$$Y = \frac{1}{r_c}, \; M = \frac{B}{A}, \; X = \frac{1}{w}, Q = \frac{1}{A}$$

is used (see more details in Ayatollahi and Akbardoost (2012)). By simplifying Eq. (6), the variations of fracture toughness K_{If} versus the specimen size can be obtained from:

$$K_{If} = \frac{f_t \sqrt{2\pi r_c}}{(1 + 3\frac{A_3^*}{A_1^*}\frac{r_c}{w})} \quad (9)$$

in which the value of r_c is determined according to specimen size from Eq. (8). Since the proposed criterion is a modified form of the MTS criterion, it is named the MMTS criterion. In the next section, the size dependent values of fracture toughness for a polycrystalline graphite reported by Sakai and Kurita (1995) will be justified using the MMTS criterion. It will be also shown that the fracture toughness of graphite obtained from specimens of different crack lengths can be predicted using the MMTS criterion.

3. Graphite experiments

Sakai and Kurita (1995)conducted several experiments on the single-edge notched beam (SENB) for investigating the dependency of size and crack length on the fracture toughness of graphite. The schematic of the SENB sample and geometric notations are shown in Fig. 1. The graphite employed by Sakai and Kurita (1995) was an isotropic polycrystalline graphite (IG- 11) with the following mechanical properties: bulk density of 1.76 g/cm^3,Young's modulus of 9 GPa and tensile strength of 27 MPa. The samples were classified into two categories: 1) the specimen having similar geometry but different sizes, 2) the specimens with similar characteristic dimension, i.e. w, but different crack lengths a. In the first category, the widths of specimens were w =2.5, 5, 7.5, 10, 12.5, 17.5, 20 mm and their crack length ratio was constant and equal to 0.5. The second category was performed on the specimens with the constant width of 12.5 mm but different crack lengths of a =1.25, 2.5, 3.75, 5, 6.25, 7.5, 8.75, 10, 11.25 mm. For all specimens, the thickness t was 10 mm and the span to width ratio S/w was equal to 4.The dimensions of samples and loading conditions for the first and second test categories are listed in Table 1 and Table 2, respectively.

The fracture toughness for the SENB samples is often calculated from:

$$K_{If} = \frac{3P_u S}{2w^2 t}\sqrt{2\pi w}\, A_1^*, \quad \textbf{(10)}$$

where P_u is fracture load. Sakai and Kurita (1995) did not report the fracture loads of tested specimens and they only presented the values of fracture toughness as displayed in Table 1 and Table 2. As seen from Table 1, the fracture toughness of graphite depends on the specimen size and increases by increasing the size of specimen. Meanwhile, the fracture toughness of tested graphite is nearly constant for specimens larger than w =12.5 mm. Therefore, one can obtained the size-independent value of K_{If} by testing the specimen larger than the width of 12.5 mm. Table 2 also shows that the fracture toughness of graphite is nearly constant for specimens with crack lengths of a=1.25, 2.5, 3.75, 5, 6.25 and 8.75 mm and decreases for larger crack lengths. It means that the fracture toughness of graphite depends on the crack length when the crack length ratio is larger than 0.7.

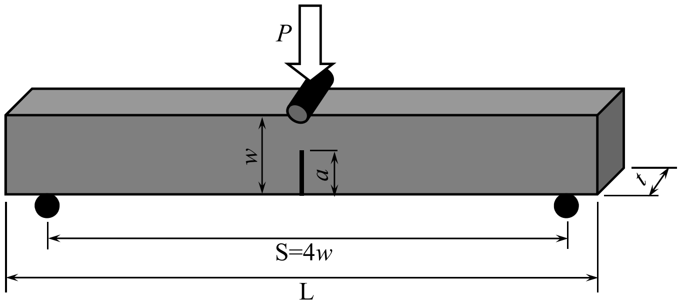

Fig. 1. The schematic of SENB specimen and its dimension notations

Table 1. Graphite specimens and their fracture parameters for size effect analysis

Category I (Size effect analysis) S/w=0.4 , a/w=0.5		
Specimen Dimensions (in mm) (L×w×t)(Sakai and Kurita 1995)	K_{If} (MPa.m$^{0.5}$) (Sakai and Kurita 1995)	r_c (mm)
20×2.5×10	0.59	0.062
30×5×10	0.62	0.075
40×7.5×10	0.65	0.085
50×10×10	0.68	0.094
60×12.5×10	0.7	0.102
80×17.5×10	0.705	0.104
90×20×10	0.71	0.106

