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An integrated model for product mix problem and scheduling considering overlapped operations

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CHRONICLE

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ABSTRACT

Product mix problem is one of the most important decisions made in production systems. Several algorithms have been developed to determine the product mix. Most of the previous works assume that all resources can perform, simultaneously and independently, which may lead to infeasibility of the schedule. In this paper, product mix problem and scheduling are considered, simultaneously. A new mixed-integer programming (MIP) model is proposed to formulate this problem. The proposed model differentiates between process batch size and transfer batch size. Therefore, it is possible to have overlapped operations. The numerical example is used to demonstrate the implementation of the proposed model. In addition, the proposed model is examined using some instances previously cited in the literature. The preliminary computational results show that the proposed model can generate higher performance than conventional product mix model.

1. Introduction

Product mix problem has been recognized as one of the most significant problems of manufacturing systems. Due to capacity constraints, it is not possible to meet demands for all products. Hence, to obtain desired profit, companies need to decide on appropriate quantities of appropriate products to be included in production plan. The integer linear programming (ILP) model of product mix problem can be represented by:

$$\text{Maximize } \sum_{i=1}^{n} (P_i - c_i) x_i \tag{1}$$

Subject to:

$$\sum_{i=1}^{n} a_{ij} x_i \le b_j \qquad j = 1,2,\dots,m \tag{2}$$

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$x_i \le d_i \qquad\qquad i = 1,2, \dots, n \qquad (3)$

$x_i \ge 0$ and x_i is integer $\qquad i = 1,2, \dots, n \qquad (4)$

where $P_i$and c_i are the selling price and the cost of product i, respectively. x_i is the decision variable representing quantity of product i. a_{ij} is the amount of resource j required to produce product i. The available capacity of resource j is shown by b_j. d_i indecates the demand of product i. n and m are the number of products and the number of resources, respectively. Let us name this model as the conventional model.

The objective function of this model is to maximize the throughput and the main constraints are limited resources and satisfying the demands. According to Linhares (2009), the problem of defining the product mix is NP-Complete, in which the increase in the number of products leads to an exponential growth in the number of possible solutions. Therefore, many researchers have proposed heuristic methods for solving this type of problem and Theory of Constraints (TOC) approach is one the most effective approaches.

Cleary, the majority of manufacturing systems are essentially multi-stage systems, which means there is a degree of dependency between different stages. Meanwhile, conventional model and existing heuristic approaches for product mix problem assume that all resources can perform, simultaneously and independently. Hence, essential attempts made on implementation of the solution, for avoiding the resource conflict or meeting the prerequisites of the operations, which are ignored by the existing approaches, lead to inevitable idleness of resources and going off the schedule.

In this paper, by considering assumptions of the Theory of Constraints, which deal with process and transfer batches, an integrated model for product mix problem and scheduling (IPMPS) is proposed. The proposed model provides realistic solutions in terms of scheduling.

The rest of this paper is organized as follows: Section 2 presents a review of literature on TOC-based product mix problem. Section 3 describes the problem under consideration and presents a mixed integer programming model. Section 4 is devoted to numerical example. Section 5 deals with computational results. Finally, the work is concluded in Section 6.

2. Literature review

Theory of Constraints is a management approach introduced by Goldratt in 1984. Goldratt defined a simple Five Focusing Steps (5FS) process for achieving continuous improvement. TOC's 5FS are as follows:

1. Identify the system's constraint(s).
2. Decide how to exploit the system's constraint(s).
3. Subordinate everything else to the above decision.
4. Elevate the constraint(s).
5. If, in the previous steps, a constraint has been broken, go back to Step 1.

These five steps are explained in detail in the literature (see for example, Aryanezhad et al., 2010; Badri & Aryanezhad, 2011). The product mix problem is one important application of the TOC's 5FS (Hsu & Chung, 1998).

