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A multi-attribute ranking approach based on net inferiority and superiority indexes, two weight vectors, and generalized Heronian means

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A B S T R A C T

In this paper, we propose a three-phase multi-attribute ranking approach having as outcomes of the modeling phase what we refer to as *net superiority* and *inferiority indexes.* These are defined as bounded differences between the classical superiority and inferiority indexes. The suggested approach herein named *MANISRA* (**M**ulti-**A**ttribute **N**et **I**nferiority and **S**uperiority based **R**anking **A**pproach) employs in the aggregation phase a bi-parameterized family of *compound averaging operators* (CAOPs) referred to as *generalized Heronian OWAWA (GHROWAWA)* operators having the usual OWAWA operators as special instances. Note that the new defined operators are built by using a composition of an arbitrary bi-parameterized binary Heronian mean with the weighted average (WA) and the ordered weighted averaging (OWA) operators. Also, note that the current developed MANISRA method generalizes the superiority and inferiority ranking (SIR-SAW) method which is known to coincide with the quite popular PROMETHEE II method when the net flow rule is used. With net superiority and inferiority indexes and GHROWAWA operators, we are better equipped to rank *rationally* pre-specified alternatives. The basic formulations, notations, phases and interlocking tasks related to the proposed approach are presented herein and its feasibility and effectiveness are shown in a real problem.

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1. Introduction

Quite often the decision processes of multi-attribute decision making (MADM) methods are composed of three phases, i.e., *modeling*, *aggregation* and *exploitation* phases. In the modeling phase, marginal utility functions, local priorities, regret and rejoicing values, degrees of preference, degrees of satisfaction, inferiority and superiority indexes, etc., are produced to serve as *input arguments* in the aggregation phase. In the present work, we advocate the use of *net inferiority* and *superiority indexes* obtained by from the traditional indexes introduced by Xu (2001). The new defined indexes are reliable and more-informative than the usual ones. In the aggregation phase, *averaging operators* are used to summarize the input arguments produced in the modeling phase. Different types of averaging operators could be found in the academic literature: (1) s*imple averaging operators*, e.g., the weighted average (WA) operator, the weighted geometric averaging (WGA) operator, the generalized weighted averaging (GWA) operator, the quasi-weighted averaging (Quasi-WA) operator, the ordered weighted averaging (OWA) operator (Yager, 1988; Yager & Kacprzyk, 1997; Yager et al., 2011; Emrouznejad & Marra, 2014), the ordered weighted geometric averaging (OWGA) operator (Xu & Da, 2002), the

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generalized ordered weighted averaging (GOWA) operator (Yager, 2004), the quasi-ordered weighted averaging (Quasi-OWA) operator (Fodor et al., 1995), and (2) *compound averaging operators* (CAOPs), e.g., the weighted ordered weighted averaging (WOWA) operator (Torra, 1995), the hybrid averaging (HWA) operator (Xu & Da, 2003), the double weighted ordered averaging (MO2P) operator (Roy, 2007), the ordered weighted averaging-weighted average (OWAWA) operator (Merigo, 2012), the semi-uninorm based ordered weighted averaging (SUOWA) operator (Llmazares, 2015), etc.
The above CAOPs unifying the operators WA and OWA in the same formulation exploit the so-called *importance weights* (or, attribute weights) and *preferential weights* (or, rank weights) in order to make the most of the *aggregation mechanisms* of both operators. In addition, according to Reimann et al. (2017), the operators WA and OWA *represent differently* the preferences of decision makers. It is equally important to remind that the importance weights are associated with WA and that the preferential weights are associated with OWA. Additionally, according to Labreuche (2016), the aforementioned types of weighting coefficients *could be provided* by decision makers. It is also of crucial importance to point out, at this stage, that the *validity* of the results of most of the CAOPs so far mentioned has often been *questioned*, mainly because of major violations of *desirable 'natural' requirements* (e.g., endpoint-preservation, monotonicity in the arguments, monotonicity in the weights and internality, etc.). Note that OWAWA operators (see Merigo (2012) for a detailed presentation) are *appealing* because they satisfy all the desirable requirements, and especially because they take into account the degree of importance that each operator has in the formulation of the resulting CAOP. Thus, in order to summarize the aforesaid net inferiority and superiority indexes in the aggregation phase of our approach, we advocate the use of a bi-parameterized family of CAOPs which will be referred to as generalized Heronian OWAWA (GHROWAWA) operators having the OWAWA operators as special instances (see Subsection 2.2). In exploitation phase, a *choice*, *ranking* or *sorting* problem could be envisaged (Roy, 1996). In this work, we deal with the crisp multi-attribute ranking problem of pre-specified alternatives.