Table 2. Graphite specimens and their fracture parameters for crack length effect analysis

Category II (crack length effect analysis) S/w=0.4 , w=12.5 mm	
Specimen Dimensions (in mm) (L×a×t)(Sakai and Kurita 1995)	K_{If}(MPa.m$^{0.5}$) (Sakai and Kurita 1995)
60×1.25×10	0.7
60×2.5×10	0.68
60×3.75×10	0.73
60×5×10	0.72
60×6.25×10	0.7
60×7.5×10	0.69
60×8.75×10	0.69
60×10×10	0.65
60×11.25×10	0.63

4. Results and discussion

In order to employ the MMTS criterion for size and crack length effects studies, the dimensionless form of the coefficients of first two terms in crack tip stress field , i.e. A_1^* and A_3^*, should be known. The non-dimensional parameters A_1^* and A_3^*, which are functions of S/w and a/w, can be determined analytically for simple geometries and numerically for complicated cracked bodies. Karihaloo and Xiao (2001) proposed the following equations for calculating the values of A_1^* and A_3^* for three-point bend specimens with S/w of 4:

$$A_1^*(\alpha) = \frac{\sqrt{\alpha}\,(1.9 + 0.41\alpha + 0.51\alpha^2 - 0.17\alpha^3)}{\sqrt{2\pi}\,(1-\alpha)^{3/2}(1+3\alpha)} \tag{11}$$

$$A_3^*(\alpha) = 0.6534 - 9.2406\alpha + 49.515\alpha^2 - 153.97\,\alpha^3 + 233.48\alpha^4 - 148.73\alpha^5 \tag{12}$$

where α in Eq. (11) and Eq. (12) is crack length ratio, i.e. $\alpha = a/w$. These equations will be used for predicting the fracture onset of graphite specimens by taking into account the effects of specimen size and crack length.

4.1. Size effect analysis

After calculating the values of A_1^* and A_3^* in Eq.(9), the value of critical distance should be determined. The values of r_c for each tested specimens are obtained from Eq. (7). For using this formula, the tensile strength (f_t) and mode I fracture toughness (K_{If}) should be known. The tensile strength of IG-11 graphite f_t was found to be 27MPafrom(Sakai and Kurita 1995). The mode I fracture toughness (K_{If}) was also extracted from Table 1 related to the size of specimen. The values of r_c calculated from Eq. (7) for each tested samples are shown in Table 1. According to this Table, the values of r_c depends on the specimen size and increases for bigger samples. Therefore, the size dependency of r_c should be considered. Eq. (8) is employed here for describing the variations of r_c versus the specimen size. For using Eq. (8), the values of A and B should be determined from a linear regression of the test results. The linear regression plot, as shown in Fig. 2, gives A = 0.106 mm and B = 1.806 mm. It is noteworthy that the values of A and B are calculated based on the first four test data and the experimental results for other specimen sizes (i.e. w =12.5, 17.5 and 20 mm) are assumed to be unknown and will be predicted using the MMTS criterion. After the calculation of A and B, the critical distance can be written as a function of specimen size. Fig. 3 illustrates the variations of r_c versus the specimen size obtained from Eq. (8).

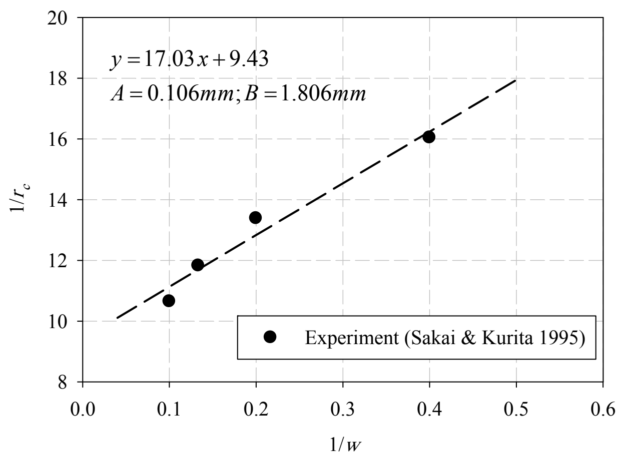

Fig. 2. Linear regression for calculating A and B in Eq. (8)