The year 1990 can be considered as the year in which the first attempts were made for solving the product mix problem with a TOC-based approach. In this year, the TOC algorithm was proposed by Goldratt. Many researchers verified the initial algorithm and found it an easy-to-use and efficient method (Luebbe & Finch, 1992; Patterson, 1992; Spencer & Cox, 1995; Finch & Luebbe, 2000).

After some time, opposing views on the use of algorithm emerged. Those who were against the use of the algorithm, had succeeded to prove its inefficiency through several examples. They had showed that in two circumstances, TOC is unable to find the optimum solution (Ray et al., 2010):

1. In problems which aim to select up to one new product among several new products, for adding to their production line (Lee & Plenert, 1993).
2. In problems in which multiple bottlenecks exist and bottlenecks are not utilized properly (Plenert, 1993).

Ferdendal and Lea (1997) proposed revised TOC (RTOC) algorithm to identify the optimal mix in multiple bottlenecks environment where the TOC algorithm could not do so. Hsu and Chung (1998) put the system's resources into the categories of capacity constraint resources and three levels of non-capacity constraint resources. They proposed a recursive algorithm for solving the product mix problem, which is similar to the dual-simplex method with bounded variables.

Onwubolu and Mutingi (2001a) used a genetic algorithm (GA) for solving large-scale product mix problems. They stated that TOC algorithm and ILP method are only capable of solving small size problems in reasonable computation time, while their proposed algorithm is capable of solving both large-scale and small-scale problems in acceptable CPU times. Onwubolu and Mutingi (2001b) proposed another algorithm based on GA for the case of multiple constrained resources. Comparing the results of TOC, RTOC, and ILP, they stated that although not in all small size problems GA can find the best solution, using it would ensure obtaining a high quality solution in reasonable time. Onwubolu (2001) developed an algorithm based on tabu search (TS). Comparing the solutions of this new algorithm with the solutions of TOC, RTOC, and ILP, he concluded that in reasonable amounts of time, the new algorithm can provide optimal or near-optimal solutions to the problems of existing literature and those large-scale problems that are randomly generated.

Proving RTOC's incapability of finding the best solution, through a counterexample, Aryanezhad and Komijan (2004) proposed an improved algorithm. Chung et al. (2005) for solving a product mix problem with two types of products in a semiconductor industry, first simulated 9 solutions and calculated the output of 8 criteria and then, chose the best solution through analytic hierarchy process (AHP) and analytic network process (ANP) approaches. Mishra et al. (2005) developed a tabu search and simulated annealing hybrid approach. They showed TOC's inability to find the best solution, through an example. They also stated that their proposed algorithm would find an appropriate solution. However, since the work load of one of the resources (resource 40 in their paper) exceeded its capacity, their obtained solution was infeasible. Komijan and Sadjadi (2005) used group decision-making approach for solving the problem. They first considered each bottleneck as a decision maker and calculated products weights based on throughput and late delivery cost. They then, determined the best product mix, through a mathematical model.

Chaharsooghi and Jafari (2007) proposed a simulated annealing (SA) algorithm for determining the product mix. They compared the results of their algorithm with those of TOC, RTOC, ILP, TS, and GA. The results indicated that in half of the small size problems, the optimum solution was obtained and among the six large-scale problems, in five problems the obtained solutions were better than the results of TS and GA.

Tsai and Lai (2007) developed a method based on the improved algorithm for optimizing a joint products further processing decision. Using the MATLAB fuzzy toolbox, Bhattacharya and Vasant (2007) and Bhattacharya et al. (2008) determined the product mix for different levels of satisfaction of decision maker.

Susanto et al. (2007) used fuzzy multi-objective linear programming for solving the product mix problem in an ice-cream manufacturing company with three products. Considering the non-integer solutions as acceptable, they determined the product mix that would ensure at least 75% of the potential maximum profit as well as a total waste that is lower than 30% of the optimum amount. Suharto et al. (2008) studied the problem of the ice-cream company with probabilistic constraints. Ray et al. (2008) discussed a state in which it was possible to both produce and outsource products.

