Decision Science Letters 2 (2012) 35–44

Contents lists available at GrowingScience

Decision Science Letters

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Developing a multi stage mathematical production management model for auto industry

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C H R O N I C L E

Article history:
Received July 25, 2012
Accepted October 10, 2012
Available online
October 18 2012

Keywords:
Production management
Inventory management
Khorasan razavi Iran Khodro
Iranian Company

A B S T R A C T

In this paper, we establish an integrated mathematical production management model based on five parts insisted of finished goods production in production sites, inventory control of finished goods in production and distribution facilities, semi-finished goods production in producer's facilities, inventory control of raw materials in production sites and Inventory control of semi-finished products. The main goal of developed model is to minimize total cost of company in the mentioned parts. Finally, the model is validated with 7 real-world case studies from auto-industry in one of the largest Iranian car producer company, Iran Khodro co and the empirical results are illustrated at the end of the paper.

1. Introduction

The most important roles of a production management model are integrating and all needed stages of products producing process and executing them in an ordered and coordinated manner. In each stage in which some values are added to a manufactured product, insisted of supplying raw materials and intermediate components, manufacturing the finished products, packaging, transporting, warehousing, and logistics. Developing a mathematical production model because of its precision essence can be so applicable for industries. So mathematical modeling of a coordinated production management is still a largely undeveloped area of research. Sharma and Agrawal (2009) considered a multi-stages serial production system with the use of probabilistic demand situations for the end product. They considered the demand situations as binomial, exponential, lognormal and Poisson. Jung et al. (2008) considered a stock management for a multi-stage supply chain in the petrochemical, chemical and pharmaceutical industries, and they demonstrated the effectiveness of theirs safety stock management model on a realistically scaled polymer supply chain problem. The deterministic optimization problem of a profit-maximizing firm is studied by Jouini et al. (2007).

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doi: 10.5267/j.dsl.2012.10.003

Fontes Dalila et al. (2007) addressed investment decisions in production systems by using real options. He tried to develop and implement stochastic dynamic programming models both for fixed and flexible capacity systems. Lejeune (2006) recommended a mathematical production management among a three-stage supply chain model, in which a sustainable inventory–production–distribution plan over a multi-period horizon is designed and he tried to solve it with a new mathematical method. Merdivenci Atici, and Uysal (2007) investigated the optimal production and inventory paths of HMMS type models on complex time domains. To solve problem they used a new mathematical theory with Time scale calculus name. Ouyang et al. (2008) presented an integrated inventory model with variable production and price-sensitive demand rates. They attempted to offer a best policy that aims at maximizing the joint total profit while the trade credit and freight rate are linked to the order quantity, simultaneously. A specification of strictly increasing and decreasing returns to scale in multi-output technologies was proposed by Boussemart et al. (2008), they established some necessary and sufficient conditions to characterize strictly increasing and decreasing returns to scale for a large class of technologies.

In This paper, we develop a mathematical production management model with decreasing costs main goal in following sections. (1) Raw materials holding costs, (2) semi-finished products production costs, (3) semi-finished products holding costs. (4) End products production costs. and (5) end products holding costs.

The remainder of the paper is organized as follows: In Section 2. We will introduce the notation used, explain the hypotheses of the model and formulate the integrated model and In Section 3. We will report the numerical results of the methodology, which is applied to a real-life industrial problem faced by a large production management in Iran khodro khorasan Razavi Company.

2. The Proposed model

2.1. Five production management components

In this paper, we develop a mathematical five-segment production management model presented in Fig. 1. The first stage, i.e., inventory control of Raw materials, semi-finished products and end products, is in charge of the procurement of raw materials and/or component, the second stage, i.e., production, represents the manufacturing and/or assembly of the finished and semi-finished goods. The customer demand and lead times, defined as the sum of the traveling, loading and unloading times, are known, and vary from period to period. There is no backlog; the demand, which cannot be satisfied from on-hand inventory is lost.