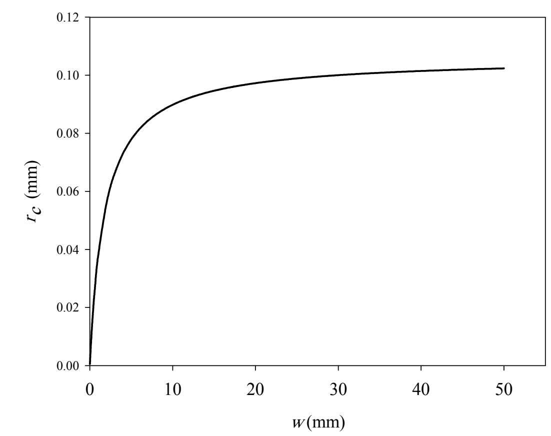

Fig. 3. The variations of critical distance versus the specimen size for graphite

Now, the size-dependent behavior of K_{If} can be presented using Eq. (9)as an asymptotic curve. Fig. 4 displays the variations of K_{If} relative to the specimen size together with the experimental results. As shown in this Figure, the MMTS criterion can provide good estimates for the fracture toughness of specimens whit width of w =12.5, 17.5 and 20 mm which were assumed to be unknown. Moreover, Fig. 4 indicates that the material fracture resistance K_{If} for infinitely large specimens is independent of specimen size and approaches a nearly constant value. The size-independent value of K_{If} can be obtained directly from Eq. (9) by taking $w$as infinity. According to Eq. (9), when $w \to \infty$ the value of K_{If} is calculated as 0.7 MPa.m$^{0.5}$. This value of K_{If} , which is denoted by K_{Ic} can be considered as the inherent fracture toughness of IG-11 graphite. One might argue that the value of K_{Ic} for IG-11 is less than those values obtained from the specimen sizes of w=17.5 and 20 mm (see Table 1). This is because K_{Ic} was calculated based on the linear regression of 4test data and the more accurate value of K_{Ic} can be achieved by using all test results. If all test data were used in the linear regression for determining the values of A and B, the value of K_{Ic} would be calculated as K_{Ic}=0.723 MPa.m$^{0.5}$.

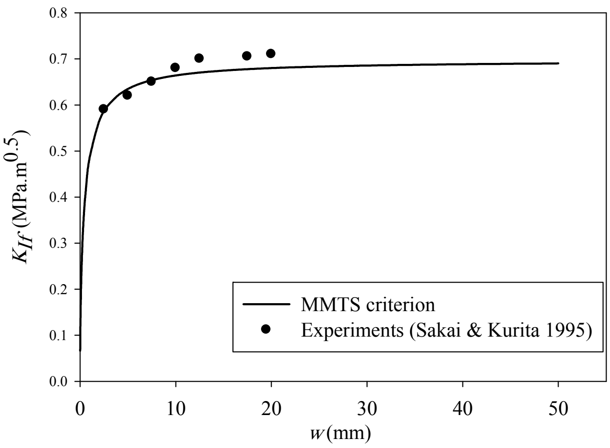

Fig. 4. The variations of fracture toughness of graphite with respect to the specimen size

4.2. Crack length effect analysis

The MMTS criterion or Eq. (6)is employed here to predict the fracture toughness of graphite obtained from the second test category. The specimens in the second test category have the same width but different crack lengths (see Table 2). By considering the tensile strength f_t as a constant material property, the left hand side of Eq. (6) would be a constant value for any given material. Thus, the following relation can be written for two specimens with different crack length ratios:

$$f_t = \frac{K_{If}}{\sqrt{2\pi r_c}}(1+3\frac{A_3^*}{A_1^*}\frac{r_c}{w})\bigg|_{(a/w)_1} = \frac{K_{If}}{\sqrt{2\pi r_c}}(1+3\frac{A_3^*}{A_1^*}\frac{r_c}{w})\bigg|_{(a/w)_2} \tag{13}$$

In order to derive a relation between the values of mode I fracture toughness for two specimens with different crack lengths, Eq. (13) can be rewritten as:

$$\frac{K_{If}\big|_{(a/w)_1}}{K_{If}\big|_{(a/w)_2}}=\frac{\left.\frac{1}{\sqrt{2\pi r_c}}(1+3\frac{A_3^*}{A_1^*}\frac{r_c}{w})\right|_{(a/w)2}}{\left.\frac{1}{\sqrt{2\pi r_c}}(1+3\frac{A_3^*}{A_1^*}\frac{r_c}{w})\right|_{(a/w)_1}} \tag{14}$$

