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Modes I-II-III stress intensity factors of a semi-elliptical surface crack at a round bar under torsion loading by FEM and DBEM

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A B S T R A C T

The corner point singularity of surface cracks by finite element method (FEM) has become a numerical concern decades ago. The literature showed that the stress intensity factors (SIFs) at the corner points were often excluded. Further, most SIFs were reported for larger ratios of the crack depth over cylinder diameter. This paper presents the SIFs (Modes I, II and III) of a semi-elliptical surface crack at a solid round bar under torsion. The tetrahedral and hexahedral elements were used in the finite element modelling. The effects of the loading mode and the crack aspect ratio on the corner point singularity were discussed. The tetrahedral meshing was generally observed to be more suitable for modelling relatively small surface cracks, particularly in respect to the corner point singularity. For all loading modes, the SIFs away from the corner points of using the tetrahedral meshing were found to have fairly good agreement with those by dual boundary element method (DBEM).

1. Introduction

The formation of flaws/cracks requires an extra effort and downtime allocated for maintenance/repair. Since flaws/cracks cannot be removed, a suitable method has to be planned to measure and estimate the integrity of cracked components under given service conditions. Systematic approaches have to be arranged to assess a crack and its effects, and to evaluate the acceptability for being used in the service period. The stress intensity factor is known to be a major characterization and prediction parameter (Aliabadi & Rooke, 1991). The concept of SIF was first introduced in 1957 (Williams 1957). The linear elastic fracture mechanics (LEFM) concept has been broadly applied in real-world engineering structural problems particularly to evaluate the behaviors of cracked components. The roles of numerical fracture mechanics on the advancement of linear elastic fracture mechanics have been essential, and the method becomes a practical tool to the structural engineers/designers. Kuna (2013) described that the evaluation of fracture and damage processes is critically substantial in the development and dimensioning of an engineering component in order to ensure its structural integrity. Any failures may cause serious consequences for the injuries/lives, environmental damages and lost revenues. The analyses of notches, cracks and similar defects under service conditions are critically important. Solutions of the stress intensity factors of a surface crack at a solid round bar are required for the estimation of the residual strength and the acceptability of crack sizes as stated in the practice codes such as API 579-1/ASME FFS-1 (for fitness for service) and BS 7910 (for acceptability of defects/flaws) and. A number of SIF solutions for various crack geometries have been developed since decades ago.

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A number of researchers have reported fracture analysis works using different methods, materials and geometries (Aliha and Ayatollahi 2008, Aliha and Ayatollahi 2010, Aliha and Ayatollahi 2012, Aliha and Ayatollahi 2013, Abd-Elhady 2013, Predan *et al*. 2013, Citarella *et al*. 2014, Zakavi *et al.* 2019, Abd-Elhady 2020). The pioneer application of finite element analysis to evaluate the SIFs of 2D cracked bodies was described by Dixon and Pook (1969). The role of finite element analysis on the development of LEFM is greatly noticed. Wawrzynek and Ingraffea (2003) reported the numerous works on the singular crack-tip stress and strain fields by the FEM. The singularity issue around a 3D corner point at a crack front intersecting a free surface has attracted attention. The early concepts pertaining to the corner point singularities were reported by Benthem (1997, 1980), Bažant and Estenssoro (1979) and later by Pook (1994). A number of related works on 3D corner point singularities by FEA have been found in literature (Shivakumar and Raju 1990, De Matos and Nowell 2008, Kotousov 2010, Lazzarin and Zappalorto 2012, Kotousov *et al*. 2012, Kotousov *et al*. 2013, Pook 2013, Pook *et al*. 2014, Pook *et al*. 2015, Pook *et al*. 2016, Pook *et al*. 2017, Aliha *et al.* 2019, Bahmani *et al.* 2020, Teh *et al*. 2021).

Aliabadi (1997) has reviewed the application of the BEM to the fracture mechanics. He mentioned that the BEM has been applied to fracture mechanics since the 1970s. The key concept of BEM as a solution approach is adopted by employing the weighting functions called the fundamental solutions that satisfy the governing equation. Only the boundary surfaces enclosing the 3D solid domain need to be divided into elements (Aliabadi 1997). The application of dual integral equations was initially reported by Watson (1986) for 2D problems utilizing the displacement equation and its normal derivative. Next, a DBEM formulation for 2D problems was presented by Portela, *et al*. (1992). An early study on 3D crack problems by DBEM was described by Gray et al. (1990). Next, Mi and Aliabadi (1992) and Cisilino and Aliabadi (1999) presented further newly developed DBEM formulations for 3D crack problems.

