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Fuzzy BWANP multi-criteria decision-making method

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A B S T R A C T

Fuzzy Analytical Network Process (F-ANP) method is able to consider the complex relationships among different levels of decisions, transactions, and feedbacks of criteria and alternatives to calculate the weights of the elements. The large number of pair-wise comparisons in F-ANP and also difficulties in understanding the way of comparisons for the expert, have reduced the efficiency and practicality of this method. In this paper, in order to eliminate the above-mentioned problems, it has been tried to provide an approach using the Fuzzy Best-Worst method, called F-BWANP. The proposed method, requires less comparison data and leads to more consistent comparisons, which means that more reliable results can be obtained, while making it much easier for responding by experts. Finally, in order to describe the proposed method and evaluate its capability, a numerical example is provided.

1. Introduction

Saaty (1996), when seeking a solution for limitations of Analytic Hierarchy Process (AHP) and its inability in applying dependencies between criteria and factors, developed another approach, which was known as Analytic Network Process (ANP). Analytic Network Process approach is an extension of AHP or in other words, its general form. AHP models the decision structure through indirect hierarchical relationships among the criteria; but, ANP provides the possibility to evaluate more complex internal relationships among the criteria. The development of this process was aimed at providing more realistic conditions for decision-making, without considering the assumptions about one-way hierarchical relationship among decision levels (Sipahi & Timor, 2010). In the other words, AHP considers the one-way relationships among decision levels, while ANP, considers the mutual relationships among decision levels and features in a more general situation (Agarwal et al., 2006). Therefore, ANP can be applied as an effective tool in situations, in which the transactions among elements of a system form a network (Saaty, 2001). ANP uses the relative scales based on pair-wise comparisons. However, it does not apply the limited hierarchical structure similar to the AHP, and models the decision-making problem using the systematic approach with feedback. Although the Fuzzy ANP was also introduced as a more accurate method for modeling the complicated decision environments, the following problems can be seen in it (Yu & Tzeng, 2006):

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- It is difficult to provide a correct network structure even for experts, and different structures lead to different results.
- To form a super-matrix all criteria, have to be pair-wise compared with regard to all other criteria, which is also difficult and somewhat unnatural, as we ask themselves questions of the type: "How much is a criterion A more important than a criterion B with regard to a criterion C?"
- Large number of pair-wise comparisons: to calculate eigenvectors, pair-wise comparisons are required, resulting in a significant increase in pair-wise comparisons.

In this paper, the F-BWANP method is presented as the alternative of F-ANP that while having a rational procedure, can possible cover the problems of above-mentioned method.

2. Literature review

Analytic Network Process can be applied in many areas (Saaty, 2005; Vargas, 2006; Saaty & Brandy, 2009). ANP is further used in areas including, Decision making, Evaluation, selection QFD, Planning and Development, Priority and Ranking, and Forecasting. There are many research works in this area and we address some of them (Hülle et al., 2013). In their work, Chen et al. (2006) used ANP method for generating a location selection model to determine the best location out of a choice of three alternatives for a biotech park in Taiwan. They suggested two ANP models that consider the environmental issues, and then, the two method were combined to select the best plan out of the three ones. Cheng et al. (2005) used the ANP and AHP to select the best shopping mall location.

Chen and Chen (2009) examined the critical factors, affecting the quality improvement in the Taiwanese banking industry. Aznar et al. (2010) applied ANP to evaluate the urban properties. Chen et al. (2008) proposed a method in which the ANP can be used in form of a knowledge-framed analytic network process (KANP) to evaluate contractor candidates in an open competition to procure a construction project. ANP has also been used in the area of planning and development. In their work, Lee et al. (2008) and Chen et al (2008) used ANP for product development. ANP has also been used for energy policy planning by Hämäläinen and Seppäläinen (1986).