The dimensionless parameters A_1^* and A_3^* in Eq. (14) can be respectively obtained from Eq. (11) and Eq. (12) for the SENB specimens with different crack length ratio α. Table 3 shows these parameters for each specimen in the second test category. Since all specimens in this test category have the same width of 12.5 mm, the critical distance r_c in Eq. (14) is taken to be as 0.102 mm according to the value obtained from SENB specimen of w=12.5 mm. By replacing the values of A_3^*/A_1^* corresponding to the specimens of the second test category and the value of r_c=0.102 mm in Eq. (14), the fracture toughness K_{If} for these specimens are estimated as shown in Table 3. It should be noted that the reference fracture toughness in Eq. (14) is taken to be 0.7 MPa.m$^{0.5}$ obtained from specimen with the width of w=12.5 mm.

Table 3. The values of dimensionless parameters and fracture toughness predicted by MMTS criterion for graphite specimens in the second test category

Specimen Dimensions (L×a×t) (in mm)	$\alpha=a/w$	A_1^*	A_3^*	A_3^*/A_1^*	K_{If} (MPa.m$^{0.5}$)		
					Experiment (Sakai and Kurita 1995)	MMTS	Error (%)
60×1.25×10	0.1	0.221	0.092	0.418	0.7	0.672	4
60×2.5×10	0.2	0.312	-0.120	-0.385	0.68	0.685	-1
60×3.75×10	0.3	0.405	-0.290	-0.715	0.73	0.690	5
60×5×10	0.4	0.527	-0.520	-0.988	0.72	0.695	3
60×6.25×10	0.5	0.706	-0.890	-1.261	0.7	0.700	0
60×7.5×10	0.6	1.000	-1.629	-1.629	0.69	0.707	-2
60×8.75×10	0.7	1.559	-3.303	-2.119	0.69	0.716	-4
60×10×10	0.8	2.895	-6.985	-2.413	0.65	0.721	-11
60×11.25×10	0.9	8.275	-14.437	-1.745	0.63	0.709	-12

Table 3 indicates that the MMTS criterion can provide good estimates for the fracture toughness obtained from specimens with the crack length ratios of α=0.1 to 0.7. It is also shown in Table 3 that there is a discrepancy of more than 10% between the values of fracture toughness obtained experimentally from specimens with crack length ratios of 0.8 and 0.9 and those values predicted by the MMTS criterion. The main reason for this discrepancy can be that the critical distance employed in this section is taken to be constant for different crack length ratios. The previous studies (Bazant & Planas 1998; Wittmann & Hu 1991) have shown that the fracture process zone around the crack tip is affected by the back boundary when $\alpha > 0.7$. Therefore, the MMTS criterion cannot predict the accurate fracture toughness for specimens with $\alpha > 0.7$.

It is noted that the stress field around the crack tip is affected by the back boundaries when α is larger than 0.7 and thus the William's series expansions cannot characterize the near tip stress components. Therefore, the laboratory specimens with $\alpha > 0.7$ are not usually employed for determining the fracture properties of materials(Hu & Wittmann, 2000). Moreover, the real graphite structures will be failed before the crack length ratios reaches to 0.7 and, therefore, the components with $\alpha > 0.7$ are practically invalid. Consequently, one can concluded that the MMTS criterion is able to predict the onset of fracture in real graphite structures.

5. Conclusions

The size and crack length effects on the fracture toughness of graphite was studied using the modified maximum tangential stress (MMTS) criterion which takes into account the A_3 term in the

Williams series expansion in addition to the conventional singular term of stress. The critical distance r_c is also assumed to be size dependent in the MMTS criterion when the size effect on fracture toughness is investigated. It was shown that the theoretical estimates obtained from the MMTS criterion are in good agreement with the experimental results reported in the previous studies. To analyze the crack length effect on the mode I fracture resistance of graphite by using MMTS, the critical distance r_c was taken to be the same value for specimens with different crack lengths. Comparison between the fracture toughness predicted by MMTS and those experimental values reported in previous studies indicated that the MMTS criterion can provide good estimates for fracture toughness of specimen with crack length ratios less than 0.7, i.e. $\alpha \leq 0.7$. Since the crack tip is close to the back boundaries for $\alpha > 0.7$(which affects the stress field around the crack tip), the MMTS criterion is not able to predict accurately the value of K_{If} obtained from the experimental results.

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