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A multiple criteria decision making for raking alternatives using preference relation matrix based on intuitionistic fuzzy sets

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C H R O N I C L E

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A B S T R A C T

Ranking various alternatives has been under investigation and there are literally various methods and techniques for making a decision based on various criteria. One of the primary concerns on ranking methodologies such as analytical hierarchy process (AHP) is that decision makers cannot express his/her feeling in crisp form. Therefore, we need to use linguistic terms to receive the relative weights for comparing various alternatives. In this paper, we discuss ranking different alternatives based on the implementation of preference relation matrix based on intuitionistic fuzzy sets.

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1. Introduction

Saaty and Sagir (2009) discussed that rank preservation and reversal are an unresolved problem in the field of economics and utility theory and these issues have come into concentration since the Analytic Hierarchy Process (AHP) was developed because it uses paired comparisons that inevitably make the priorities of the alternatives interdependent.

Saaty and Sagir (2009) summarized some essential issues, which play key roles in rank preservation and reversal with counterexamples to demonstrate that preserving rank in all situations could be wrong. Szmidt Kacprzyk (2001) proposed a non-probabilistic-type entropy measure for intuitionistic fuzzy sets. Atanassov (1994) offered different operators over the interval valued intuitionistic fuzzy sets and their basic properties were studied. Atanassov and Gargov (1989) presented a generalization of the notion of intuitionistic fuzzy set in the spirit of ordinary interval valued fuzzy sets.

Qian and Feng (2008) presented an intuitionistic weight generation approach from intuitionistic preference relations. They considered the consistency and priority method of intuitionistic preference relationship and defined the concepts of intuitionistic vector, certain intuitionistic vector, interval

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intuitionistic vector, normalized intuitionistic vector, consistent intuitionistic preference relation and satisfactory consistent intuitionistic preference relationship. The also built programming techniques for estimating interval intuitionistic priority vector from intuitionistic preference relations and provided two instances to demonstrate the validity and practicality of their methods. According to

Qian et al. (200) studied the consistency issue of the interval complementary comparison matrix based on the consistency of interval complementary comparison matrix. They also defined perfect consistency, strong consistency, consistency and satisfactory consistency and discussed the relationships among all definitions. They also proposed one method for examining strong consistency, three techniques for examining consistency and one method for examining satisfactory consistency.

Xu (2007) introduced a technique for performing comparison between two intuitionistic fuzzy values and developed some aggregation operators based on score function and accuracy function, such as the intuitionistic fuzzy weighted averaging operator, intuitionistic fuzzy ordered weighted averaging operator, and intuitionistic fuzzy hybrid aggregation operator, for combining intuitionistic fuzzy values and establish various properties of these operators.

Bustince and Burillo (1996) recapitulated the definition given by Atanassov (1986) of intuitionistic fuzzy sets as well as the definition of vague sets and compared both definitions. Wang and Xin (2005) introduced the axiom definition of distance measure between intuitionistic fuzzy sets (IFSs) and some distance measures were proposed and corresponding proofs were given. They also analyzed the relationships between similarity measure and distance measure of IFSs and the distance measures of IFSs were applied to pattern recognitions.

Wang and Chin (2006) presented an eigenvector method (EM) to produce interval or fuzzy weight estimate from an interval or fuzzy comparison matrix, which differs from Csutora and Buckley's Lambda-Max techniques in different perspectives. First, the proposed EM generated a normalized interval or fuzzy eigenvector weight estimate through the solution of a linear programming technique, while the Lambda-Max technique uses a series of non-normalized interval eigenvector weight estimate. The other thing is that the EM solves the principal right eigenvector of an interval or fuzzy comparison matrix, directly while the Lambda-Max technique requires transforming a fuzzy comparison matrix into a series of interval comparison matrices by applying α-level sets and the extension principle and therefore requires the solution of a series of eigenvalue problems. Finally, the Lambda-Max technique requires the help of the principal right eigenvector of a crisp comparison matrix to detect the final interval weights, while the EM does not need this condition. They also reported that not all interval or fuzzy comparison matrices could generate normalized interval or fuzzy eigenvector weights. There are some circumstances where the EM cannot be used and we analyze the aggregation of local interval or fuzzy weights into global interval or fuzzy weights and discuss the findings.