ANP was used by Cheng and Li (2005) for prioritizing a set of projects. Suitable enterprise architecture was presented by Wadhwa et al. (2009) for virtual enterprises and virtual manufacturing focused on agility. In order to model the mutual relationship between different decision areas for prioritizing the enterprise-wide flexibility dimensions, ANP was used. Lee et al. (2008) tried to improve the technology foresight by using ANP. Crowe and Lucas-Vergona (2007) investigated the problem of excessive illegal immigration. They used ANP in order to create a decision model based on economic, social, political and environmental factors to make decision among six alternatives. Zoffer et al. (2008) studied an issues related to the conflict in Middle East and a possible road-map to the Middle East peace process. ANP was used by the authors in order to evaluate the conflict around the world and to synthesize judgement for finding an optimal conflict solution. Wu et al. (2009) used ANP for its ability to integrate the relationships among decision levels. Blair et al. (2002) analyzed expert judgement regarding prediction of the resumption of the American Economic development's growth, using the ANP. Chang et al. (2009a) investigated a manufacturing model for predicting the presence of a silicon wafer using an ANP framework. In order to improve clients' satisfaction, Buyukozkan et al. (2004) applied QFD for translating their needs into technical design requirements. In order to prioritize the design requirements as a part of the house of quality ANP approach was used. Pal et al. (2007) proposed an integrated method using ANP and QFD. This approach was used to determined and prioritize the engineering needs about a cast part to select a suitable, rapid prototype-based route to tool manufacturing. As it was seen, ANP was considered by many researchers and has been used in various fields. Some researchers have sought to combine this method with other methods for better use of ANP, resulting in ISM-ANP and D-ANP methods that tries to improve the relations matrix in ANP (Chang et al., 2013), or GP-ANP that attempts to obtain better results from ANP (Chang et al., 2009b). But, the issue which is challenging in all the mentioned methods is the large number of comparisons and calculations and the difficulty of responding by the experts. In this paper, the F-BWANP method is provided, through which, while achieving more relatable results, the pair-wise comparisons would be facilitated and reduced.

3. The proposed F-BWANP method

Like the F-ANP, F-BWANP first calculates Eigenvectors and then, a super-matrix is formed, but, the difference between the two methods is how to calculate eigenvectors. F-ANP uses the pair-wise comparisons of F-AHP to calculate eigenvectors, resulting in significant increase in pair-wise comparisons. F-BWANP has eliminated the problem and uses the F-BWM comparisons in order to calculate eigenvectors (Guo & Zhao, 2017) that needs less comparison data, while leading to a more reliable comparison, and it means that F-BWANP gives more reliable answers. Therefore, to calculate the eigenvector, first, the best (most important) and worst (least important) criterion should be determined and then, the preference of the best criterion over all the other criteria ($\tilde{a}_{Bj}$) and also the preference of all the other criteria over the worst criterion ($\tilde{a}_{jW}$) are determined and the criteria's weight is calculated according to the F-BWM method. In the other words, all elements of F-AHP pair-wise comparisons matrix are not needed to calculate the eigenvector , and only one row and one column of it are needed, namely the row and column representing $\tilde{a}_{Bj}$ and $\tilde{a}_{jW}$. In F-BWANP, only this row and column is calculated. After determining $\tilde{a}_{Bj}$ and $\tilde{a}_{jW}$, the model is formulated in form of a linear programming problem and solved. In this approach, the comparisons are considerably reduced.

Steps of F-BWANP method

1. The decision problem is decomposed into its decision elements and structured into a hierarchy that includes an overall goal, criteria, sub criteria, and alternatives, with the number of levels varying depending on the complexity of the problem and the number of factors to be considered.

2. Using pair-wise comparisons:

Table 1
Transformation rules of linguistic variables ofdecision-makers

Linguistic terms	Membership function
Equally importance	(1 1 1)
Weakly important	(2/3 1 3/2)
Fairly important	(3/2 2 5/2)
Very important	(5/2 3 7/2)
Absolutely important	(7/2 4 9/2)

2.a. Determine the best (most important) criterion and Execute the fuzzy reference comparisons for the best criterion. By using the linguistic terms of decision-makers listed in Table 1, the fuzzy preferences of the best criterion over all the criteria can be determined. Then, the obtained fuzzy preferences are transformed to TFNs according to the transformation rules shown in Table 1. The obtained fuzzy Best-to-Others vector is (Sadjadi & Karimi, 2018):

$$\tilde{A}_B = (\tilde{a}_{B1}.\tilde{a}_{B2}.\dots.\tilde{a}_{Bn}) \quad \text{where} \quad \tilde{a}_{Bj} = (l_{Bj}.m_{Bj}.u_{Bj}) \tag{1}$$