Hasuike and Ishii (2009a, 2009b) allowing non-integer solutions for the amount of production, discussed a case in which the objective coefficients follow a normal distribution and the amounts of each product's need of resources are fuzzy numbers. Wang et al. (2009) used TOC and immune algorithms for solving the product mix problem. By solving the published problems of literature, they indicated that their suggested method reaches the optimum solution in all cases, while, TS and GA are only capable of finding the optimum solution in 20% and 50% cases, respectively. They admitted TOC and ILP's more effectiveness in obtaining the optimum solution in small size problems (with less than 5 products and 10 resources). However, they asserted that their proposed approach is more effective in solving the real-world problems, which are large-scale problems (with more than 100 products and 50 resources), and obtains high quality solutions in reasonable computation times.

Rezaie et al. (2009, 2010) implemented particle swarm optimization (PSO) method for solving the product-mix problem and claimed that their approach provides better solutions in both small size and large-scale problems. However, the results of their comparison were only based on a single example with 5 products and 6 resources. Nazari-Shirkouhi et al. (2010) used imperialist competitive algorithm (ICA) for solving the problem. They declared obtaining optimum solution through an example with three products and four resources and claimed that in other examples similar results would be obtained.

Karakas et al. (2010) developed an ABC-based approach for solving the problems with three activity levels and fuzzy constraints. Allowing non-integer solutions for production amounts, they determined the product mix. Badri and Ghazanfari (2011) developed an algorithm based on harmony search for finding the optimum solution of the problem. Comparing the results of their proposed algorithm with those of the TOC, TS, and PSO algorithms, they concluded that their algorithm is superior. Chellappa and Manemaran (2011) used the Borda method for prioritizing the products. However, they did not determine the product mix.

Susanto and Bhattacharya (2011) used compromise fuzzy multi-objective linear programming in order to determine the product mix of a chocolate manufacturing company with eight products. In their approach, they assumed the objective coefficients are fuzzy numbers. They also considered non-integer solutions as acceptable. Azadegan et al. (2011) reviewed the literature on fuzzy logic applications in manufacturing and developed an algorithm based on fuzzy RTOC. Also, some algorithms have been proposed for solving the problems with fuzzy processing time and capacity (Ghazinoory et al., 2012; Kaveh et al., 2013).

Tanhaei and Nahavandi (2011) considered the two objectives of maximizing the throughput and bottleneck exploitation and used goal programming for determining the product mix. Ray et al. (2010) developed an algorithm based on TOC and AHP. Wang et al. (2014) discussed this algorithm and found the cases in which it fails to provide the optimum solution.

Sobreiro and Nagano (2012) proposed a method based on TOC and knapsack problem which they claimed was superior to RTOC of Fredendall and Lea (1997) and the improved algorithm of Aryanezhad and Komijan (2004) in terms of the quality of solutions and running times. Tanhaei and Nahavandi (2013) developed an algorithm for solving product mix problem in two-constraint resources environment. Through an example with five products and six resources, they showed the superiority of their algorithm to TOC and tabu search algorithm. They asserted that its solutions are as good as the solutions of RTOC and ILP.

de Soza et al. (2013) developed an algorithm which determines its initial solution based on RTOC presented by Fredendall and Lea (1997) and defined the production mix based on Barnard's factor. Sobreiro et al. (2013) proposed 'Cut SM', an algorithm for maximizing the throughput per day. Finally, Badri et al. (2014) considered product mix problem with interval parameters and proposed a multi-criteria decision-making approach to determine the product mix based on TOC.

With respect to the above literature review, in the following an integrated model for product mix problem and scheduling is discussed.

3. Model formulation

In this section, a mathematical model is developed based on mixed integer linear programming. This model aims to determine the product mix in a way that would go on schedule. In this problem m resources (machines) are used for producing n products. As mentioned in the introduction section, Theory of Constraints is one of the most effective approaches for determining the product mix. Process batches and transfer batches are dealt differently by TOC. In proposed model, the amount of production is selected for the size of process batches, while, the transfer batches can have smaller sizes. By having the transfer batch be less than the process batch, it is possible to have several resources processing on a product simultaneously. Therefore, each operation of certain product may be overlapped with another operation of the same product on the next resources.