In this paper, we present a hybrid method to rank different alternatives based on fuzzy AHP and IFV method. The organization of this paper first presents details of terms and necessary definitions in section 2 and section 3 while section 4 presents details of the proposed model and the paper ends with concluding remarks to summarize the contribution of the paper.

2. Assumptions and definitions

There are different kinds of fuzzy analytical hierarchy process (AHP) and the proposed study of this paper uses the one presented by Chang (1996). The method first uses triangular fuzzy numbers for pairwise comparison and then the synthetic extend value S_i of the pairwise comparison is presented and by implementing the principle of the comparison of fuzzy numbers we have

$V(M_1 \geq M_2) = 1 \quad \text{iff} \quad m_1 \geq m_2$

and

$V(M_1 \geq M_2) = \text{hgt}(M_1 \cap M_2) = \mu_{M_1}(d),$

and the weight vectors for each element can be represented based on the following,

$d(A_i) = \min V(S_i \geq S_k), k = 1, \cdots, n, k \neq i.$

The proposed model of this paper uses the concept of triangular fuzzy numbers based on the following definition,

Definition 1. $M \in F(R)$ is a fuzzy number if there exists $x_0 \in R$ such that $\mu_M(x_0) = 1$ and for any $\alpha \in [0,1]$ we have $A_\alpha = [x, \mu_{A_\alpha}(x) \geq \alpha]$.

In this paper we adopt the regular arithmetic operations from Chang (1996).

Definition 2. Let $M_{g_i}^1, \cdots, M_{g_i}^m$ be values of extent analysis of ith objective for m objectives. Then the value of fuzzy synthetic extend can be expressed as follows,

$$S_i = \sum_{j=1}^{m} M_{g_i}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^j \right]^{-1}.$$

The first task of fuzzy AHP technique is to find relative importance of each pair of factors in the same hierarchy. We first build $A = (a_{ij})_{n \times m}$ in triangular fuzzy term $a_{ij} = (u, m, l)$. In addition the inverse relationship is defined as $a_{ij}^{-1} = (1/u, 1/m, 1/l)$. Interested readers are referred to read Chang (1996) for more details.

3. Intuitionistic fuzzy sets

Definition 3. $\alpha_A(x) = (\mu_\alpha(x), v_\alpha(x))$ is considered intuitionistic fuzzy sets (IFV) when we have $\mu_\alpha(x), v_\alpha(x) \epsilon [0,1], \mu_\alpha(x) + v_\alpha(x) \leq 1$. Based on the value of $\pi_\alpha(x)$ IFV can be represented as $\alpha_A(x) = (\mu_\alpha(x), v_\alpha(x), \pi_\alpha(x))$ such that $\mu_\alpha(x), v_\alpha(x), \pi_\alpha(x) \epsilon [0,1]$, $\pi_\alpha(x) + \mu_\alpha(x) + v_\alpha(x) = 1$. For the sake of simplicity we represent $\alpha = (\mu, v, \pi)$ as $\alpha = (\mu, v)$.

Definition 4. The vector $X = (x_1, \ldots, x_n)$ is called IF vector when we have,

$$\begin{cases} \sum_{i=1}^{n} u_i + \pi_i \leq 1 \\ \sum_{i=1}^{n} v_i + \pi_i \leq n - 1 \end{cases} \quad j = 1, \ldots., n$$

We may now define IF matrix (Wang et al., 2011) based on the following notation,

$M = (M_{IJ})_{m \times n} = ((u_{IJ}, v_{IJ}))_{m \times n}$

Definition 5. An Intuitionistic Fuzzy Comparison Matrix(IFCM) or Intuitionistic Fuzzy Preference Relation(IFPR) can be defined as follows,

$M = (M_{it})_{m \times s} = (({u_{it}}^M, {v_{it}}^M))_{m \times s} \quad , N = (N_{tj})_{s \times n} = (({u_{tj}}^N, {v_{tj}}^N))_{s \times n}$

In addition, for multiplying two matrices M and N we use $C = (C_{ij})_{m\times n} = ((u_{ij}{}^{C}, v_{ij}{}^{C}))_{m\times n}$ as follows,

$$C_{ij} = \underset{t=1}{\overset{s}{\oplus}} (M_{it} \otimes N_{tj})$$

Next we need to define eigenvalue of IFCM.