The central originality of this work is to demonstrate how the new defined net superiority and inferiority indexes, two weight vectors and the bi-parameterized generalized Heronian means ***can be put together*** *to establish an original and useful multi-attribute ranking approach which generalizes the SIR-SAW and PROMETHEE II methods.*

Thus, this work is intended to develop a ranking approach herein referred to as **M**ulti-**A**ttribute **N**et **I**nferiority and **S**uperiority based **R**anking **A**pproach (MANISRA) which exploits in the aggregation phase the above-mentioned CAOPs to summarize the aforesaid net superiority and inferiority indexes produced in the modeling phase to get the *overall net superiority and* ***i****nferiority indexes* from where the choice-worthiness grades of predetermined alternatives are derived. The remainder of this paper is structured as follows. In the Sections 2 and 3, we present the material essential for the understanding of the basic philosophy of the MANISRA method. In Section 4, we illustrate the suggested approach by means of a real-world logistics service provider (LSP) ranking problem. And, in Section 5, we conclude the article with some remarks and ideas for future research.

2. Mathematical tools

2.1 Basic problem

To begin, the problem formulation can be set out as follows.

Given:

1. m feasible alternatives $\{A_i, i = 1, \ldots, m\}$,
2. n relevant attributes $\{g_j, j = 1, \ldots, n\}$,
3. A $m \times n$ performance table, $[a_{ij}]$, where a_{ij} denotes the attribute value of alternative A_i with respect to attribute g_j,
4. An importance weight vector $P = (p_1, p_2, \ldots, p_n)$ satisfying $p_j \in [0, 1]$ and $\sum_{j=1}^{n} p_j = 1$,

5. A preferential weight vector $W = (w_1, w_2, \dots, w_n)$ such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$,
6. A parameters $\beta \in [0,1]$,
7. A parameter $\omega \in [0, \infty]$.

Goal:

Rank the predetermined alternatives using their net inferiority and superiority indexes along with CAOPs whose formulas will be set out (hereafter, Subsection 2.3).

2.2 Definitions related to input arguments

2.2.1 The generalized criteria

Let a_{lj} and a_{kj} be the respective attribute values of two alternatives A_l and A_k with respect to a given cardinal attribute g_j, then the difference $d_{lk} = a_{lj} - a_{kj}$ is meaningful. Additionally, given $f_j(d_{lk})$ an appropriate generalized criterion function (Brans & Vincke, 1985; Brans et al., 1986), the intensity of preference of A_l over A_k given g_j is $P_j(A_l, A_k)$ where $P_j(A_l, A_k) = f_j(a_{lj} - a_{kj}) = f_j(d_{lk})$. Also, if $\mathbb{R}$ stands for the set of real numbers, the function $f_j(d_{lk})$ is a non-decreasing function from $\mathbb{R}$ to [0,1] such that $f(d_{lk}) = 0$ for $d_{lk} \leq 0$. Six *generalized criteria* were introduced in (Brans & Vincke, 1985; Brans et al., 1986) as shown in Table 1. The parameters Δ and Δ' presented in Table 1 are respectively preference and indifference thresholds.

Table 1
Generalized criteria

Type 1 True-criterion	Type 2 Quasi criterion	Type 3 Criterion with linear preference
$f_j(d_{ik}) = \begin{cases} 1 & if\ d_{ik} > 0 \\ 0 & if\ d_{ik} \leq 0 \end{cases}$	$f_j(d_{ik}) = \begin{cases} 1 & if\ d_{ik} > \Delta \\ 0 & if\ d_{ik} \leq \Delta \end{cases}$	$f_j(d_{ik}) = \begin{cases} 1 & if\ d_{ik} > \Delta \\ \frac{d_{ik}}{\Delta} & if\ 0 < d_{ik} \leq \Delta \\ 0 & if\ d_{ik} \leq 0 \end{cases}$
Type 4 Level criterion	**Type 5 Criterion with linear**	**Type 6 Gaussian criterion preference indifference area**
$f_j(d_{ik}) = \begin{cases} 1 & if\ d_{ik} > \Delta \\ \frac{1}{2} & if\ \Delta' < d_{ik} \leq \Delta \\ 0 & if\ d_{ik} \leq \Delta' \end{cases}$	$f_j(d_{ik}) = \begin{cases} 1 & if\ d_{ik} > \Delta \\ \frac{d_{ik} - \Delta'}{\Delta - \Delta'} & if\ \Delta' < d_{ik} \leq \Delta \\ 0 & if\ d_{ik} \leq \Delta' \end{cases}$	$f_j(d_{ik}) = \begin{cases} 1 - \exp(\frac{-d_{ik}^2}{2\sigma^2}) & if\ d_{ik} > 0 \\ 0 & if\ d_{ik} \leq 0 \end{cases}$

2.2.2 Net inferiority and superiority indexes

First, we remind below the definitions of inferiority and superiority indexes introduced by Xu (2001), then we define the net inferiority and superiority indexes .

