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# An AHP-GRA method for asset allocation: A case study of investment firms on Tehran Stock Exchange

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A B S T R A C T

During the past five decades, there have been tremendous efforts to offer different methods for portfolio management. The primary objective of many of these methods is to provide a trade-off between risk and reward. The proposed study of this paper uses analytical hierarchy process (AHP) and grey relational analysis to offer a method for portfolio management. The proposed method of this paper uses a statistical sample consists of 16 firms whose shares were trading during the fiscal year of 2010 on Tehran Stock Exchange. The study uses AHP and gray relational analysis to assign weight to each firm. We also use a linear programming technique to model the resulted problem by considering some realistic constraints.



## 1. Introduction

Investment has been a major concern on many managerial problems and when it comes to uncertainty, many investors face some challenges (Miller, 1999; Macharis et al., 2004). Ju-Long (1982) considered stability and stabilization of a grey system whose state matrix is triangular and presented the displacement operator and established transfer. Many investment models involve multiple criteria decision making problems and we need to use preference measure methods to handle such problems (Lee et al., 1999; Dong et al., 2008; Hsia et al., 2004; Huang et al., 2008). Gondzio and Grothey (2007) exploited the structure of optimization problems and showed how portfolio optimization problems with sizes measured in millions of constraints and decision variables, featuring constraints on semi-variance, skewness or non-linear utility functions in the objective, could be solved with the state-of-the-art solver. Tanaka et al. (2000) proposed two types of portfolio selection models based on fuzzy probabilities and possibility distributions, respectively, rather than conventional probability distributions in Markowitz's model (Markowitz, 1952; Markowitz et al., 2000).

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Since fuzzy probabilities and possibility distributions were computed based on possibility grades of security data offered by experts, investment experts' knowledge could be reflected. Jia and Dyer (1996) presented a standard measure of risk and risk-value models. Ince and Trafalis (2006) looked at portfolio optimization problem by arguing that the USA equity market could not be efficient. They formulated the problem as a classification problem by implementing state of the art machine learning techniques such as minimax probability machine (MPM) and support vector machines (SVM). The implementation of MPM technique reported a bound on the misclassification probabilities. On the other hand, SVM detected a hyperplane, which maximizes the distance between two classes but they stated that both methods proved similar results for short-term portfolio management. Some of portfolio optimization problems can be formulated as NP-Hard problem where we may need to use metaheuristics to solve the resulted problems (Rolland, 1997).

Loraschi et al. (1995) presented distributed genetic algorithms with an application to portfolio selection problems. Inuiguchi and Tanino (2000) considered portfolio selection under independent possibilistic information. Some of the portfolio selection problems are involved with integration of multi criteria decision making such as analytical hierarchy process (Saaty, 1980, 1994; Tung, & Tang, 1998). There are also some cases where we wish to foretaste stock price using forecasting techniques (Tang et al., 2002). According to Lahmiri (2012), in financial industry, the accurate prediction of the stock market is a major challenge to optimize and update portfolios and also to make an assessment of several financial derivatives. Artificial neural networks and technical analysis are becoming widely used by industry experts to predict stock market moves. Lahmiri used various technical analysis measures and resilient back-propagation neural networks to forecast the price level of five major developed international stock markets, namely the US S&P500, Japanese Nikkei, UK FTSE100, German DAX, and the French CAC40. They compared four different technical analysis measures including indicators, oscillators, stochastics, and indexes. The out-of-sample simulation results demonstrated a strong evidence of the effectiveness of the indicators category over the oscillators, stochastics, and indexes. Besides, he reported that combining all these measures lead to an increase of the prediction error. In sum, technical analysis indicators seem to provide valuable information to predict the S&P500, Nikkei, FTSE100, DAX, and the CAC40 price level.