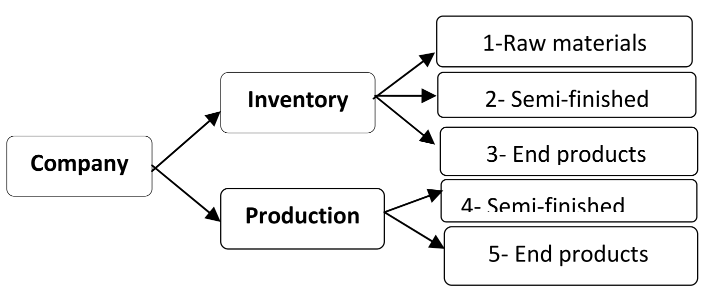

Fig. 1. Five parts production management model

The production management program, which is defined in this paper, follows below schemes,

- ✓ The ending inventory level required at each node and at each period in its three mentioned parts;

- ✓ The production scheme, i.e., the quantity of semi-finished and finished products and raw materials to be produced or entered to each facility at each production facility and at each period;

2.2 Model formulation and notation

In the next subsections, we define the objective function, decision variables and parameters of developed model.

Decision variables

The decision variables involved in costs minimization of the production model are as follows,

$\boldsymbol{P_{it}}$	Quantity of finished products, provided by facility i at time t,
$\boldsymbol{b_{it}}$	Quantity of raw materials provided by facility i at time t,
o_{it}	Demand for semi-finished products at node i and time t,
s_{it}	Products inventory level at facility i at the end of period t.
w_{it}	Raw materials inventory level in facility i at the end of period t.
B'_{it}	Quantity of raw materials which are entered in facility i at time t.
oin_{it}	Semi-finished product inventory level at facility i and time t.

2.2.1 Objective function

The objective function is defined for minimizing the holding costs (Raw materials, semi-finished products and finished product, parts 1,2 and 3 in objective function) and production costs (for finished and semi-finished product, parts 4 and 5 in objective function). The production costs vary with the production level at the facilities. The holding costs are proportion to the stock levels of products and raw materials at all nodes in production management. Generally, this objective function is designed for minimization of company's costs in its mentioned five parts. The objective function is formulated as follows:

$$\min \underbrace{\sum_{t\in T}\sum_{i\in I}\left(h'_{it}.(B'_{it}+b_{it}+w_{it})\right)}_{\text{1- Raw materials holding}}+\underbrace{\sum_{t\in T}\sum_{i\in I}\left(h''_{it}.(oin_{it}+o_{it})\right)}_{\text{2- Semi-finished holding costs}}$$

$$+\underbrace{\sum_{t\in T}\sum_{i\in I}\left(h_{it}.(P_{it}+s_{it})\right)+\sum_{t\in T}\sum_{j\in J}\left(h_{jt}\cdot s_{jt}\right)}_{\text{3- Finished products holding costs in producers and distributors nodes}}$$

$$\underbrace{\sum_{t\in T}(p_{it}.P_{it})}_{\text{4-Finished products production}}+\underbrace{\sum_{t\in T}\sum_{i\in I}o_{it}.O_{it}}_{\text{5- Semi-finished product production}}$$

where I is the set of manufactures production facilities, J is the set of distributors, T is the set of time-periods in the developed model planning horizon, p_{it} is the unit production cost for finished products in facility i at time t. O_{it} is the unit production cost for semi-finished products in facility i at time t. h'_{it} is the unit holding costs for raw materials in facility i at time t, h''_{it} is unit holding costs for semi-finished products in facility i at time t, h_{it} is the unit holding cost at node k and time t.

2.2.2. Constraints

Below, we list the constraints that must be satisfied over the planning horizon. The constraints should be defined to cover following parts:

- Raw materials
  Sustainability, balance, Production, Storage,
- Semi-finished products
  Balance, production, storage
- Finished products.
  Inventory, production, balance

- *raw materials constraints*:

Sustainability:

$$z_{jt} \geq z_{jt-1}, \qquad j \in J, t \in T \tag{1}$$

$$s_{it} \geq s_{it-1}, \qquad i \in I, t \in T \tag{2}$$

$$w_{it} \geq w_{it-1}, \qquad i \in I, t \in T \tag{3}$$

In which index t presents the periods of planning horizon. A sustainable plan requires the inclusion of the constraints (1)–(3), which enforce that the end inventory levels of products and raw materials be at least equal to the before ones. The omission of this type of constraints would result in the construction of a *myopic* plan (Heyman. ET, all. 1984) that may be characterized by unacceptably low, possibly equal to 0, end inventory levels, and leave the production plan in an undesirable state for the future.