The following are the main assumptions of the problem:

- The demand, price, and the process times of each product as well as the cost of its raw materials are given variables.
- The available capacity of each resource is given.
- All resources and raw materials are available from the beginning.
- The operations performed on each product should follow a predetermined sequence.
- No product receives the same operation more than once.
- Only one type of each resource exists.
- At any given moment, each resource can perform operations on maximum one product.
- Setup times are included in the processing times.
- Preemption is not allowed.
- Only integer solutions are considered feasible for this system.

One basic question remains. What products and in what quantities should be produced so that we can obtain the maximum profit?

3.1 Notation

The notations used in the proposed model are listed as follows:

Indices:

i, i' index of product

j, j' index of resource

Parameters:

n number of products

m number of resources

$r_{ijj'}$ Binary parameter, it is equal 1 if process on resource j is predecessor for

	process on resource j' for product i, and zero otherwise
TB_i	transfer batch size of product i
M	a large real number (M → ∞)
t_{ij}	processing time of product i on resource j
D_i	demand of product i
p_i	selling price of the product i
m_i	raw material cost of product i
AC_j	available capacity of resource j,
OE	operating expenses,

Decision variables:

Q_i	decision variable representing the produced units of product i
$I_{ijj'}$	waiting time between the operation of preceding resource j and succeeding resource j' for product i
S_{ij}	start time of product i on resource j
$Y_{ii'j}$	a binary variable that is equal to 1 if product i is processed before i' on resource j , 0 otherwise

3.2 The proposed mathematical model

The integrated model for product mix problem and scheduling (IPMPS) is as follows:

$$\text{Maximize} \sum_{i=1}^{n} Q_i p_i - \sum_{i=1}^{n} Q_i m_i - OE \tag{5}$$

Subject to:

$$S_{ij} + t_{ij} Q_i \le S_{i'j} + M\left(1 - Y_{ii'j}\right) \qquad \forall i, j, i' \ne i \tag{6}$$

$$S_{i'j} + t_{i'j} Q_{i'} \le S_{ij} + M Y_{ii'j} \qquad \forall i, j, i' \ne i \tag{7}$$

$$I_{ijj'} = \begin{cases} 0 & , if\ t_{ij} \le t_{ij'} \\ \left(t_{ij} - t_{ij'}\right)(Q_i - TB_i), & if\ t_{ij} > t_{ij'} \end{cases} \qquad \forall i, j, j' \ne j, r_{ijj'} = 1 \tag{8}$$

$$S_{ij'} \ge S_{ij} + t_{ij} TB_i + I_{ijj'} \qquad \forall i, j, j' \ne j, r_{ijj'} = 1 \tag{9}$$

$$S_{ij} + t_{ij} Q_i \le AC_j \qquad \forall i, j \tag{10}$$

$$Q_i \le D_i \qquad \forall i \tag{11}$$

$$Q_i \ge 0, \text{int} \qquad \forall i \tag{12}$$

$$S_{ij} \ge 0 \qquad \forall i, j \tag{13}$$

$$Y_{ii'j} \in \{0,1\} \qquad \forall i, j, i' \ne i \tag{14}$$

Eq. (5) is the objective function, which aims to maximize the net profit. The first, second and the final parts relate to the sales revenue, the total cost of raw materials, and operating expenses, respectively. Eq. (6) and Eq. (7) define resource conflict constraints. These two constraints are Either-Or constraints. This means that for each resource, one of the constraints automatically becomes satisfied and thus leaves the calculations. Eq. (8) refers to the sum of times that product i should wait in order to be operated by resource j' , before which a transfer batch was processed by resource j. Eq. (9) is the constraint that ensures the operations performed on each product follow the predetermined sequence. Eq. (10) and Eq. (11) deal with the limitations of resource capacities and the products demands, respectively. Eq. (12) makes sure obtained production quantities are positive integers. To help the reader better understand Eq. (8), two cases are discussed:

Case 1. $\boldsymbol{t_{ij} \leq t_{ij'}}$.