At the initial stage of crack growth, the crack front of a surface crack is observed to form a semi-circular/elliptical front (Athanassiadis *et al*. 1981, Mackay and Alperin 1985, Forman and Shivakumar 1986, Lorentzen *et al*. 1986, Hojfeldt and Ostervig 1986, Caspers and Mattheck 1987). In particular, small surface planar flaws (with nearly semi-elliptical shapes) are typically observed in the cracked round bars as reported by Lorentzen *et al*. (1986). The SIF solutions for a surface crack in solid round bars by FEM have been presented in literature (Raju and Newman 1986, Carpinteri 1992, Carpinteri 1993, Carpinteri and Brighenti 1996a, Carpinteri and Brighenti 1996b, Fonte and Freitas 1997, Lin and Smith 1997, Lin and Smith 1998, Fonte and Freitas 1999, Guo *et al*. 2003, Shin and Cai 2004, Carpinteri and Brighenti 2006, Toribio *et al*. 2009a, Ismail *et al*. 2012, Citarella *et al*. 2014, Toribio *et al*. 2014). Toribio *et al*. (2009b) reviewed the stress intensity factors of surface cracks at solid round bars subjected to tension. However, in literature, most SIF solutions were presented for larger ratios of the crack depth over round bar diameter, and the SIF solutions at the corner points were often excluded. Shin and Cai (2004) reported the stress intensity factors at the corner points for some larger crack sizes. Most modelling works on a surface crack at solid round bars by FEA used 20-node brick elements for the region away from the crack front and 15-node special wedge elements applied to the cracked region. In particular, Guo *et al*. (2003) applied the 20-node hexahedron elements and singular elements by shifting the mid-side nodes to their 1/4-point position. Fonte and Freitas (1999) approximated the corner points using the nearest nodes to the corner points to obtain the approximated stress intensity factors. The analyses of the cracked solid round bar by the DBEM based codes have been documented in literature as reported by Citarella *et al*. (2014), Joseph *et al*. (2014, 2020), Chandra *et al*. (2014, 2016) and Ramezani *et al*. (2018a, 2018b), in which the SIF solutions at the corner points were presented.

Any updates on the fracture behaviors, especially on the corner point singularities and the SIF solutions for moderately small surface cracks, are desirable. The SIFs for small crack sizes are key information used to analyze the stable stage of crack growths. This paper presents the stress intensity factors (Modes I, II and III) of a semi-elliptical surface crack at a round bar under torsion. The finite element based software of ANSYS® 2019 and the dual-boundary element software package of BEASY® are used for carrying out the simulations.

2. Materials and Method

The *J*-integral method is chosen to calculate the stress intensity factors. Poisson's ratio $\upsilon = 0.3$ is used in all models. The stress intensity factors are presented in the normalized values K/K_o in which K_o is defined as

$$K_o = \tau\sqrt{\pi . a}, \tag{1}$$

where τ is any maximum nominal shearing stress due to an applied torsion; a is the crack depth (a = 1 length unit). The elements and other parameters used in FEM and DBEM are depicted in Fig. 1.

It has been known that the SIFs along the crack front of a surface crack at a smooth round bar subjected to a torsion loading would be approximately zero (0) for the inclination angles of 0° (Mode I) and 45° (Modes II and III). Thus, to have non-zero SIFs along the crack front and to have a mixed-mode loading under torsion, a slanted surface crack with an inclination angle of 22.5° is chosen and introduced at a solid round bar as depicted in Fig. 1a. The finite element models are developed by using ANSYS® 2019 with 10-node tetrahedron and 20-node hexahedral elements (Figs. 1b and 1c). The surface elements used in DBEM modelling are shown in Fig. 1d. The crack aspect ratio *a/c* defined in ANSYS® 2019 and BEASY® is shown in Fig. 1e. Points *A* and *C* respectively show the deepest and corner points at a crack front. As the crack aspect ratio *a/c* changes, the intersection angle of the crack front would change (see Fig. 1e.) The numerical issue in three-dimensional crack analyses by FEM has been reported at the corner points intersecting the free boundaries (Pook 1994). The singularity power at the corner points is influenced by the intersection angle (Fig. 1e) and the Poisson's ratio (Benthem 1997, Benthem 1980, Bažant and Estenssoro 1979, and Pook 1994). The stress states around a corner point are the stress results from the stress intensity factors and corner point singularities as reported by Pook *et al*. (2017).