Definition 2.1 The inferiority index (I-index) $I_j(A_i)$ and superiority index (S-index) $S_j(A_i)$ are respectively defined by

$$I_j(A_i) = \sum_{K=1}^{m} P_j(A_K, A_i) = \sum_{K=1}^{m} f_j(a_{kj} - a_{ij}) = \sum_{k=1}^{m} f_j(d_{ki}), \quad (1)$$

$$S_j(A_i) = \sum_{K=1}^{m} P_j(A_i, A_K) = \sum_{K=1}^{m} f_j(a_{ij} - a_{kj}) = \sum_{k=1}^{m} f_j(d_{ik}). \quad (2)$$

Using the so defined indexes, we now introduce the net inferiority index (net I-index) $I_j^*(A_i)$, and the net superiority index (net S-index) $S_j^*(A_i)$ as follows.

Definition 2.2 The net I-index and net S-index of alternative $A_i$with respect to attribute g_j are respectively defined by

$$\mathrm{I}_j^*(\mathrm{A}_i) = \mathrm{I}_j(\mathrm{A}_i) \ominus \mathrm{S}_j(\mathrm{A}_i), \quad (3)$$
$$\mathrm{S}_j^*(\mathrm{A}_i) = \mathrm{S}_j(\mathrm{A}_i) \ominus \mathrm{I}_j(\mathrm{A}_i), \quad (4)$$

where $\ominus$ denotes the bounded-difference operator defined by Zadeh (1975). Note that the net I-index is a *cost indicator* (the lower the better), whereas the net S-index is a *benefit indicator* (the higher the better). In addition, they lie in the closed real interval I = [0, m-1].
From now on, we will associate with each alternative A_i a pair of descriptive *n*-dimensional profiles:
1. The *profile of net I-indexes*

$$\mathrm{I}^*(\mathrm{A}_i) = (\mathrm{I}_1^*(\mathrm{A}_i),\ \mathrm{I}_2^*(\mathrm{A}_i), \ldots, \mathrm{I}_n^*(\mathrm{A}_i)) \quad (5)$$

2. The *profile of net S-indexes*

$$\mathrm{S}^*(\mathrm{A}_i) = (\mathrm{S}_1^*(\mathrm{A}_i),\ \mathrm{S}_2^*(\mathrm{A}_i), \ldots, \mathrm{S}_n^*(\mathrm{A}_i)) \quad (6)$$

2.3 Definitions related to averaging aggregators

Assume $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n) \in I^n$, to produce a summary of the components of the *n* -vectors *x* and *y*, we will be exclusively concerned with using some specific CAOPs. Thus, we next turn our attention to a presentation of the CAOPs of interest.

2.3.1 Averaging operators involved

The *inner* averaging operators considered here are the familiar weighted average (WA_P) operator and the non-conventional ordered weighted averaging (OWA_W) operator (Yager, 1988). The weighted average (WA_P) operator is one of the most popular aggregation operators found in the literature. It has been extensively used in a great number of applications including statistics, economics and engineering. It can be defined as follows.

Definition 2.3 A weighted average (WA_P) operator acting on the interval *I* having an associated *n*-dimensional importance weight vector *P* is defined to be the mapping $\mathrm{WA}_P : I^n \to I$ such that

$$\mathrm{WA}_P(\mathrm{x}) = \sum_{j=1}^{n} \mathrm{p_j}\, \mathrm{x_j}. \quad (7)$$

The ordered weighted averaging (OWA_W) operator is an aggregation operator that provides a parameterized family of aggregation operators between the minimum and the maximum values. It can be defined as follows.

Definition 2.4 An ordered weighted averaging (OWA_W) operator acting on the interval *I* and having an associated n-dimensional preferential weight vector W is defined to be the mapping $\mathrm{OWA}_W : I^n \to I$ such that

$$\mathrm{OWA}_W(\mathrm{x}) = \sum_{j=1}^{n} \mathrm{w_j}\, \mathrm{x_{(j)}}, \quad (8)$$

where $\mathrm{x_{(j)}}$ stands for the *j*th largest element among the $\mathrm{x_j}$s. Let us now recall the definition of the OWAWA operator introduced by Merigo (2012).