Definition 6. Let M and x be a matrix and vector of IFCM, respectively with $x = ((u_i^x, v_i^x))_{n\times 1}$ then we have

$$M \otimes x = \lambda \otimes x,$$

where λ is the eigenvalue and x represents eigenvector, respectively. In this study, we use different types of IFV.

Definition 7.

Let $a = (u_a, v_a), b = (u_b, v_b)$ be two IFV numbers, then the preference of these two numbers are performed as follows,

$$P(a > b) = \frac{\max\{0, 1 - v_a - u_b\} - \max\{0, u_a - (1 - v_b)\}}{\pi_a + \pi_b}$$

where $\pi_a = 1 - u_a - v_a, \pi_b = 1 - u_b - v_b$. In addition when $P(a > b) > P(b > a)$ we say a has a degree of preference over b and it is denoted as $a \overset{P(a>b)}{\succ} b$.

4. The proposed model

The proposed model of this paper uses the following steps for ranking different alternatives.

Step 1. Form hierarchies: the hierarchy is formed by decision makers. Suppose there are n levels of hierarchy where the highest level, the first level or the lowest level options, *n* is called the depositary. Note that there are n_i components or criteria in level *i*.

Note that each member (*u*, *v*) in $M_j^{(i)}$ maintains two components, where the first one determines the degree of certainty of criterion *u* compared with other criteria and degree of certainty of *v* to prefer the first criterion to the previous one where $u + v \leq 1$. Note that it is not always easy task to convert regular numbers to IFV and in case of confusion, interested readers can use the methods suggested by Wang and Chin (2006).

Step 2. Perform pair-wise comparison based on the fuzzy AHP method explained earlier,

Step 3. Compute eigenvalue and eigenvectors associated with the pairwise comparison and prepare the following matrix,

$$M^{(i)} = \begin{bmatrix} x_{11}^{(i)} & \cdots & x_{1,n_{i-1}}^{(i)} \\ \vdots & \ddots & \vdots \\ x_{n_i,1}^{(i)} & \cdots & x_{n_i,n_{i-1}}^{(i)} \end{bmatrix}$$

Note that when $M_j^{(i)}$ maintains consistent information it is an straightforward task to find eigenvalues as well as eigenvectors of $x_j^i = (x_{1j}^i, \dots, x_{n_i,j}^i)^T$ based on the implementation of Wang and Chin (2006).

Step 4. Combine all components after calculating eigenvalues on vectors Note that $x_j^i = (x_{1j}^i, \dots, x_{n_i,j}^i)^T$ based on $x = M^{(2)} \otimes \dots \otimes M^{(n)}$.

The implementation of this method is able to handle uncertainty very easily since it receives vague figures based on fuzzy numbers.

5. Conclusion

In this paper, we have presented a new method to rank different criteria based on using preference relation matrix based on intuitionistic fuzzy sets. The proposed model of this paper uses fuzzy AHP based on the fuzzy method presented by Chang (1996). We have implemented the technique originally developed by Wang and Chin (2006) to calculated the eigenvalue and eigenvectors for generating normalized interval and fuzzy weights. Recently, there have been special interest in using robust optimization techniques to handle uncertainty associated with input parameters and we believe the idea of this paper can be extended in this area. Many robust optimization techniques assume that input parameters follow uniform distribution and make sure that the final solution is robust against changes on users' feedback. We leave this area of research for interested researcher as future work.

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