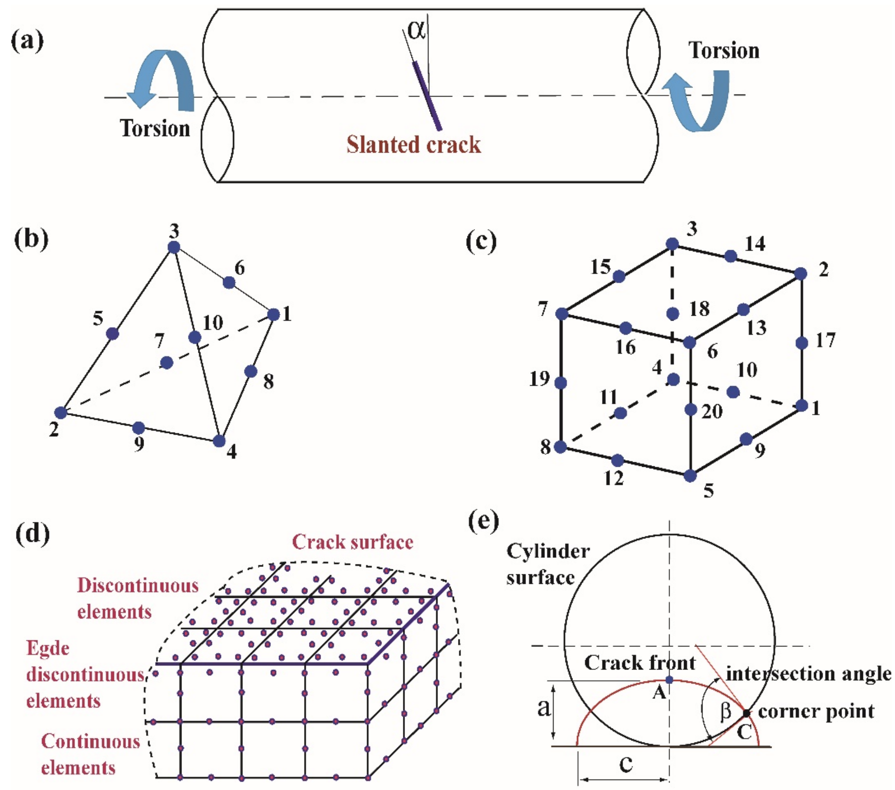

Fig. 1. (a) A slanted crack at a solid round bar under torsion; (b) A 10-node tetrahedral element (FEM); (c) A 20-node hexahedral element (FEM); (d) Elements used in DBEM; (e) Crack parameters and intersection angle.

The 10-node tetrahedron and 20-node hexahedron elements used around the crack region are depicted in Fig. 2a. The meshing at the fracture affected zone around the crack front and corner points may be sized by defining the largest contour radius. The largest contour radius (Fig. 2a) is empirically taken to be 0.25-0.5 *a* (the crack depth) to produce ultra-fine meshing around the corner points and crack front. Figure 2b shows the section views of tetrahedral and hexahedral meshing around the crack regions.