2.b. Determine the worst (least important) criterion and Execute the fuzzy reference comparisons for the worst criterion. By using the linguistic evaluations of decision-makers listed in Table 1, the fuzzy preferences of all the criteria over the worst criterion can be determined, and then they are transformed to TFNs according to the transformation rules listed in Table 1. The fuzzy Others-to-Worst vector can be obtained as:

$$\tilde{A}_W = (\tilde{a}_{1W}.\tilde{a}_{2W}.\dots.\tilde{a}_{nW}) \quad \text{where} \quad \tilde{a}_{jW} = (l_{jW}.m_{jW}.u_{jW}) \tag{2}$$

3. Determine the optimal fuzzy weights. The optimal fuzzy weight for each criterion is the one where, for each fuzzy pair $\widetilde{w}_B/\widetilde{w}_j$ and $\widetilde{w}_j/\widetilde{w}_W$, it should have $\widetilde{w}_B/\widetilde{w}_j = \tilde{a}_{Bj}$ and $\widetilde{w}_j/\widetilde{w}_W = \tilde{a}_{jW}$. To satisfy these conditions for all j, it should determine a solution where the maximum absolute gaps $\left| \frac{\widetilde{w}_B}{\widetilde{w}_j} - \right.$

$\tilde{a}_{Bj}\,\Big|\, and \,\Big|\, \frac{\tilde{W}_j}{\tilde{W}_W} - \tilde{a}_{jW}\,\Big|$ for all j are minimized. Therefore, we can obtain the constrained optimization problem for determining the optimal fuzzy weights $(\tilde{w}_1^*.\,\tilde{w}_2^* \ldots.\,\tilde{w}_n^*)$ as follows:

$$\min \max_j \left\{ \left| \frac{\tilde{W}_B}{\tilde{W}_j} - \tilde{a}_{Bj} \right| . \left| \frac{\tilde{W}_j}{\tilde{W}_W} - \tilde{a}_{jW} \right| \right\}$$

$$\begin{cases} \sum_{j=1}^{n} R(\tilde{w}_j) = 1 \\ l_j^w \le m_j^w \le u_j^w \\ l_j^w \ge 0\,. \quad j = 1.2. \ldots . n \end{cases} \tag{3}$$

where

$$\tilde{W}_B = (l_B^w.\,m_B^w.\,u_B^w) \quad . \quad \tilde{W}_j = (l_j^w.\,m_j^w.\,u_j^w) \quad . \quad \tilde{W}_W = (l_W^w.\,m_W^w.\,u_W^w)$$
$$\tilde{a}_{Bj} = (l_{Bj}.\,m_{Bj}.\,u_{Bj}) \quad . \quad \tilde{a}_{jW} = (l_{jW}.\,m_{jW}.\,u_{jW})$$

Eq. (3) can be transferred to the following nonlinearly constrained optimization problem:

$$\min \quad \tilde{\xi}$$

$$\begin{cases} \left| \frac{\tilde{W}_B}{\tilde{W}_j} - \tilde{a}_{Bj} \right| \le \tilde{\xi} \\ \left| \frac{\tilde{W}_j}{\tilde{W}_W} - \tilde{a}_{jW} \right| \le \tilde{\xi} \\ \sum_{j=1}^{n} R(\tilde{w}_j) = 1 \\ l_j^w \le m_j^w \le u_j^w \\ l_j^w \ge 0\,.\, j = 1.2. \ldots . n \end{cases} \tag{4}$$

where $\tilde{\xi} = (l^{\xi}.\,m^{\xi}.\,u^{\xi})$

Considering $l^{\xi} \le m^{\xi} \le u^{\xi}$, we suppose $\tilde{\xi}^* = (k^*.\,k^*.\,k^*)\,.\,k^* \le l^{\xi}$, then Eq. (4) can be transferred as:

$$\min \quad \tilde{\xi}^*$$

$$\begin{cases} \left| \frac{(l_B^w.\,m_B^w.\,u_B^w)}{(l_j^w.\,m_j^w.\,u_j^w)} - (l_{Bj}.\,m_{Bj}.\,u_{Bj}) \right| \le (k^*.\,k^*.\,k^*) \\ \left| \frac{(l_j^w.\,m_j^w.\,u_j^w)}{(l_W^w.\,m_W^w.\,u_W^w)} - (l_{jW}.\,m_{jW}.\,u_{jW}) \right| \le (k^*.\,k^*.\,k^*) \\ \sum_{j=1}^{n} R(\tilde{w}_j) = 1 \\ l_j^w \le m_j^w \le u_j^w \\ l_j^w \ge 0\,.\, j = 1.2. \ldots . n \end{cases} \tag{5}$$

Table 2
Consistency index (CI)

Linguistic terms	Equally importance	Weakly important	Fairly important	Very important	Absolutely important
$\tilde{a}_{BW}$	(1 1 1)	(2/3 1 3/2)	(3/2 2 5/2)	(5/2 3 7/2)	(7/2 4 9/2)
CI	3	3.8	5.29	6.69	8.04

The obtained consistency index (CI) with regards to different linguistic terms of decision-makers for fuzzy BWM are listed in Table 2. We then calculate the consistency ratio, using $\tilde{\xi}^*$ and the corresponding consistency index, as follows:

$$CR = \frac{\tilde{\xi}^*}{CI} \tag{6}$$

4. form the super-matrix. The general form of the super-matrix can be described as follows:

$$W = \begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \\ e_{11} \dots e_{1m_1} & e_{21} \dots e_{2m_2} & \cdots & e_{n1} \dots e_{nm_n} \end{array} \\ \begin{array}{cc} & e_{11} \\ & e_{12} \\ C_1 & \vdots \\ & e_{1m_1} \\ & e_{21} \\ C_2 & e_{22} \\ & \vdots \\ & e_{2m_2} \\ & \vdots \\ & e_{n1} \\ C_n & e_{n2} \\ & \vdots \\ & e_{nm_n} \end{array} & \begin{bmatrix} W_{11} & W_{12} & & W_{1n} \\ W_{21} & W_{22} & & W_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ W_{n1} & W_{n2} & & W_{nn} \end{bmatrix} \end{array}$$

where C_m denotes the mth cluster, e_{mn} denotes the nth element in the mth cluster, and W_{ij} is the principal eigenvector of the influence of the elements compared in the jth cluster to the ith cluster. In addition, if the jth cluster has no influence on the ith cluster, then $W_{ij} = 0$. After forming the super-matrix, the weighted super-matrix is derived by transforming all column sums to unity exactly. Next, we raise the weighted super-matrix to limiting powers such as Eq. (7) to get the global priority vectors or so-called weights:

$$\lim_{k\to\infty} W_w^k \tag{7}$$

In addition, if the super-matrix has the effect of cyclicity, the limiting super-matrix is not the only one. There are two or more limiting supermatrices in this situation and the Cesaro sum would be calculated to get the priority. The Cesaro sum is formulated as:

$$\lim_{k\to\infty} \left(\frac{1}{N}\right) \sum_{r=1}^{N} W_r^k \tag{8}$$

to calculate the average effect of the limiting super-matrix (i.e., the average priority weights) where W_r denotes the rth limiting super-matrix. Otherwise, the super-matrix would be raised to large powers to get the priority weights (Saaty, 1996).

4. Case study

In order to evaluate the capabilities of the proposed method, a case study is provided. To this end, the performance of companies in the area of product development is evaluated (in USA 2017). Criteria and decision-making alternatives are as follows:

Table 3
The criteria and companies

	Company
A1	TechAhead
A2	Parangat Technologies
A3	OpenXcell
A4	LeewayHertz

Criteria	
C1	Financial factors
C2	Behavioral-cultural factors
C3	Environmental factors
C4	Organizational factors
C5	Management factors
C6	Risk factors

In the following, calculations of F-BWANP approach are provided. Before implementing the method, the relation matrix of criteria is extracted using the ISM method, which is described as follows (see Table 4). The matrix represents the internal dependencies of criteria to calculate W22.