Definition 2.5 An OWAWA operator acting on the interval I and having a compensation parameter β, an n-dimensional importance weight vector P, and an n-dimensional preferential weight vector W is defined to be the mapping $\mathrm{M}_{\beta}^{P,W}: I^{n} \rightarrow I$ such that

$$\mathrm{M}_{\beta}^{P,W}(x) = \beta \times \mathrm{OWA}_{W}(x) + (1-\beta) \times \mathrm{WA}_{P}(x)\,. \tag{9}$$

Before introducing the generalized Heronian OWAWA operator, we need to recall the definition of generalized Heronian mean in the sense of Janous (2001).

Definition 2.6 Let a and b be two non-negative real numbers. The generalized Heronian mean $\mathrm{HM}_{\omega}(a,b)$ of a and b is defined by

$$\mathrm{HM}_{\omega}(a,b) = \begin{cases} \dfrac{a+b+\omega\sqrt{ab}}{2+\omega}, & 0 \le \omega < \infty \\ \sqrt{ab}, & \omega = \infty \end{cases} \tag{10}$$

So, we now can introduce what we call a *bi-parameterized generalized Heronian mean* as follows.

Definition 2.7 Let a and b be two non-negative real numbers. The bi-parameterized generalized Heronian mean $\mathrm{HM}_{\beta,\omega}(a,b)$ of a and b is taken as

$$\mathrm{HM}_{\beta,\omega}(a,b) = \begin{cases} \dfrac{\beta\, a + (1-\beta) b + \omega\sqrt{\beta\,(1-\beta)\,ab}}{1+\omega\sqrt{\beta\,(1-\beta)}}, & 0 \le \omega < \infty \\ \sqrt{ab}, & \omega = \infty \end{cases} \tag{11}$$

and, based on Definition 2.7, we now can define the generalized Heronian OWAWA (GHROWAWA) operator as follows.

Definition 2.8 A generalized Heronian OWAWA (GHROWAWA) operator acting on the interval I and having two parameters β and ω, and an n-dimensional importance weight vector P, and an n-dimensional preferential weight vector W is defined to be the mapping $\mathrm{H}_{\beta,\omega}^{P,W}: I^{n} \rightarrow I$ such that

$$\mathrm{H}_{\beta,\omega}^{P,W}(x) = \mathrm{HM}_{\beta,\omega}(\mathrm{OWA}_{W}(x), \mathrm{WA}_{P}(x)) \tag{12}$$

Let us explain briefly the working of the above CAOP. The CAOP $\mathrm{H}_{\beta,\omega}^{P,W}$ is built as the composition of an arbitrary bi-parameterized binary Heronian mean with the classical weighted average (WA_P) operator and the non-conventional ordered weighted averaging (OWA_W) operator. More precisely, the aggregation arguments and the importance weights are "synthesized" by applying an WA_P operator. In addition, the aggregation arguments and the preferential weights are "synthesized" by applying an OWA_W operator. Then the values returned by these two averaging operators are merged by means of a binary bi-parameterized Heronian mean. Note that the above CAOP has, among others, the following special cases:

- $\mathrm{H}_{\beta,\omega}^{P,W}(x) = \mathrm{WA}_{P}(x)$, if $\beta = 0$.
- $\mathrm{H}_{\beta,\omega}^{P,W}(x) = \mathrm{OWA}_{W}(x)$, if $\beta = 1$.
- $\mathrm{H}_{\beta,\omega}^{P,W}(x) = \mathrm{M}_{\beta}^{P,W}(x)$, if $\omega = 0$.
- $\mathrm{H}_{\beta,\omega}^{P,W}(x) = \dfrac{\mathrm{OWA}_{W}(x) + \mathrm{WA}_{P}(x) + \omega\sqrt{\mathrm{OWA}_{W}(x) \times \mathrm{WA}_{P}(x)}}{2+\omega}$, if $\beta = \frac{1}{2}$.
- $\mathrm{H}_{\beta,\omega}^{P,W}(x) = \sqrt{\mathrm{OWA}_{W}(x) \times \mathrm{WA}_{P}(x)}$, if $\omega = \infty$.

It is note-worthy at this level that the GHROWAWA operators considered above fulfill, among other possible properties, the following desirable 'natural' requirements:

1. Endpoint-preservation

$H_{\beta,\omega}^{P,W}(\underbrace{0, 0, \ldots, 0}_{n-times}) = 0$ and $H_{\beta,\omega}^{P,W}(\underbrace{m-1, m-1, \ldots, m-1}_{n-times}) = m - 1$.

2. Monotonicity in the arguments

$x \leq y$ implies $H_{\beta,\omega}^{P,W}(x) \leq H_{\beta,\omega}^{P,W}(y)$ for all x and $y \in I^n$.

3. Internality property

$MIN(x) \leq H_{\beta,\omega}^{p,w}(x) \leq MAX(x)$ for all $x \in I^n$.