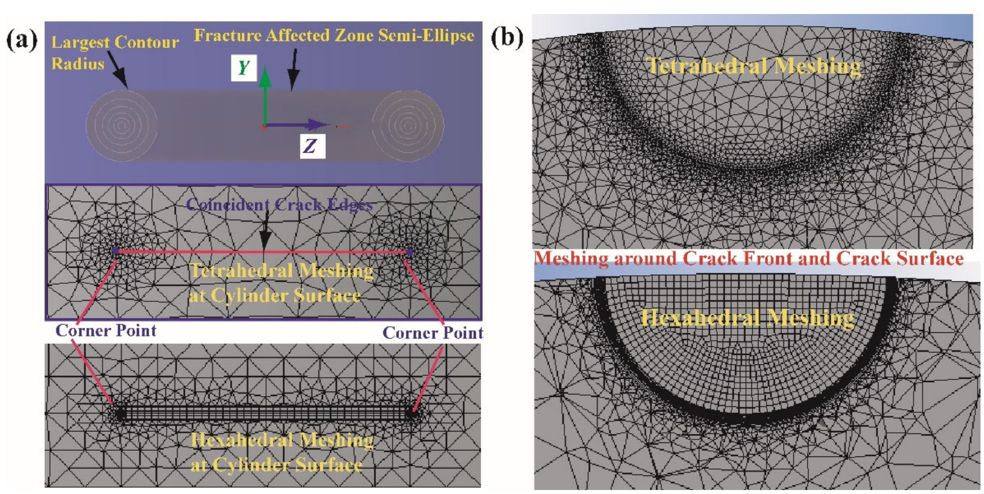

Fig. 2. (a) Meshing strategy using tetrahedron and hexahedron elements; (b) Section views of meshing around the crack front and crack surfaces.

The DBEM uses the 2D edge discontinuous quadrilateral elements to treat the singularities problems. Thus, the singularities at the corner points in 3D crack problems can be more suitably treated by the DBEM. The DBEM used in BEASY® employs two independent boundary integral equations (Mi and Aliabadi 1992, Cisilino and Aliabadi 1999) for treating the crack surfaces. The 3D crack modelling by DBEM using the elements depicted in Fig. 1d can be described as follows (Mi and Aliabadi 1992, Cisilino and Aliabadi 1999):

- The discontinuous elements are used for the crack surfaces.
- The edge discontinuous quadrilateral or triangular elements are used to model surfaces that are intersecting a crack surface.
- The upper crack surface is treated by the displacement integral equation, meanwhile the traction integral equation is used to treat the lower crack surface.

3. Results and Discussion

In this section, the SIFs are plotted over the normalized location points at the crack front of which 0 and 1 denote the corner points. To show the ability of tetrahedron elements in respect to the corner point singularity by FEM, the comparisons of SIFs of a semi-elliptical surface crack with those by hexahedron elements are made. For this purpose, the crack parameters for *a/d* = 0.025, 0.05 and 0.1 with the crack aspect ratio of *a/c* = 1 and 2 are chosen. The normalized SIFs K_I/K_o, K_{II}/K_o and K_{III}/K_o along the crack front are depicted in Fig. 3. Figures 3a and 3b show the normalized Mode I SIFs for α = 45°. In general, the FEM and DBEM results are observed to be in good agreement. The SIF calculations by FEM using the hexahedron element are found to have a singularity issue at the corner points. This issue is likely due to the meshing shape of the hexahedron elements at the intersection between the crack front and free surface. For the opening mode (Mode I) SIFs at the corner points, the tetrahedron element produces satisfactory estimations and shows a good agreement with those by DBEM. Teh *et al*. (2021) reported that less singularity issue at the corner points was observed at *a/c* = 1 or larger.

The normalized Mode II SIFs for α = 0° are presented in Figs. 3c and 3d. Mode II SIFs by FEM and DBEM for *a/c* = 1 are fairly shown to be in good agreements. Meanwhile, some offsets of the SIFs for *a/c* = 2 are observed at some distances next to the corner points. This phenomenon may be due to the twisting effect as the ratio of the crack depth over the crack length is getting larger. However, for the shearing mode (Mode II), good agreements between the FEM (using both tetrahedral and hexahedral elements) and DBEM results are noticed at the corner points.

Fig. 3e and Fig. 3f show the normalized Mode III SIFs for α = 0°. For the tearing mode (Mode III), the FEM calculations of using the hexahedron element seem to have a problem for modelling small crack sizes. This simply indicates that the hexahedron meshing would have a singularity issue when the ratio of *a/c* is large enough, and the issue is getting lesser as the ratio of *a/d* increases. For small crack sizes (*a/d*), the tetrahedron element demonstrates good SIF calculations at the crack front away from the corner points as compared to those by DBEM. For the tearing mode, the offsets are clearly observed at the corner points. Similar feature was observed at the case under tension loading (Teh *et al*., 2021).