Table 4
Final reachability matrix

	C1	*C2*	*C3*	*C4*	*C5*	*C6*
C1	1	0	0	1*	1*	1*
C2	1	1	0	1*	1*	1*
C3	1	1	1	1*	1	1*
C4	1*	0	0	1	1*	1*
C5	1*	0	0	1*	1	1*
C6	0	0	0	0	0	1

Calculation of matrix W21:

The matrix W21 is the eigenvector, representing the importance of criteria with regard to the goal. According to the experts, the most important criterion is C8 and the least important criterion is C11 that their comparison with other criteria is provided in Table 5. The calculations related to determining the Weights of matrix W21 are provided in Table 7.

Table 5
Pair-wise comparisons of criteria with the best and the worst criterion

	C1	C3	C4	C5	C6
BEST: C2	(5/2 3 7/2)	(5/2 3 7/2)	(3/2 2 5/2)	(2/3 1 3/2)	(7/2 4 9/2)

	C1	C3	C4	C5
WORST: C6	(2/3 1 3/2)	(2/3 1 3/2)	(3/2 2 5/2)	(5/2 3 7/2)

Table 6
Modeling and solving the model

min k	$\|l_1 - 0.67*u_6\| \leq k*u_6$	k	0.296548
$\|l_2 - 2.5*u_1\| \leq k*u_1$	$\|m_1 - 1*m_6\| \leq k*m_6$	l_1	0.08567
$\|m_2 - 3*m_1\| \leq k*m_1$	$\|u_1 - 1.5*l_6\| \leq k*l_6$	m_1	0.099898
$\|u_2 - 3.5*l_1\| \leq k*l_1$	$\|l_3 - 0.67*u_6\| \leq k*u_6$	u_1	0.101532
$\|l_2 - 2.5*u_3\| \leq k*u_3$	$\|m_3 - 1*m_6\| \leq k*m_6$	l_2	0.283938
$\|m_2 - 3*m_3\| \leq k*m_3$	$\|u_3 - 1.5*l_6\| \leq k*l_6$	m_2	0.324368
$\|u_2 - 3.5*l_3\| \leq k*l_3$	$\|l_4 - 1.5*u_6\| \leq k*u_6$	u_2	0.32525
$\|l_2 - 1.5*u_4\| \leq k*u_4$	$\|m_4 - 2*m_6\| \leq k*m_6$	l_3	0.08567
$\|m_2 - 2*m_4\| \leq k*m$	$\|u_4 - 2.5*l_6\| \leq k*l_6$	m_3	0.098396
$\|u_2 - 2.5*l_4\| \leq k*l_4$	$\|l_5 - 2.5*u_6\| \leq k*u_6$	u_3	0.101532
$\|l_2 - 0.67*u_5\| \leq k*u_5$	$\|m_5 - 3*m_6\| \leq k*m_6$	l_4	0.1475
$\|m_2 - 1*m_5\| \leq k*m_5$	$\|u_5 - 3.5*l_6\| \leq k*l_6$	m_4	0.1475
$\|u_2 - 1.5*l_5\| \leq k*l_5$	$1/6*l_1+4/6*m_1+1/6*u_1+\ldots=1$	u_4	0.170496
$\|l_2 - 3.5*u_6\| \leq k*u_6$	$l_1 \leq m_1 \leq u_1, \ldots, l_6 \leq m_6 \leq u_6$	l_5	0.247766
$\|m_2 - 4*m_6\| \leq k*m_6$	$l_1 > 0, \ldots, l_6 > 0$	m_5	0.250179
$\|u_2 - 4.5*l_6\| \leq k*l_6$		u_5	0.293765
		l_6	0.077377
		m_6	0.077377
		u_6	0.088635

Table 7
Deffuzified weights

CR	0.036884	for all experts
W_{C1}	0.097799	0.105799
W_{C2}	0.317777	0.287777
W_{C3}	0.096798	0.103798
W_{C4}	0.151333	0.160333
W_{C5}	0.257041	0.261041
W_{C6}	0.079253	0.081253

Calculation of matrix W22

This matrix compares the criteria based on each criterion. In this step, in order to determine the internal dependency of criteria, ISM method is used. Calculations related to the criteria's weights are based on C1 shown in Table 8 and Table 9. The operation is also performed for the other criteria, and its final result can be seen in Table 11.