As shown in Fig. (1), resource j' starts processing product i, just when the operations of resource j on the transfer batch of product i ends.

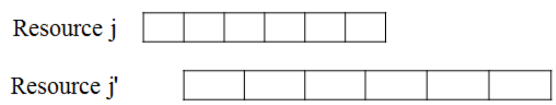

Fig. 1. If $t_{ij} \leq t_{ij'}$, then $I_{ijj'} = 0$.

Case 2. $\boldsymbol{t_{ij} > t_{ij'}}$.

As shown in Fig. 2, resource j' can only start processing product i, when a period of time $(t_{ij} - t_{ij'})(Q_i - TB_i)$ has been passed since resource j had finished processing the transfer batch of product i.

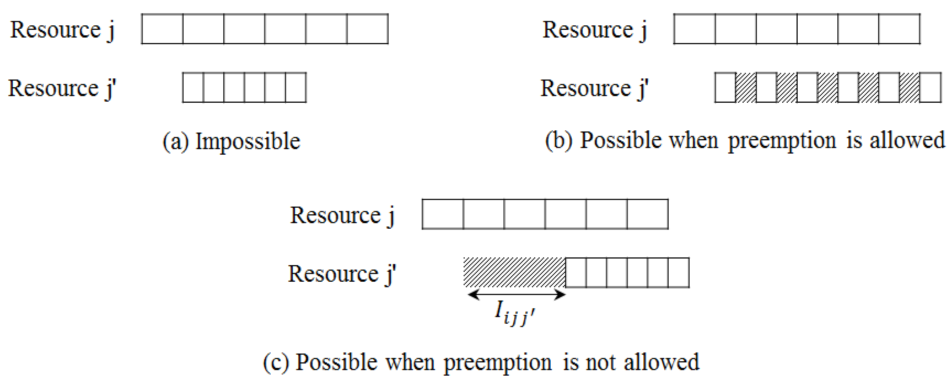

Fig. 2. If $t_{ij} > t_{ij'}$, then $I_{ijj'} = (t_{ij} - t_{ij'})(Q_i - TB_i)$.

4. Numerical Example

Assume a manufacturing company produces three products A, B and C using four resources (machines) R1, R2, R3 and R4. The selling price, the weekly demands, the processing times, the raw material costs, and the product flows through resources, are shown in Fig. 3. The available capacity of all resources in each week is 2400 minutes. The operating expense is \$3000 per week.