Flow balance of raw materials at supplier's facilities:

$$w_{it} = w_{it-1} + b_{it} - \sum_{v \in V} \sum_{\substack{i' \in I \\ i \neq i'}} q_{ii'vt} - f_{it} \times p_{it} - g_{it} \times o_{it}, \qquad i \in I, t \in T \tag{4}$$

The raw materials are consumed at the suppliers' and manufacturers' facilities. All raw materials needed by the manufacturers' facilities come from the suppliers' facilities. Denoting by b_{it} the amount of raw materials available in time t, by $q_{ii'vt}$ the amount of raw materials moved from the suppliers' facility i to the manufacturers' facility i' using v at t, by p_{it} the production at the suppliers' facility at t, by f_{it} the amount of raw materials for a unit of production, by o_{it} the demand for the semi-finished product at t, and by g_{it} the appropriate conversion coefficient, we obtain Eq. (4).

Flow balance of raw materials at manufacturers' facilities:

$$w_{it} = w_{it-1} + \sum_{v \in V} \sum_{\substack{i \in I \\ i \neq i'}} q_{ii'vt} - f_{it} \times p_{it} - g_{it} \times o_{it} \tag{5}$$

Constraints (4), (5) enforce that current raw materials and product demands at facilities be met from current production and on-hand inventories.

Production capacity of raw materials:

$$0 \leq b_{it} \leq b_i^{\max} \qquad , i \in I, t \in T \tag{6}$$

Constraint (6) limits the production of raw materials and products below a specified maximum level.

Storage capacity of raw materials at facilities:

$$0 \le w_{it} \le w_i^{\max} \qquad ,i \in I, t \in T \tag{7}$$

Constraint (7) limits from above the inventories of raw materials and products. The non-negativity Restrictions ensure that no backlog occurs.

Lack of raw materials inventory level Prevention in facilities:

$$B'_{it} + w_{it} \ge f_{it} \times po_{it} + g_{it} + po_{it} \tag{8}$$

where $\mathbf{po_{it}}$ is Company program for production of finished products at period *t* in facility *i*, $\boldsymbol{f_{it}}$ is the conversion rate of raw materials into finished-product at node *i* and time *t*, and $\boldsymbol{g_{it}}$ is the conversion rate of raw materials into semi-finished product at node *i* and time *t*.

Semi-finished products constraints:

Flow balance of semi-finished products at manufacturer's facilities:

$$oin_{it} = o_{it} + oin_{it-1} + \sum_{v \in V} \sum_{\substack{i' \in I \\ i' \ne i}} q_{ii'vt} - r_{it} \times p_{it} \tag{9}$$

Constraint (9) enforce that the current semi-finished product and its demands in facilities be met from current production and on-hand inventory.

Production capacity of semi-finished products in manufacturers' facilities

$$0 \le o_{it} \le o_i^{\max} \tag{10}$$

Storage capacity of raw materials at facilities:

$$0 \le oin_{it} \le oin_i^{\max} \tag{11}$$

Constraints (10) and (11) limit inventories and production of semi-finished products below a defined maximum level. Again, non-negativity Restrictions ensure that no backlog occurs.
Prevention of lack of semi-finished products inventory level in facilities:

$$oin_{it} + o_{it} \ge r_{it} \times po_{it} \tag{12}$$

In which $\boldsymbol{r_{it}}$, is conversion rate of semi-finished products into finished products at node i and time t.

End products constraints:

Flow balance of finished- products at facilities:

$$s_{it} = s_{it-1} + \sum_{j \in J} \sum_{v \in V} q_{ijvt}, i \in I, t \in T \tag{13}$$

Constraint (13) enforce that the current finished products and its demands in facilities or costumer node's be met from current production and on-hand inventory.

Production capacity of finished-products:

$$0 \le p_{it} \le p_i^{\max}, \qquad i \in I, t \in T \tag{14}$$

Storage capacity of finished-products at facilities:

$$0 \le s_{it} \le s_i^{\max}, \qquad i \in I, t \in T \tag{15}$$

Constraints (14), (15) limit from above the productions and inventories of finished-products in manufacturers facilities. The non-negativity restrictions ensure that no backlog occurs.

Prevention of lack of semi-finished products inventory level in facilities:

$$p_{it} + s_{it} \ge po_{it} \tag{16}$$

Shipment indivisibility:

$$x_{ijvt} \in Z^+, \qquad i \in I, j \in J, v \in V, t \in T \tag{17}$$

where $\boldsymbol{Z}^+$ denotes the set of the positive integers;

Non-negativity:

$$z_{jt}, P_{jt}, s_{jt}, w_{jt}, b_{it}, q_{ijvt} \ge 0, \qquad i \in I, j \in J, t \in T, v \in V \tag{18}$$

Operationalization of the distribution scheme:

$$x_{ijvt} = 0, \qquad i \in I, v \in V \tag{19}$$

Constraint (19) indicates the impossibility to deliver to a certain node j', at a given time t', this resulting, for example, from bad weather conditions or the closing of facilities for some time.