Table 8
Pair-wise comparisons of criteria with the best and the worst criteria based on C1

	C3	C4	C5	C6
BEST: C2	(3/2 2 5/2)	(2/3 1 3/2)	(1 1 1)	(5/2 3 7/2)

	C3	C4	C5
WORST: C6	(1 1 1)	(2/3 1 3/2)	(3/2 2 5/2)

Table 9
Modeling and solving the model-Eigenvector based on C1

Min k

$|l_2 - 1.5*u_3| \le k*u_3$
$|m_2 - 2*m_3| \le k*m_3$
$|u_2 - 2.5*l_3| \le k*l_3$

$|l_2 - 0.67*u_4| \le k*u_4$
$|m_2 - 1*m_4| \le k*m$
$|u_2 - 1.5*l_4| \le k*l_4$

$|l_2 - 1*u_5| \le k*u_5$
$|m_2 - 1*m_5| \le k*m_5$
$|u_2 - 1*l_5| \le k*l_5$

$|l_2 - 2.5*u_6| \le k*u_6$
$|m_2 - 3*m_6| \le k*m_6$
$|u_2 - 3.5*l_6| \le k*l_6$

$|l_3 - 1*u_6| \le k*u_6$
$|m_3 - 1*m_6| \le k*m_6$
$|u_3 - 1*l_6| \le k*l_6$

$|l_4 - 0.67*u_6| \le k*u_6$
$|m_4 - 1*m_6| \le k*m_6$
$|u_4 - 1.5*l_6| \le k*l_6$

$|l_5 - 1.5*u_6| \le k*u_6$
$|m_5 - 2*m_6| \le k*m_6$
$|u_5 - 2.5*l_6| \le k*l_6$

$1/6*l_2+4/6*m_2+1/6*u_2+\ldots=1$
$l_2 \le m_2 \le u_2, \ldots, l_6 \le m_6 \le u_6$
$l_2 > 0, \ldots, l_6 > 0$

k	0.5615528
l_2	0.284965
m_2	0.284965
u_2	0.3433966
l_3	0.1606736
m_3	0.1606736
u_3	0.1606736
l_4	0.172637
m_4	0.1824882
u_4	0.2353592
l_5	0.2333901
m_5	0.2333901
u_5	0.2333901
l_6	0.1168633
m_6	0.1168633
u_6	0.1451313

Table 10
Deffuzified weights

CR	0.0839391	for all experts
W_{C2}	0.2947036	0.2732036
W_{C3}	0.1606736	0.1756736
W_{C4}	0.1896582	0.1916582
W_{C5}	0.2333901	0.2248901
W_{C6}	0.1215746	0.1345746
CR	0.0839391	for all experts

Table 11
Results of calculating the matrix W22

	C1	C2	C3	C4	C5
C1	0	0	0	0	0
C2	0.2732036	0	0.4216322	0.5125032	0.432556
C3	0.1756736	0.3713472	0	0.281425	0.1807155
C4	0.1916582	0.40550633	0.3165542	0	0.223558
C5	0.2248901	0	0	0	0
C6	0.1345746	0.22314927	0.2618136	0.2060718	0.1631705

As a sample, the procedures performed for W21 and W22 are mentioned. The same calculations were applied for W23 and W32. Finally, the weights were obtained and the super-matrix was completed. The placement of the four obtained matrices into the initial super-matrix is presented in Table 12. The limiting super-matrix can be seen in Table 13.