Gharakhani and Sadjadi (2013) investigated advanced optimization technique for portfolio problem introduced by Black and Litterman to study the shortcomings of Markowitz standard Mean-Variance optimization. Black and Litterman proposed a new technique to estimate asset return. They presented a way to incorporate the investor's views into asset pricing process. Since the investor's view about future asset return was always subjective and imprecise, we may represent it by using fuzzy numbers and the resulting model is multi-objective linear programming. Therefore, Gharakhani and Sadjadi proposed a model to analyze through fuzzy compromise programming approach using appropriate membership function. For this purpose, they introduced the fuzzy ideal solution concept based on investor preference and indifference relationships using canonical representation of proposed fuzzy numbers by means of their corresponding$\alpha$-cuts. A real world numerical example was also presented in which MSCI (Morgan Stanley Capital International Index) was chosen as the target index. The results were reported for a portfolio consisting of the six national indices. The performance of the proposed models was compared using several financial criteria.

## 2. The proposed method

Markowitz, H. (1952) is believed to be the first who introduced the idea of portfolio optimization. His model tries to find asset allocation based on the following mathematical model,

$$\begin{aligned} &\min Z: -\lambda(\sum_{i=1}^{N} X_i E_i) + \sum_{i=1}^{N}\sum_{j=1}^{N} X_i X_j C_{ij} \\ &\text{subject to} \quad \sum_{i=1}^{N} X_i = 1 \\ &\qquad\qquad \lambda, X_i \geq 0 \end{aligned} \tag{1}$$

In model (1), $X_i$ and $E_i$ are the amount of investment and return on asset *i,* respectively. $C_{ij}$ is the covariance between asset *i* and asset *j*. The first term in the objective function is associated with portfolio return; the second term determines portfolio risk and $\lambda$ determines the trade-off between these two terms. The first constraint is called budget constraint while the second constraint $X_i \geq 0$ specifies that there is no short selling.

Jia and Dyer (1996) argued that Markowitz model does not consider many existing constrains with the model such as liquidity, limitation on buy/sell, etc. In addition, when we add cardinality constraint to model (1), we may face more complicated problem.

### *2.1. Analytical hierarchy process*

Saaty (1980) is believed to be the first who introduced the idea of analytical hierarchy process (AHP) as the first multi criteria decision making technique. Fig. 1 demonstrates different components of ranking various alternatives using this technique.

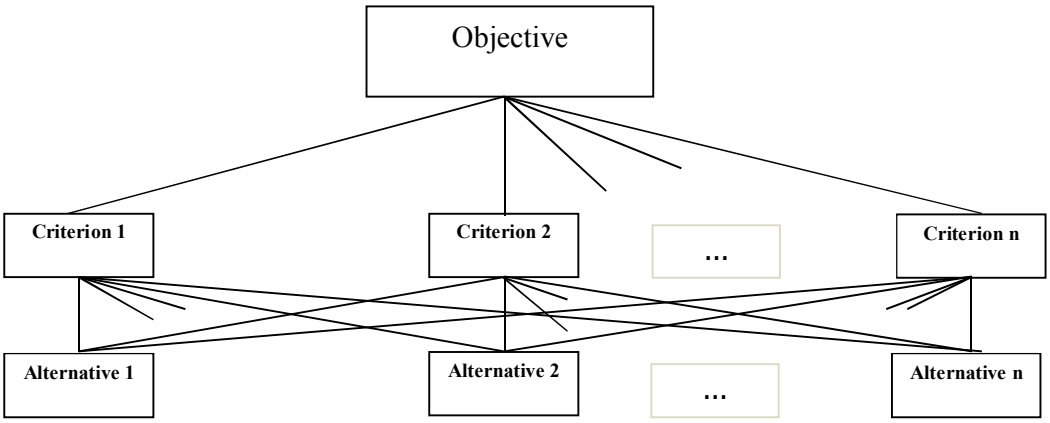


**Fig. 1.** The structure of AHP

The basic structure of AHP is based on pairwise comparison of various alternative where decision maker (DM) gives his/her relative importance of one alternative versus another one based on some linguistic terms, which could be transferred to some Likert numbers from one to nine. The method finds average row-column and using a consistency ratio attempts to determine whether the comparisons are consistent or not.