2.2.4. Notation (sets and parameters)

In this formulation, we define the following sets and indices so that using them in developed model:

I = {1, 2, …, i, . . . ,¡} set of production facilities (manufacturers and suppliers),
J = {1, 2, . . . , j, . . . , c} set of distributors,
K = I + J = {1, 2, … , k, … , i + c} set of nodes (suppliers and manufacturers, and distributors),
T = {1, 2, … , t , … , s} set of time periods in the planning horizon,
The parameters used in the formulation of the model are listed below:

$\boldsymbol{p_{it}}$ Unit production cost at facility i and time t;

po_{it}	The company's plan for producing goods in facility i and period t;
h_{it}	Unit holding cost of finished-products at facility i and time t;
O_{it}	Unit production cost of semi-finished products at facility i and time t;
h'_{it}	Unit holding cost of raw materials at facility i and time t;
h''_{it}	Unit holding cost of semi-finished products at facility i and time t;
r_{it}	Conversion rate of semi-finished product into finished-products at node i and time t;
f_{it}	Conversion rate of raw materials into finished-products at node i and time t;
g_{it}	Conversion rate of raw materials into semi-finished products at facility i and time t;
$O_{it}^{\max}$	Maximum production capacity for semi-finished products at facility i and time t;
$P_i^{\max}$	Maximum range of production capacity for finished-products at facility i;
$w_i^{\max}$	Maximum range of storage capacity for raw materials in facility i;
$oin_i^{\max}$	Maximum range of storage capacity for semi-finished products in facility i;
$s_i^{\max}$	Maximum range of storage capacity for end products in facility i.

3. The results

3.1. Test laboratory

As a case study for model proposed, we use the data provided by one of the largest Iranian companies. This company (Iran khodro khorasan Razavi co.) assembles three kinds of cars, Peugeot Pars, Peugeot 405Glx and Peugeot 405Gli. A one-year planning horizon with monthly time-periods is considered. In this research, we use Lindo (Standard Operational Research Optimization) software to reach the Numerical Results.

In the case study of Iran khodro co, we have tested our Model seven times. In every test, we have used the proposed model in formation of cars both single and combinatorial as follow,

Peugeot 405Glx;
Peugeot Pars;
Peugeot 405Gli ;
Peugeot Pars, Peugeot 405Glx and Peugeot 405 Gli.
Peugeot Pars and Peugeot 405Glx;
Peugeot Pars and Peugeot 405Gli;
Peugeot 405Gli and Peugeot 405Glx;

The results of these tests are illustrated in Tables 1 to Table 7. Table 8 shows add up results. All tests results are classified in two categories (costs and savings) and five sections for each category based on five parts of developed model involve. Note that the all costs are denoted in terms of Tumans, which is equivalent to 10 Rials.

Table 1
Costs and Savings (Peugeot 405Glx)

Costs			Savings	
Part	Real value(Tuman)	Predicted value	Tuman	%
1-End product assembly costs	27, 080, 000, 000t	26, 837, 600, 000t	242,400, 000	0.895
2-holding costs of End products	120, 544,000t	100, 506, 160t	20, 047, 840	16.62
3-Semi-finished products assembly costs	8, 720, 000t	8, 123, 040t	596, 960	6.845
4-Holding costs of raw material	138, 007, 600t	129, 312, 400t	8, 695,200	6.30
5-Holding costs of semi-finished products	10, 480, 000t	8, 393, 120t	4,467,200	19.91

Table 2

Costs and Savings (PEUGEOT PARS)

Costs			Savings	
Part	Real value(Tuman)	Predicted value	Tuman	%
1-End product assembly costs	40,620,000,000	40, 252, 400, 000t	376, 600, 000	0.904
2-holding costs of End products	180, 816, 000	150, 644, 240t	30, 169, 760	16.68
3-Semi-finished products assembly costs	13, 080, 000t	12, 164, 560t	915, 440	6.998
4-Holding costs of raw material	207,011,400t	190, 218, 600t	16, 792, 800	8.11
5-Holding costs of semi-finished products	15, 720, 000t	12, 680, 000t	3, 040, 000	19.33