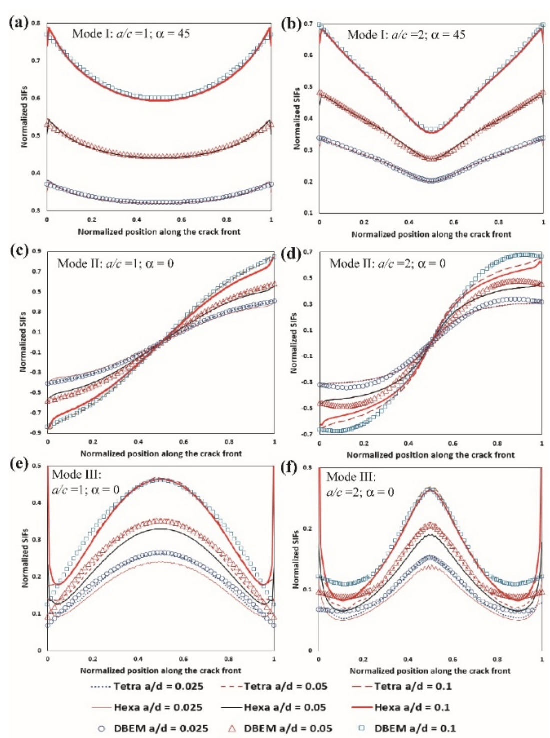

Fig. 3. Normalized SIFs by FEM and DBEM for different modes, *a/c* and *a/d*

As mentioned in Section 2, in order to have non-zero SIFs along the crack front, a slanted surface crack (with an inclination angle of 22.5°) is introduced to demonstrate a mixed-mode loading under torsion. The small crack sizes of *a/d* = 0.025 and 0.05 are chosen. Figure 4 presents the normalized Mode I SIFs by FEM and DBEM for different *a/c* and *a/d*. A surface crack with a smaller crack aspect ratio (e.g. *a/c* = 0.333) would have a smaller crack front intersection angle (Teh *et al*. 2021), causing a greater singularity problem at the corner points (see Fig. 4a). The normalized Mode I SIFs (*a/c* = 0.333) away from the corner points by FEM and DBEM are generally found to be in good agreements. Similar features are also observed for the larger *a/c* (Figs. 4b, 4c and 4d). As mentioned earlier, the SIF calculations by FEA of using the hexahedron element will have offsets at the corner points. For the opening mode and relatively small *a/c*, the hexahedral meshing promotes the corner point singularities more severely than that of using the tetrahedral meshing. Similar to Mode I SIFs, Mode II SIF calculations by FEM (*a/c* = 0.333) show offsets at the corner points (Fig. 5a). The hexahedral meshing fairly produces satisfactory Mode II SIFs at the corner points (Figs. 5b and 5c). Both tetrahedral and hexahedral meshing generate similar Mode II SIFs for larger *a/c* (see Figs. 3c, 3d and 5d).

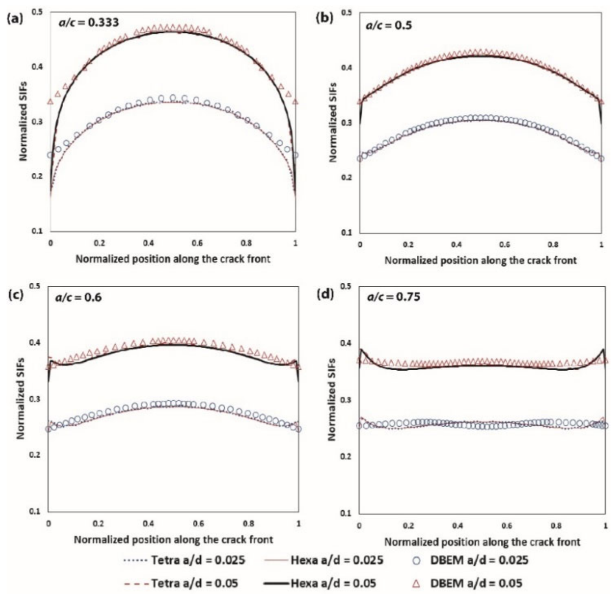

Fig. 4. Normalized Mode I SIFs by FEM and DBEM for different *a/c* and *a/d* at $\alpha = 22.5^{o}$

