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Selection of materials using multi-criteria decision-making methods with minimum data

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A B S T R A C T

Selection of material for a specific engineering component, which plays a significant role in its design and proper functioning, is often treated as a multi-criteria decision-making (MCDM) problem where the most suitable material is to be chosen based on a given set of conflicting criteria. For solving these MCDM problems, the designers do not generally know what should be the optimal number of criteria required for arriving at the best decisive action. Those criteria should be independent to each other and their number should usually limit to seven plus or minus two. In this paper, five material selection problems are solved using three common MCDM techniques to demonstrate the effect of number of criteria on the final rankings of the material alternatives. It is interesting to observe that the choices of the best suited materials solely depend on the criterion having the maximum priority value. It is also found that among the three MCDM methods, the ranking performance of VIKOR (Vlse Kriterijumska Optimizacija Kompromisno Resenje) method is the best.

1. Introduction

Multi Selecting the most appropriate materials for diverse engineering applications is often regarded as a multi-criteria decision-making (MCDM) problem for which the relevant decision matrix needs to be constructed consisting of a finite set of feasible alternatives and a predetermined number of criteria (Jahan et al., 2010). Wrong and improper selection of material for a given product/component may lead to its early failure, causing loss of profit and reputation of the manufacturing organizations (Edwards, 2006). Thus, utmost care should always be taken for short-listing the pertinent criteria affecting the material selection decision and reaching at the best course of action. The deployment of any MCDM method for solving a decision-making problem usually involves the followings steps, i.e. a) determination of the relevant conflicting criteria and feasible alternatives, b) measurement of the relative importance of the considered criteria and impact of the alternatives on those criteria, and c) determination of the performance measures of the alternatives for ranking. In the domain of materials

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selection, the earlier researchers have mainly emphasized on the application of various MCDM techniques, and subsequent determination of the performance scores to evaluate and rank the candidate material alternatives. The first step is completely ignored by the past researchers and selection of the pertinent criteria for different material selection problems is observed to be entirely an arbitrary process (Yurdakul and İç, 2003).

Athawale and Chakraborty (2012) stressed on the fact that the main focus should be on the selection of the relevant criteria and alternative materials for subsequent development of the decision matrix, not on the selection of the MCDM method to be adopted. Selection of the exact number of criteria required for almost accurate ranking of the materials still remains a challenge to be properly addressed by the researchers. Based on Miller's theory (Miller, 1965), it is always suggested that the decision maker should consider seven plus or minus two criteria for a given decision-making problem. It states that 'seven plus or minus two represents the greatest amount of information an observer can give us about an object on the basis of an absolute judgment' (Yurdakul and İç, 2009). On the other hand, Olson (1996) claimed that the considered criteria should be independent and the decision hierarchy should contain the minimum number of criteria. Thus, it is claimed that selection of criteria requires application of formal processing to obtain an independent set of approximately seven plus or minus two criteria in MCDM applications (Yurdakul and İç, 2009). But, accumulation of the relevant data for those seven plus or minus two criteria may often create problems to the designers, and analysis of the MCDM problems with higher number of criteria may reduce the solution accuracy and lengthen the computational time for solving those problems.

In this paper, a maiden attempt is made to investigate the effect of the number of criteria on the ranking performance of three commonly adopted MCDM methods and determine the value of the minimum number of criteria required to arrive at the best decisive action for all the three methods.

2. Methodology adopted

The earlier researchers have already applied different MCDM techniques for dealing with the material selection problems arising out of various engineering applications. Amongst those MCDM methods, TOPSIS (technique for order preference by similarity to ideal solution) (Milani et al., 2005; Shanian & Savadogo, 2006[a]; Thakker et al., 2008; Rao & Davim, 2008; Rathod & Kanzaria, 2011; Chauhan & Vaish, 2012; Jahan et al., 2012), VIKOR (Vlse Kriterijumska Optimizacija Kompromisno Resenje) and comprehensive VIKOR (Rao, 2008; Chatterjee et al., 2009; Jahan et al., 2011; Bahraminasab & Jahan, 2011; Girubha & Vinodh, 2012), ELECTRE (elimination and choice expressing the reality) (Shanian & Savadogo, 2006[b]; Shanian & Savadogo, 2006[c]; Chatterjee et al., 2009; Cavallini et al., 2013) and PROMETHEE (preference ranking organization method for enrichment evaluation) (Jiao et al., 2011; Chatterjee & Chakraborty, 2012; Çalışkan et al, 2013; Peng & Xiao, 2013) are the most celebrated ones.