Table 3

Costs and Savings (PEUGEOT 405GLI)

Costs			Savings	
Part	Real value(Tuman)	Predicted value	Tuman	%
1-End product assembly costs	26, 984, 000, 000t	26, 762, 570, 000t	221, 430, 000	0.82
2-holding costs of End products	120, 544,000t	100, 556, 160t	19, 977, 840	16.573
3-Semi-finished products assembly costs	8, 526, 000t	7, 921, 040t	604, 960	7.095
4-Holding costs of raw material	138, 665, 750t	128, 513, 900t	10, 151,850	7.32
5-Holding costs of semi-finished products	9, 480, 000t	7, 783, 120t	1, 696, 880	17.89

Table 4

Costs and Savings (Peugeot 405Glx and Peugeot Pars)

Costs			Savings	
Part	Real value(Tuman)	Predicted value	Tuman	%
1-End product assembly costs	67,700,000,000 t	67, 158, 400, 000t	541, 600,000	0.8
2-holding costs of End products	301, 360, 000t	251, 390, 400t	49, 969, 600	16.58
3-Semi-finished products assembly costs	21, 800, 000t	20, 807, 600t	992, 400	4.525
4-Holding costs of raw material	345, 019, 000t	324, 031, 000t	20, 988, 000	6.08
5-Holding costs of semi-finished products	26, 200, 000t	21, 732, 800t	4, 467, 200	17.05

Table 5

Costs and Savings (Peugeot 405Gli and Peugeot Pars)

Costs			Savings	
Part	Real value(Tuman)	Predicted value	Tuman	%
1-End product assembly costs	67, 605, 680,000	67,088,005,900	517, 674,100	0.765
2-holding costs of End products	281,980,700	239,256, 400	42, 724, 300	15.15
3-Semi-finished products assembly costs	20, 780, 000	19, 669, 100	1, 110, 900	5.34
4-Holding costs of raw material	338, 704, 500	319, 056, 000	19, 648, 500	5.8
5-Holding costs of semi-finished products	25, 410, 000	21, 320, 300	4, 089, 700	16.09

Table 6

COSTS AND SAVINGS (PEUGEOT 405GLI AND PEUGEOT 405GLX)

Costs			Savings	
Part	Real value(Tuman)	Predicted value	Tuman	%
1-End product assembly costs	56, 987, 805, 000t	56, 571, 005, 900t	416, 799, 100	0.731
2-holding costs of End products	210, 004, 700t	176, 390, 400t	33, 614, 300	16
3-Semi-finished products assembly costs	19, 780, 000t	18, 396, 100t	1, 383, 900	7
4-Holding costs of raw material	316, 504, 900t	298, 956, 000t	17, 548, 900	5.54
5-Holding costs of semi-finished products	21, 432, 400t	18, 038, 300t	3, 394, 100	15.83

Table 7
COSTS AND SAVINGS (PEUGEOT 405GLI AND PEUGEOT 405GLX AND PEUGEOT PARS)

Costs			Savings	
Part	Real value(Tuman)	Predicted value	Tuman	%
1-End product assembly costs	94, 684, 000, 000t	93, 997, 570, 000t	686, 430, 000	0.725
2-holding costs of End products	421, 904, 000t	358, 946, 560t	62, 957, 440	14.92
3-Semi-finished products assembly costs	30, 326, 000t	29, 128, 640t	1, 197, 360	3.948
4-Holding costs of raw material	483, 684, 750t	458, 944, 900t	24, 739, 850	5.11
5-Holding costs of semi-finished products	35, 680, 000t	30, 415, 920t	5, 264, 080	14.75

For more clearness we add up our results in Table 8.

Table 8
The summary of the result of testing the performance of the proposed model for seven products

Test number	Case(s)	Average savings	Average Percent of savings
1	Peugeot 405Glx	273,826,880	11.31
2	Peugeot Pars	427,518,000	12.22
3	Peugeot 405Gli	253,861,530	11.69
4	Peugeot 405Glx&Pars	618,017,200	11
5	Peugeot 405Gli&Pars	585,247,500	10.13
6	Peugeot 405Glx&Gli	472,740,300	9.85
7	Peugeot 405Glx,Gli&Pars	780,588,730	9.69

In Table 8, we show the average value of each test and in Fig. 1 we try to show these values.