4. Idempotency

The operator $H_{\beta,\omega}^{P,W}$ is idempotent. That is, $H_{\beta,\omega}^{P,W}(\underbrace{t, t, \ldots, t}_{n-times}) = t$ for all $t \in I$.

5. Monotonicity in the weights

Suppose that $x_{j_0} \leq y_{j_0}$ for a given j_0. If for an importance weight vector *P*, we have $H_{\beta,\omega}^{P,W}(x) \leq H_{\beta,\omega}^{P,W}(y)$ for x and $y \in I^n$ then we will also have $H_{\beta,\omega}^{P,W}(x) \leq H_{\beta,\omega}^{P,W}(y)$ where P' stands for the importance weight vector resulting from a positive increase of the importance weight p_{j_0} with proportional decrease of other weights.

6. Nonnegative responsiveness

Letting $x' \in I^n$ stand for the n-vector resulting from a positive increase of the component x_{j_0} of the *n*-vector x for a given j_0 then we will have $H_{\beta,\omega}^{P,W}(x) \leq H_{\beta,\omega}^{P,W}(x')$.

7. Homogeneity

The operator $H_{\beta,\omega}^{P,W}$ is homogeneous. That is, we have $H_{\beta,\omega}^{P,W}(\lambda\, x) = \lambda\, H_{\beta,\omega}^{P,W}(x)$ for all $x \in I^n$ and all $\lambda > 0$ such that all $\lambda\, x \in I^n$.

8. Continuity

$H_{\beta,\omega}^{P,W}$ is a continuous function in each argument.

Based on the material and ideas presented in this section, we now move on to present in the next section the basic definitions and interlocking tasks which are essential to fully understand the way of working of the MANISRA method.

3.The MANISRA method' way of working

Let $H_{\beta,\omega}^{P,W}$ denote any compound averaging operator defined as above, we now can state the following basic definitions used to develop the mechanics of the MANISRA method.

Definition 3.1 The overall net superiority index (written: $ONS_{\beta,\omega}^{P,W}(A_i)$) of alternative A_i (for $i = 1\ to\ m$) is defined as

$$ONS_{\beta,\omega}^{P,W}(A_i) = H_{\beta,\omega}^{P,W}(S^*(A_i)). \tag{13}$$

The overall net superiority index of any alternative is obtained by synthesizing its profile of net S-indexes.

Definition 3.2. The overall net inferiority index (written: $ONI_{\beta,\omega}^{P,W}(A_i)$) is given by

$$ONI_{\beta,\omega}^{P,W}(A_i) = H_{\beta,\omega}^{P,W}\big(I^*(A_i)\big). \tag{14}$$

The overall net inferiority index of any alternative is the result of the aggregation of its profile of net I-indexes. In addition, knowing that the overall net inferiority and superiority indexes lie in the closed real interval [0, m-1], we now can give the formulation of the choice-worthiness grade of any given alternative A_i as follows.

Definition 3.3 The choice $-$ worthiness grade of any alternative A_i (for $i = 1\ to\ m$) is a number between 0 and 1 (written: $CWG_{\beta,\omega}^{P,W}(A_i)$) obtained by using the Eq.(15) below:

$$\mathrm{CWG}^{P,W}_{\beta,\omega}(A_i)) = \frac{\mathrm{ONS}^{P,W}_{\beta,\omega}(A_i) - \mathrm{ONI}^{P,W}_{\beta,\omega}(A_i) + m - 1}{2\,(m-1)} \tag{15}$$

Note that the choice − worthiness grade thus defined is calculated as a normalized difference between the overall net superiority and inferiority indexes of any given alternative A_i.

Statement. If $\beta = 0$, then the methods MANISRA, SIR-SAW and PROMETHEE II yield the same rankings.

Proof. We already know that the SIR-SAW and PROMETHEE II methods produce the same rankings when the net flow rule is used (see Xu, 2001). So, it suffices to show that the MANISRA method with $\beta = 0$ and the SIR-SAW method when the net flow rule is used produce the same rankings. Or, if $\beta = 0$ then $H^{P,W}_{\beta,\omega}(x) = WA_P(x)$. So, $\mathrm{ONS}^{P,W}_{\beta,\omega}(A_i) = H^{P,W}_{\beta,\omega}(S^*(A_i)) = WA_P(S^*(A_i)$ and $\mathrm{ONI}^{P,W}_{\beta,\omega}(A_i) = H^{P,W}_{\beta,\omega}(I^*(A_i)) = WA_P(I^*(A_i)$.