Table 12
Unweighted Super-matrix

	G	*C1*	*C2*	*C3*	*C4*	*C5*	*C6*	*A1*	*A2*	*A3*	*A4*
G	-	-	-	-	-	-	-	-	-	-	-
C1	*0.1058*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.1182*	*0.1080*	*0.1284*	*0.0981*
C2	*0.2878*	*0.2732*	*0.0000*	*0.4216*	*0.5125*	*0.4326*	*0.0000*	*0.3013*	*0.3096*	*0.2931*	*0.3176*
C3	*0.1038*	*0.1757*	*0.3713*	*0.0000*	*0.2814*	*0.1807*	*0.0000*	*0.1174*	*0.1071*	*0.1277*	*0.0971*
C4	*0.1603*	*0.1917*	*0.4055*	*0.3166*	*0.0000*	*0.2236*	*0.0000*	*0.1547*	*0.1530*	*0.1564*	*0.1514*
C5	*0.2610*	*0.2249*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.2249*	*0.2410*	*0.2088*	*0.2566*
C6	*0.0813*	*0.1346*	*0.2231*	*0.2618*	*0.2061*	*0.1632*	*0.0000*	*0.0835*	*0.0814*	*0.0856*	*0.0793*
A1	-	*0.1565*	*0.1423*	*0.1797*	*0.1610*	*0.1945*	*0.1840*	-	-	-	-
A2	-	*0.1768*	*0.1655*	*0.1246*	*0.1450*	*0.0935*	*0.1079*	-	-	-	-
A3	-	*0.4602*	*0.4710*	*0.4154*	*0.4432*	*0.5134*	*0.5049*	-	-	-	-
A4	-	*0.2065*	*0.2213*	*0.2803*	*0.2508*	*0.1987*	*0.2032*	-	-	-	-

Table 13
Limiting Super-matrix

	G	C1	C2	C3	C4	C5	C6	A1	A2	A3	A4
G	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
C1	0.0425	0.0425	0.0425	0.0425	0.0425	0.0425	0.0425	0.0425	0.0425	0.0425	0.0425
C2	0.1887	0.1887	0.1887	0.1887	0.1887	0.1887	0.1887	0.1887	0.1887	0.1887	0.1887
C3	0.1064	0.1064	0.1064	0.1064	0.1064	0.1064	0.1064	0.1064	0.1064	0.1064	0.1064
C4	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
C5	0.0870	0.0870	0.0870	0.0870	0.0870	0.0870	0.0870	0.0870	0.0870	0.0870	0.0870
C6	0.0880	0.0880	0.0880	0.0880	0.0880	0.0880	0.0880	0.0880	0.0880	0.0880	0.0880
A1	0.0610	0.0610	0.0610	0.0610	0.0610	0.0610	0.0610	0.0610	0.0610	0.0610	0.0610
A2	0.0486	0.0486	0.0486	0.0486	0.0486	0.0486	0.0486	0.0486	0.0486	0.0486	0.0486
A3	0.1708	0.1708	0.1708	0.1708	0.1708	0.1708	0.1708	0.1708	0.1708	0.1708	0.1708
A4	0.0824	0.0824	0.0824	0.0824	0.0824	0.0824	0.0824	0.0824	0.0824	0.0824	0.0824

As it can be seen, ranking of criteria and alternatives are shown in Tables 14.

Table 14
Ranking of criteria and alternatives

Criteria	F-BWANP Weights	Ranking
C2	0.1887	1
C4	0.1250	2
C3	0.1064	3
C6	0.0880	4
C5	0.0870	5
C1	0.0425	6

Alternatives	F-BWANP Weights	Ranking
A3	0.1708	1
A4	0.0824	2
A1	0.0610	3
A2	0.0486	4

5. Discussion and conclusions

In this paper, some problems of F-ANP method were described and then, F-BWANP method was proposed as the alternative. The proposed method, requires less comparison data and leads to more consistent comparisons, which means that more reliable results can be obtained. F-BWANP is a vector-based method that requires fewer comparisons compared to the F-ANP matrix-based method. For F-BWANP, we only need to have 2n-3 comparisons while for F-ANP, n(n-1)/2 comparisons are needed. In this paper, it has been shown that the proposed method is preferred to F-ANP due to the significant decrease in pair-wise comparisons and calculations and also calculating more reliable final weights.

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