### *2.2 Grey Relational Grade*

Consider $X_0$ as reference and *N* alternatives with *k* criteria as follows,

$$X_0 = \{X_0(1), X_0(2), \ldots\ldots, X_0(j), \ldots.X_0(k)\}$$
$$X_1 = \{X_1(1), X_1(2), \ldots\ldots, X_1(j), \ldots.X_1(k)\}$$
$$\vdots$$
$$X_i = \{X_0 = \{X_0(1), X_0(2), \ldots\ldots, X_0(j), \ldots.X_0(k)\} \quad (2)$$
$$\vdots$$
$$X_N = \{X_i(1), X_i(2), \ldots\ldots, X_i(j), \ldots.X_i(k)\}$$

Grey relational coefficient are calculated as follows,

$$\gamma_{0i} = \frac{\Delta\min + \xi\Delta\max}{\Delta X_{0i} + \xi\Delta\max}, \quad (3)$$

where $\Delta X_{0i}$ is the absolute difference between $X_0$ and $X_i$ in $k_{th}$ criterion, $\Delta X_{0i} = |X_0(k) - X_i(k)|$. In addition, $\Delta\max = \max_i \max_j \Delta X_{0i}$ and $\Delta\min = \min_i \min_k \Delta X_{0i}$. Finally, grey relational degree is calculated as follows,

$$\Gamma_{0i} = \sum_{j=1}^{k} w_j \gamma_{0i}, \quad (4)$$

where $w_j$ is the weight of criterion $j$ and we may use $W_j = \frac{1}{k}$. Finally, all relationships must be normalized as follows,

$$x_i^*(j) = \frac{x_i(j) - \min_j x_i(j)}{\max_j x_i(j) - \min_j x_i(j)}, \quad (5)$$

$$x_i^*(j) = \frac{\max_j x_i(j) - x_i(j)}{\max_j x_i(j) - \min_j x_i(j)}, \quad (6)$$

and we use Eq. (5) in this paper for our calculations.

## 3. The case study

In this paper, we have gathered the information from Tehran Stock Exchange. The proposed method of this paper uses a statistical sample consists of 16 firms whose shares were trading during the fiscal year of 2010 on Tehran Stock Exchange. First, we have asked some experts to perform AHP on important criteria and let us find the relative importance of all criteria. Table 1 summarizes the results of our survey,

**Table 1**
The summary of important factors along with relative weights

| Index | Share price | EPS | DPS | Share benefit | Management | Operating profit | Technology | P/E | Firm size |
|---|---|---|---|---|---|---|---|---|---|
| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
| Weight | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.05 | 0.07 | 0.04 |
| Index | Equity size | EVA | β | Current ratio | Quick ratio | Inventory turnover | Weighted mean | Rumors | International rules |
| | C10 | C11 | C12 | C13 | C14 | C15 | C16 | C17 | C18 |
| Weight | 0.03 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 | 0.04 | 0.03 |

EPS: Earning per share, DPS: Dividend per share, EVA: Economic value added

The proposed study considers the information of 16 investment firms denoted by A1 to A16. Table 2 demonstrates details of weights (C1-C16) associated with different firms (A1-A16).