Therefore we will have $\mathrm{ONS}^{P,W}_{\beta,\omega}(A_i) - \mathrm{ONI}^{P,W}_{\beta,\omega}(A_i) = H^{P,W}_{\beta,\omega}(S^*(A_i)) - H^{P,W}_{\beta,\omega}(I^*(A_i))$
$= WA_P(S^*(A_i)) - WA_P(I^*(A_i)) = \sum_{j=1}^{n} p_j\,(S_j(A_i) \ominus I_j(A_i))$ - $\sum_{j=1}^{n} p_j\,(I_j(A_i) \ominus S_j(A_i))$

Or, for any two real numbers *a* and *b*, we have $(a \ominus b)$ - $(b \ominus a)$ = a - b.

Thus, we will have $\sum_{j=1}^{n} p_j \left(S_j(A_i) \ominus I_j(A_i)\right) - \sum_{j=1}^{n} p_j \left(I_j(A_i) \ominus S_j(A_i)\right) =$
$\sum_{j=1}^{n} p_j \left[\left(S_j(A_i) \ominus I_j(A_i)\right) - \left(I_j(A_i) \ominus S_j(A_i)\right)\right] = \sum_{j=1}^{n} p_j \left(S_j(A_i) - I_j(A_i)\right) = \sum_{j=1}^{n} p_j\, S_j(A_i) - \sum_{j=1}^{n} p_j\, I_j(A_i) = \varphi^{>}(A_i) - \varphi^{<}(A_i)$. This proves that
$\mathrm{ONS}^{P,W}_{\beta,\omega}(A_i) - \mathrm{ONI}^{P,W}_{\beta,\omega}(A_i) = \varphi^{>}(A_i) - \varphi^{<}(A_i)$ (i.e., the net flow score of A_i).

As a consequence when $\beta = 0$, the MANISRA and SIR-SAW methods yield the same rankings.

To rank predefined multi-attribute alternatives, the MANISRA method proceeds as follows:

Modeling phase tasks

1. To compute the binary intensities of preference.
2. To compute the inferiority and superiority indexes.
3. To compute the net inferiority and superiority indexes.

Aggregation phase tasks

1. To select a suitable GHROWAWA operator.
2. To compute the overall net inferiority and superiority indexes.

Exploitation phase tasks

1. To compute the choice-worthiness grades of the various alternatives.
2. To rank the alternatives according to their choice-worthiness grades.

We are now ready to illustrate the suggested approach by means of the real problem presented hereafter.

4. Illustrative example

The present real problem is meant to give the reader a feel about the applicability of the MANISRA method on ways of working. To achieve this end, we will compare the ranking provided by the MANISRA method with those obtained by the SIR methods: SIR-SAW and SIR-TOPSIS (Xu, 2001), SIR-VIKOR (Valahzaghard et al., 2011), and SISINA (Hidouri & Rebaï, 2018). Moreover, note that the firm's senior management provided us with the relevant data needed to solve the multi-attribute ranking problem at hand. Throughout this section the firm of interest will be denoted SGB and the fourteen (14) competing logistics service providers (LSPs) will be denoted P_k (for k = 1, 2, ..., 14). Now, let us present the problem description.

4.1 The problem description

SGB is a medium-sized firm localized in Sousse a city in the central-east of Tunisia. This firm is specialized in the manufacturing of all types of electronic weighing scales and in metal construction of industrial buildings since the year 2007. At present, SGB has a favorite LSP (denoted STU) who may not be readily available at certain times. LSP STU has fourteen competitors, namely: EI (LP_1), MDC (LP_2), CPM (LP_3), R2K (LP_4), CGM (LP_5), GM (LP_6), JM (LP_7), PRS (LP_8), SOQ (LP_9), REV (LP_{10}), SM (LP_{11}), SDM (LP_{12}), GAB (LP_{13}), and SC (LP_{14}). In addition, the firm has no choice but to switch to one of the fourteen competing LSPs whenever required. Each LSP is evaluated in terms of the ratings according to a bundle of five prescribed attributes using two weight vectors. The five prescribed attributes are: Responsiveness (g_1), Price (g_2), Delivery time (g_3), Services (g_4), and Quality (g_5). Moreover, the respective importance weights are $p_1 = 0.50$, $p_2 = 0.20$, $p_3 = 0.15$, $p_4 = 0.10$, and $p_5 = 0.05$, whilst the respective preferential weights are $w_1 = 0.25$, $w_2 = 0.25$, $w_3 = 0.17$, $w_4 = 0.17$, and $w_5 = 0.16$. The LSPs ratings are measured on a 0-10 scale as shown in Table 2 below.