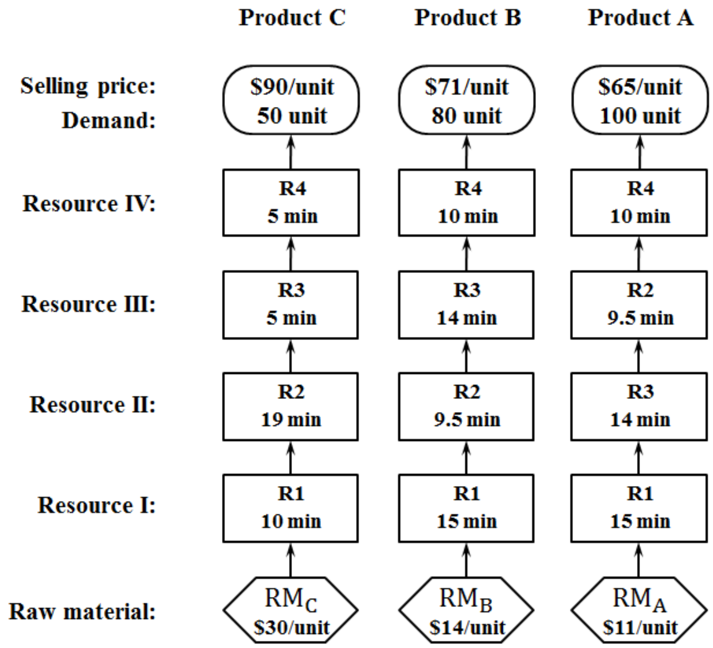

Fig. 3. Data for product mix problem

The solution of the conventional model, discussed in section 1, for this problem includes 34A, 80B and 69C. Assuming the size of transfer batches is one, as it can be observed in Fig. (4), 2789 minutes are needed for the production to become complete. Taking the available capacity (2400 minutes at last) into consideration, the feasible solution changes into 34, 49 and 69 for the quantities of the products A, B and C, respectively. Considering the operating expense of $3000, the net profit is calculated as follows:

Net Profit = 34 × (65 – 11) + 49 × (71 – 14) + 69 × (90 – 30) – 3000 = $5769

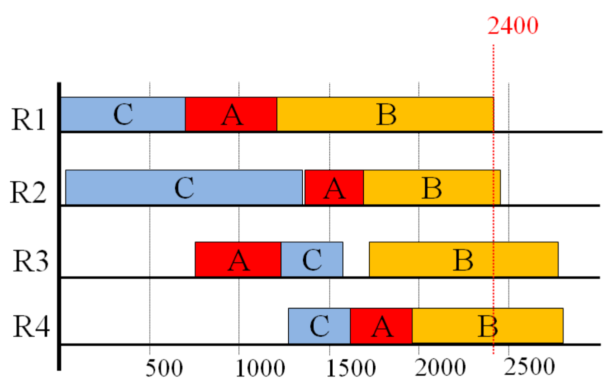

Fig. 4. Gantt chart for the conventional model solution (34A, 80B and 69C)

The problem was also solved through the proposed IPMPS. The obtained solution is 55A, 51B and 55C with a net profit of $6177.

Net Profit = 55 × (65 – 11) + 51 × (71 – 14) + 55 × (90 – 30) – 3000 = $6177

As shown in Fig. 5, producing the obtained product mix lasts for 2400 minutes and can be performed on schedule. It also leads to a 7.07% raise in the company's net profit.

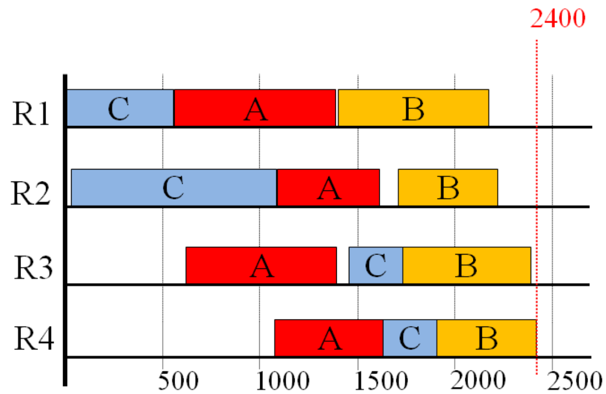

Fig. 5. Gantt chart for the proposed IPMPS solution (55A, 51B and 55C).

5. Computational results

The mentioned example in the previous section is solved for various sizes of transfer batches. The net profit is calculated for each case and the results are shown in Fig. 6. As it can be observed, increase in the size of transfer batches has non-increasing effect on the obtained net profits. It is also clear that the net profit of the mentioned problem is most sensitive to the changes in the size of transfer batches of product B, while it is least sensitive to the changes in the size of transfer batches of product C.