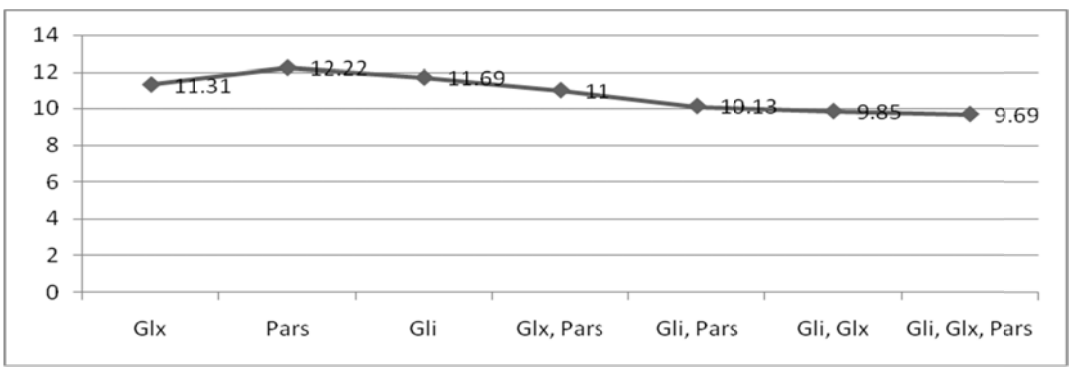

Fig. 1. The average cost saving associated with each case study

4. Conclusion

Production management and analysis involves determining a number of decisions such as: 1) the appropriate inventory levels for components and finished products, 2) safety production programming for all products. In a large enterprise with a production management involving hundreds of items, a detailed analysis becomes virtually impossible when each item is considered individually. The use of an integrated production management becomes necessary in this regard.

In this research, a comprehensive methodology has been presented a new model of production management under different restrictive assumptions. This model has parts which cover some famous issues in industrial engineering, such as inventory control of components, production planning of them and warehousing. We have formulated cost function that included all the costs of raw materials ordering, inventories and deliveries of finished and semi-finished products to our facilities. For developing a mathematical model of production management in five parts we used a linear operational research planning, validated on a real-industrial life problem faced by a large Iranian production companies, the developed methodology proposed is very effective as it can be seen in table 7 which shows these savings in all parts of our model.

As a review, in this paper, first, we introduced the parts of our model, next we tried to test it in a large production case study and eventually, we showed numerical results of our study in Iran khodro khorasan razavi company. In the case presented above, there was reduction in the costs of all parts by using new proposed model.

References

Boussemart, J.P., Briec, W., Peypoch, N., & Tavéra, C. (2008). α-Returns to scale and multi-output production technologies. *European Journal of Operational Research*, 197(1), 332-339.

Jouini, E., Chazal, M., & Tahraoui, R. (2007). Production planning and inventories optimization: A backward approach in the convex storage cost case. *Journal of Mathematical Economics*, 44, 997–1023.

Fontes, Dalila B.M.M. (2007). Fixed versus flexible production systems: A real options analysis. *European Journal of Operational Research*, 188, 169-184.

Heyman, D.P., & Sobel, M.J. (1984). *Stochastic Models in Operations Research. Volume II: Stochastic Optimization*. McGraw-Hill Book Company New York, NY.

Jung, J.Y., Blau, G., Pekny, J.F., Reklaitis, G.V., & Eversdyk, D. (2008). Integrated safety stock management for multi-stage supply chains under production capacity constraints. Computers & Chemical Engineering, 32(11), 2570-2581.

Lejeune, M.A. (2006). A variable neighborhood decomposition search method for supply chain management planning problems. *Journal of Operational Research*, 175, 959-976.

Merdivenci Atici, F., & Uysal, F. (2007). A production–inventory model of HMMS on time scales. Journal of *Applied Mathematics Letters*, 21, 236-243.

Ouyang, L.Y., Ho, C.H., & Su, C.H. (2008). Optimal strategy for an integrated system with variable production rate when the freight rate and trade credit are both linked to the order quantity. *Journal of Production Economics*, 115(1), 151-162.

Sharma, S., & Agrawal, N. (2009). Selection of a pull production control policy under different demand situations for a manufacturing system by AHP-algorithm. *Computers & Operations Research*, 36(5), 1622-1632.

Uphoff, N., Kassam, A., & Stoop, W. (2007). A critical assessment of a desk study comparing crop production systems: The example of the ‘system of rice intensification ’versus‘ best management practice. *Journal of Field Crops Research*, 108, 109-114.