Table 2
Rating table

Attribute	LP_1	LP_2	LP_3	LP_4	LP_5	LP_6	LP_7	LP_8	LP_9	LP_{10}	LP_{11}	LP_{12}	LP_{13}	LP_{14}
g_1	9	0	1	7	0	1	5	8	8	5	7	7	5	0
g_2	8	6	7	10	6	6	7	8.5	8.5	7	6	6	6	5
g_3	9	0	2	5	0	1	5	5	8	1	0	0	0	0
g_4	5	0	0	8	0	8	3	7	6	1	0	0	0	0
g_5	5	0	0	9	1	1	9	8	8	1	7	7	7	5

In this work, we will (1) use the current developed MANISRA method to rank the fourteen competing LSPs (from most to least choice-worthy), and (2) compare the ranking produced with those provided by the four SIR methods: SISINA, SIR-SAW, SIR-TOPSIS, and SIR-VIKOR.

4.2 The ranking results

In the present problem since the preference and indifference thresholds are not provided, it becomes natural to treat all the attributes as true-criteria. Therefore the superiority and inferiority indexes defined by Xu (2001) will boil down to the superiority and inferiority scores defined in (Rebaï, 1993, 1994; Rebaï & Martel, 2000) resulting in the S-matrix and I-matrix in the Tables 3-4 given below.

Table 3
S-matrix

LSP	S-score				
	$S_1(.)$	$S_2(.)$	$S_3(.)$	$S_4(.)$	$S_5(.)$
LP_1	13	10	13	9	5
LP_2	0	1	0	0	0
LP_3	3	7	8	0	0
LP_4	8	13	9	12	12
LP_5	0	1	0	0	2
LP_6	3	1	6	12	2
LP_7	5	7	9	8	12
LP_8	11	11	9	11	10
LP_9	11	11	12	10	10
LP_{10}	5	7	6	7	2
LP_{11}	8	1	0	0	7
LP_{12}	8	1	0	0	7
LP_{13}	5	1	0	0	7
LP_{14}	0	0	0	0	5

Table 4
I-matrix

LSP	I-score				
	$I_1(.)$	$I_2(.)$	$I_3(.)$	$I_4(.)$	$I_5(.)$
LP_1	0	3	0	4	7
LP_2	11	7	8	7	12
LP_3	9	4	5	7	12
LP_4	3	0	2	0	0
LP_5	11	7	8	7	9
LP_6	9	7	6	0	9
LP_7	6	4	2	5	0
LP_8	1	1	2	2	2
LP_9	1	1	1	3	2
LP_{10}	6	4	6	6	9
LP_{11}	3	7	8	7	4
LP_{12}	3	7	8	7	4
LP_{13}	6	7	8	7	4
LP_{14}	11	13	8	7	7

And, the net S-scores matrix (S^*-matrix) and net I-scores matrix (I^*-matrix) are in the Tables 5-6 below:

Table 5
$S^* - \text{matrix}$

LSP	Net S-score				
	$S^*_1(.)$	$S^*_2(.)$	$S^*_3(.)$	$S^*_4(.)$	$S^*_5(.)$
LP_1	13	7	13	5	0
LP_2	0	0	0	0	0
LP_3	0	3	3	0	0
LP_4	5	13	7	12	12
LP_5	0	0	0	0	0
LP_6	0	0	0	12	0
LP_7	0	3	7	3	12
LP_8	10	10	7	9	8
LP_9	10	10	11	7	8
LP_{10}	0	3	0	1	0
LP_{11}	5	0	0	0	3
LP_{12}	5	0	0	0	3
LP_{13}	0	0	0	0	3
LP_{14}	0	0	0	0	0

Table 6
I^*-matrix

LSP	Net I-score				
	$I^*_1(.)$	$I^*_2(.)$	$I^*_3(.)$	$I^*_4(.)$	$I^*_5(.)$
LP_1	0	0	0	0	2
LP_2	11	6	8	7	12
LP_3	6	0	0	7	12
LP_4	0	0	0	0	0
LP_5	11	6	8	7	7
LP_6	6	6	0	0	7
LP_7	1	0	0	0	0
LP_8	0	0	0	0	0
LP_9	0	0	0	0	0
LP_{10}	1	0	0	0	7
LP_{11}	0	6	8	7	0
LP_{12}	0	6	8	7	0
LP_{13}	1	6	8	7	0
LP_{14}	11	13	8	7	2

Now for the sake of illustration, we will use the GHROWAWA operator defined by the Eq. (16) below:

$$H^{P,W}(x) = \frac{OWA_W(x) + WA_P(x) + \sqrt{OWA_W(x) \times WA_P(x)}}{3} \tag{16}$$

Moreover, we will display the results of the aggregation of the various scores in the Tables 7-8 below.