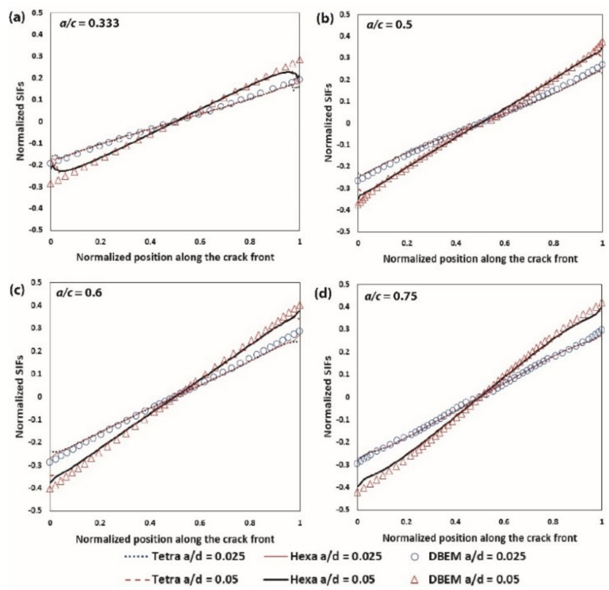

Fig. 5. Normalized Mode II SIFs by FEM and DBEM for different *a/c* and *a/d* at $\alpha = 22.5^{o}$

For the tearing mode (Mode III) and relatively small *a/c* (see Fig. 6), the SIFs away from the corner points (by FEM) are shown to be in good agreement with those by DBEM. These are in contrast to the SIFs of using the hexahedral meshing for larger *a/c* ($\alpha = 0^{o}$) as presented in Figs. 3e and 3f. In all cases for the tearing mode, singularity issue is observed at the corner points. Overall, it may be observed from Figs. 3-6 that the tetrahedral meshing seems to be preferable for modelling semi-elliptical surface cracks at round bars.

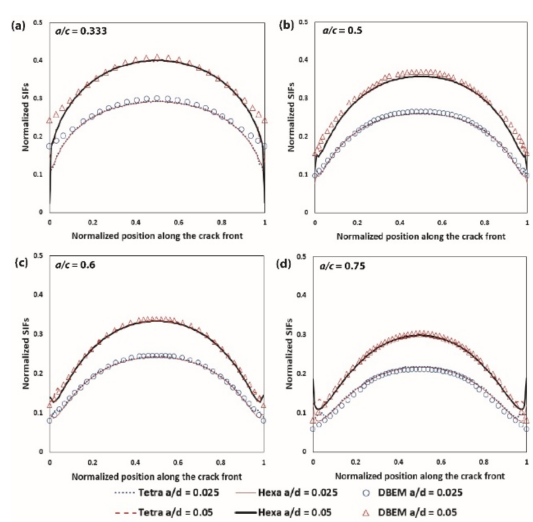

Fig. 6. Normalized Mode III SIFs by FEM and DBEM for different *a/c* and *a/d* at $\alpha = 22.5^o$

4. Conclusions

The SIFs of a semi-elliptical surface crack at a solid round bar under torsion by FEM and DBEM were presented. Some concluding remarks may be fairly drawn as follows:

● The tetrahedral meshing was generally observed to be more suitable than the hexahedral meshing for modelling relatively small surface cracks at round bars.
● In all cases, the corner point singularity issue was observed at a smaller a/c (e.g. *a/c* = 0.333).
● For all loading modes, the SIFs away from the corner points of using the tetrahedral meshing were found to have fairly good agreement with those by DBEM.
● For any given a/c, the hexahedral meshing was observed to have singularity problems at the corner points for the opening mode (Mode I) and the tearing mode (Mode III).
● For larger a/c and small crack sizes, Mode III SIFs away from the corner points of using the hexahedral meshing were shown to be deviated from those by DBEM.
● The tetrahedral meshing generally had corner point singularities for Mode III.
● Mode II SIFs for a/c > 0.33 of using both tetrahedral and hexahedral meshing generally showed good agreement with those by DBEM.

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