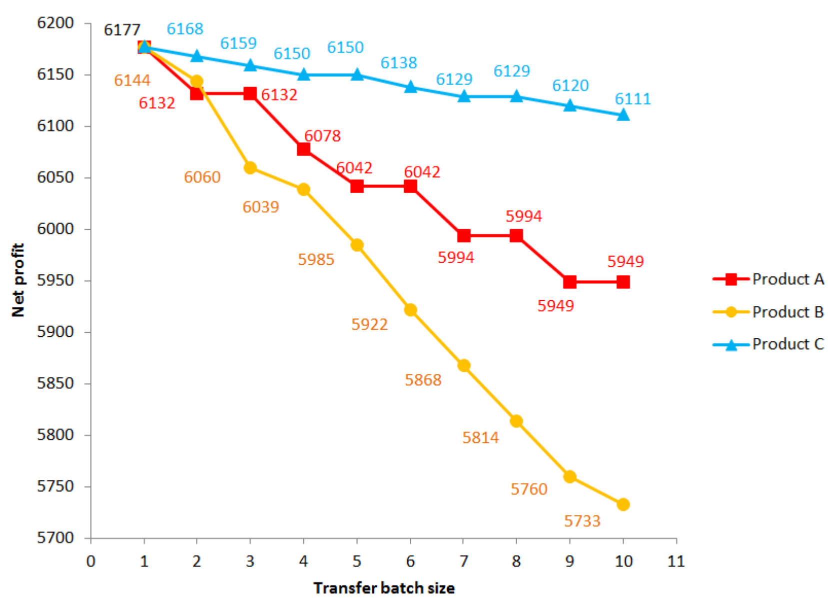

Fig. 6. Effect of transfer batch size in net profit

Seven product mix problems previously cited in the literature were solved by proposed IPMPS. The results are given in Table 1. It should be noted that in all of these problems, the size of transfer batches are considered 1. The production sequences of all products are also considered identical. Moreover, it is assumed that all resources are available for 2400 minutes.

As shown in Table 1, since the conventional model of product mix problems ignores the waiting time and the inevitable idleness of resources, it provides solutions that go off schedule. Hence, implementing the solutions of the conventional model requires capacities that exceed the available capacities. Meanwhile, the suggested IPMPS does not surpass the available capacities.

Table 1
Product mix comparison of conventional model and proposed IPMPS (since the require capacity exceeds the available one, the conventional model solution is infeasible)

No.	Author (year)	Prob. size m×n	Available capacity	Conventional model		Proposed IPMPS	
				Product mix (infeasible)	Require capacity	Product mix	Require capacity
1	Luebbe and Finch (1992)	4×2	2400	100A, 30B	2435	100A, 28B	2375
2	Lee and Plenert (1993)	4×3	2400	100P, 18Q, 18R-2	2420	100P, 16Q, 20R-2	2400
3	Lee and Plenert (1993)	4×3	2400	99P, 17Q, 20S-2	2415	100P, 16Q, 20S-2	2400
4	Luebbe and Finch (1992)	4×4	2400	100A, 50B, 5C, 0D	2435	100A, 50B, 3C, 1D	2395
5	Luebbe and Finch (1992)	4×4	2400	100A, 10B, 50C, 25D	2870	66A, 31B, 50C, 25D	2400
6	Fredendall and Lea (1997)	6×5	2400	20A, 20B, 40C, 28D, 50E	2723	20A, 23B, 40C, 18D, 52E	2388
7	Hsu and Chung (1998)	7×4	2400	51R, 38S, 50T, 100U	4245	34R, 28S, 50T, 36U	2400

6. Conclusion and further research

Most of existing approaches for solving product mix problems, assume that all resources can operate simultaneously and independently. Hence, as it was shown in case of the discussed example, their solutions cannot be fully implemented. This research proposed integrated model for product mix problem and scheduling (IPMPS). The solutions of this model can be implemented on schedule. In other words, the proposed model has determined the product mix and scheduling, simultaneously. Hence, the obtained product mix is more realistic. Furthermore, the fact that the size of transfer batches differs from the size of process batches in this model, makes overlapped operations possible. Studying the product mix problems of the published literature makes it clear that the proposed approach is superior to the conventional approach in terms of scheduling. Developing heuristic or meta-heuristic algorithms for solving large-scale problems and considering uncertainty are directions for future research.

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