**Table 2**
Relative weight of each firm based on different criteria

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 | C13 | C14 | C15 | C16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ($A_1$) | .33 | .20 | .31 | .43 | .22 | .43 | .32 | .23 | .26 | .65 | .73 | .45 | .14 | .64 | .32 | .26 |
| ($A_2$) | .11 | .37 | .76 | .16 | .33 | .54 | .24 | .10 | .62 | .16 | .82 | .14 | .62 | .14 | .62 | .72 |
| ($A_3$) | .47 | .01 | .34 | .66 | .86 | .19 | .21 | .68 | .66 | .85 | .90 | .13 | .66 | .10 | .68 | .34 |
| ($A_4$) | .64 | .14 | .18 | .28 | .58 | .78 | .23 | .39 | .76 | .51 | .69 | .24 | .85 | .56 | .17 | .62 |
| ($A_5$) | .26 | .65 | .73 | .45 | .14 | .64 | .32 | .26 | .38 | .31 | .40 | .21 | .83 | .85 | .16 | .66 |
| ($A_6$) | .98 | .75 | .16 | .73 | .69 | .41 | .56 | .89 | .13 | .89 | .49 | .23 | .08 | .37 | .85 | .76 |
| ($A_7$) | .19 | .71 | .14 | .95 | .42 | .75 | .85 | .35 | .64 | .59 | .67 | .73 | .85 | .62 | .83 | .38 |
| ($A_8$) | .17 | .23 | .76 | .32 | .42 | .39 | .37 | .15 | .96 | .74 | .25 | .16 | .98 | .48 | .08 | .13 |
| ($A_9$) | .16 | .31 | .38 | .48 | .51 | .69 | .62 | .13 | .69 | .48 | .16 | .14 | .31 | .55 | .34 | .74 |
| ($A_{10}$) | .85 | .90 | .13 | .66 | .31 | .40 | .48 | .86 | .42 | .51 | .14 | .76 | .14 | .69 | .18 | .48 |
| ($A11$) | .83 | .73 | .64 | .91 | .89 | .49 | .55 | .68 | .42 | .91 | .65 | .38 | .61 | .42 | .73 | .51 |
| ($A_{12}$) | .08 | .64 | .96 | .38 | .59 | .67 | .88 | .16 | .51 | .38 | .75 | .56 | .32 | .42 | .16 | .91 |
| ($A13$) | .85 | .14 | .69 | .97 | .74 | .25 | .29 | .64 | .31 | .97 | .16 | .85 | .62 | .51 | .14 | .16 |
| ($A_{14}$) | .72 | .83 | .96 | .54 | .48 | .16 | .12 | .34 | .89 | .54 | .66 | .37 | .66 | .75 | .76 | .14 |
| ($A_{15}$) | .18 | .76 | .98 | .43 | .28 | .71 | .27 | .43 | .16 | .37 | .28 | .24 | .85 | .54 | .38 | .65 |
| ($A_{16}$) | .86 | .12 | .31 | .49 | .23 | .64 | .85 | .34 | .66 | .62 | .45 | .21 | .69 | .43 | .64 | .62 |

Finally, Table 3 summarizes the results of grey relational analysis for the proposed study of this paper.

**Table 3**
The summary of grey relational analysis for 16 firms

| Firm | Name | Grey relational analysis | Rank |
|---|---|---|---|
| ($A_6$) | *Bank Melli Investment firm* | 0.88 | 1 |
| ($A_{14}$) | *Tokafolad Investment firm* | 0.82 | 2 |
| ($A_4$) | *Credit Union Investment firm* | 0.79 | 3 |
| ($A_{10}$) | *Khozestan development Investment firm* | 0.77 | 4 |
| ($A_{11}$) | *Toos development Investment firm* | 0.62 | 5 |
| ($A_1$) | *Iranian petrochemical Investment firm* | 0.60 | 6 |
| ($A_2$) | *Maskan Investment firm* | 0.58 | 7 |
| ($A_8$) | *Khozestan development Investment firm* | 0.55 | 8 |
| ($A_5$) | *Alborz Investment firm* | 0.53 | 9 |
| ($A_3$) | *Damavant Investment firm* | 0.49 | 10 |
| ($A_7$) | *Bahman Investment firm* | 0.48 | 11 |
| ($A_{12}$) | *Industrial development Investment firm* | 0.42 | 12 |
| ($A_{16}$) | *Civil Investment firm* | 0.37 | 13 |
| ($A_{15}$) | *Rena Investment firm* | 0.30 | 14 |
| ($A_{13}$) | *Melli development Investment firm* | 0.27 | 15 |
| ($A_9$) | *Azarbayejan Investment firm* | 0.15 | 16 |

According to the results of Table 3, Bank Melli investment firm is number one priority followed by Tokafolad investment firm, credit union and Khozestan development investment group. The ranking of various firms have indicated that the management of some firms have had better performance in the past.

## 4. Conclusion

In this paper, we have presented an empirical survey on ranking different investment firms based on various criteria. The proposed study has implemented analytical hierarchy process as well grey relational analysis to rank investment groups. The results of ranking of these investment groups can be implemented as inputs of a linear programming model where some regular constraints such as budget and lower/upper bounds are considered.

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