Table 7

The aggregation results of the net S-scores

LSP	WA_P	OWA_W	$\sqrt{WA_P \times OWA_W}$	$H^{P,W}$
LP_1	10.35	8.54	9.40	9.43
LP_2	0	0	0	0
LP_3	1.05	1.5	1.25	1.27
LP_4	7.95	10.28	9.04	9.08
LP_5	0	0	0	0
LP_6	1.2	3	1.90	2.03
LP_7	2.55	5.77	3.84	4.05
LP_8	9.35	9.01	9.18	9.18
LP_9	9.75	9.43	9.59	9.57
LP_{10}	0.7	1	0.84	0.85
LP_{11}	2.65	2	2.30	2.32
LP_{12}	2.65	2	2.30	2.32
LP_{13}	0.15	0.75	0.34	0.41
LP_{14}	0	0	0	0

Table 8

The aggregation results of the net I-scores

LSP	WA_P	OWA_W	$\sqrt{WA_P \times OWA_W}$	$H^{P,W}$
LP_1	0.1	0.5	0.22	0.27
LP_2	9.2	9.26	9.23	9.23
LP_3	4.3	2.77	3.45	3.51
LP_4	0	0	0	0
LP_5	8.95	8.09	8.51	8.52
LP_6	4.55	4.27	4.41	4.41
LP_7	0.5	0.25	0.35	0.37
LP_8	0	0	0	0
LP_9	0	0	0	0
LP_{10}	0.85	2	1.30	1.38
LP_{11}	3.1	4.77	3.85	3.91
LP_{12}	3.1	4.77	3.85	3.91
LP_{13}	3.6	4.94	4.22	4.25
LP_{14}	10.1	8.87	9.47	9.48

Below, we will show the ranking results in the Tables 9-10.

Table 9

Ranking produced by MANISRA

LSP	LP_1	LP_2	LP_3	LP_4	LP_5	LP_6	LP_7	LP_8	LP_9	LP_{10}	LP_{11}	LP_{12}	LP_{13}	LP_{14}
CWG	0.852	0.145	0.41	0.85	0.172	0.42	0.642	0.853	0.868	0.48	0.44	0.44	0.352	0.14
RANK	3	13	10	4	12	9	5	2	1	6	7	7	11	14

The respective descriptions of the notations used in Table 10 below are the following:

- DV: desirability value;

- NNFS: normalized net flow score;
- RFS: net flow score;
- RCS: net closeness coefficient using superiority indexes;
- RCI: net closeness coefficient using inferiority indexes;
- QS: ranking index using superiority indexes;
- QI: ranking index using inferiority indexes.

Table 10
Rankings of the top-7 LSPs produced by SIR methods

SISINA		SIR-SAW				SIR-TOPSIS				SIR-VIKOR			
DV	LSP	NNFS	LSP	RFS	LSP	RCS	LSP	RCI	LSP	QS	LSP	QI	LSP
0.815	LP1	0.89	LP1	0.896	LP1	0.89	LP1	0.88	LP1	0.108	LP1	0.097	LP1
0.730	LP9	0.88	LP8	0.895	LP8	0.84	LP8	0.83	LP8	0.118	LP8	0.107	LP9
0.694	LP8	0.83	LP9	0.890	LP9	0.79	LP9	0.82	LP9	0.158	LP9	0.109	LP8
0.563	LP4	0.81	LP4	0.844	LP4	0.66	LP4	0.76	LP4	0.226	LP4	0.169	LP4
0.325	LP11	0.483	LP7	0.591	LP7	0.48	LP11	0.63	LP11	0.402	LP7	0.338	LP11
0.325	LP12	0.481	LP11	0.476	LP11	0.48	LP12	0.63	LP12	0.420	LP11	0.338	LP12
0.287	LP7	0.481	LP12	0.476	LP12	0.44	LP7	0.51	LP7	0.420	LP12	0.367	LP7

4.3 Brief commentary

Studying the Tables 9-10 above, we can underline the following two key points which could be made about the produced rankings of LSPs:

1. The top-4 LSPs are absolutely the same, but their ranks may vary from one method to another.

2. The ranking delivered by the MANISRA method is different from those provided by the four SIR methods.

5. Conclusions

In the present work, we have proposed a **M**ulti-**A**ttribute **N**et **I**nferiority and **S**uperiority based **R**anking **A**pproach (MANISRA). This approach is understandable, applicable, general, orderly, transparent, flexible, and effective. Moreover, it was shown that the SIR-SAW and PROMETHEE II methods fall out as particular cases of this approach. In addition, we have treated a real problem to illustrate the applicability and effectiveness of the suggested approach. The ranking results have shown that the MANISRA method delivers a different ranking from those produced by the SIR methods: SIR-SAW, SIR-TOPSIS, SIR-VIKOR and SISINA.

Future research could be undertaken (1) in extending the way of working of the MANISRA method to solve *multi-person multi-attribute* ranking problems, (2) in providing an experimental assessment of its way of working, and (3) in applying it in different areas, including economics and finance, etc.

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