In all those adopted methodologies, it is revealed that the relative importance or priority weights assigned to the considered evaluation criteria have an immense role in obtaining the accurate rankings of the material alternatives. However, it is not clear what is the effect of those criteria weights or number of criteria in the material selection decision matrix on the solution accuracy and ranking performance of the adopted MCDM methods. Obviously, the least important criterion (having the minimum weight) has the minimum influence on the performance of the MCDM methods and intuitively, it can be claimed that the material selection results would be principally dictated by the most important criterion having the maximum weight. This paper tries to substantiate these claims with the help of five cited examples as already been solved the past researchers. In the mathematical procedure of the adopted approach, the requirement of independency between the considered criteria needs to be fulfilled first using the correlation coefficient values. Yurdakul and İç (2009) set a threshold value for the correlation coefficient as 0.65 and claimed that two criteria could be treated as entirely uncorrelated or independent if the correlation coefficient between them had a value less than

0.65. After confirming that the considered criteria are uncorrelated, the least important criterion is discarded first from consideration and the ranking of the material alternatives is obtained from the reduced set of criteria/data. The weight of the discarded criterion is equally apportioned among the remaining criteria keeping in mind that the sum of all the criteria weights must add up to one. Then, subsequently the other least important criteria are omitted maintaining the weight additivity constraint, and the candidate materials are evaluated and ranked using those further reduced set of criteria. Finally, there will be a situation when only the most important criterion with the maximum weight stays in picture and material ranking is obtained considering only that criterion (after apportionment, its weight will be one).

In this paper, the material alternatives from the five cited examples are evaluated and ranked using the reduced sets of criteria employing only VIKOR, TOPSIS and PROMETHEE methods because of their comprehensiveness and simplicity. ELECTRE method is not applied for material rankings as it has a quite complex and tedious mathematical approach.

3. Illustrative examples

In order to study the effects of criteria weights and the most important criteria on the rankings of the alternative materials, the following five illustrative examples are cited.

3.1 Example 1: cryogenic storage tank material

Manshadi et al. (2007) considered a material selection problem for a cryogenic storage tank for transporting liquid nitrogen. For transporting liquid nitrogen safely, the material to be chosen should be strong and stiff enough, in spite of having lower density and specific heat, smaller thermal expansion coefficient and thermal conductivity, adequate toughness at operating temperature, good weldability and processability. Seven criteria, i.e. toughness index (TI), yield strength (YS), Young's modulus (YM), density (D), thermal expansion coefficient (TE), thermal conductivity (TC) and specific heat (SH) were considered along with seven alternative materials from which the best suited material was selected for cryogenic storage tank design. The decision matrix is shown in Table 1.

Among the seven criteria, toughness, yield strength and Young's modulus are beneficial attributes requiring higher values; on the other hand, density, specific heat, thermal expansion coefficient and thermal conductivity are non-beneficial attributes where smaller values are always essential. While solving this material selection problem for cryogenic storage tank using a modified digital logic method, Manshadi et al. (2007) estimated the values of different criteria weights as $w_{TI} = 0.28$, $w_{YS} = 0.14$, $w_{YM} = 0.05$, $w_D = 0.24$, $w_{TE} = 0.19$, $w_{TC} = 0.05$ and $w_{SH} = 0.05$, which are used here for subsequent analyses. Using a modified digital logic method, Manshadi et al. (2007) derived the ranking of the alternative materials as 6-7-1-3-2-4-5. On the other hand, Chatterjee et al. (2011) solved the same problem employing COPRAS (complex proportional assessment) and EVAMIX (evaluation of mixed data) methods, and obtained the rankings of the considered candidate materials as 6-7-1-3-2-4-5 and 5-7-1-4-6-3-2 respectively. In all these three methods, SS 301-FH was the best chosen material and Al 5052-O was the least preferred choice for the considered application.

Table 1
Data for cryogenic storage tank material selection problem (Manshadi et al., 2007)

Sl. No.	Material	TI	YS	YM	D	TE	TC	SH
1.	Al 2024-T6	75.5	420	74.2	2.8	21.4	0.37	0.16
2.	Al 5052-O	95	91	70	2.68	22.1	0.33	0.16
3.	SS 301-FH	770	1365	189	7.9	16.9	0.04	0.08
4.	SS 310-3AH	187	1120	210	7.9	14.4	0.03	0.08
5.	Ti-6Al-4V	179	875	112	4.43	9.4	0.016	0.09
6.	Inconel 718	239	1190	217	8.51	11.5	0.31	0.07
7.	70Cu-30Zn	273	200	112	8.53	19.9	0.29	0.06

Toughness index (TI): based on UTS, yield strength (YS) and ductility (e) at -196° C = (UTS + YS)e/2; yield strength (YS): MPa; Young's modulus (YM): GPa; density (D): (g/cm^3); thermal expansion (TE): $10^{-6}/^{\circ}$C; thermal conductivity (TC): cal/cm^2/cm/$^{\circ}$C/s; specific heat (SH): cal/g/$^{\circ}$C

Table 2
Correlation matrix for cryogenic storage tank material selection problem

Criteria	TI	YS	YM	D	TE	TC	SH
TI	1.000	0.594	0.490	0.544	-0.107	-0.477	-0.484
YS	0.594	1.000	0.874	0.560	-0.711	-0.665	-0.554
YM	0.490	0.874	1.000	0.816	-0.598	-0.475	-0.717
D	0.544	0.560	0.816	1.000	-0.379	-0.312	-0.924
TE	-0.107	-0.711	-0.598	-0.379	1.000	0.613	0.600
TC	-0.477	-0.665	-0.475	-0.312	0.613	1.000	0.483
SH	-0.484	-0.554	-0.717	-0.924	0.600	0.483	1.000

Table 2 exhibits the correlation matrix between the seven criteria and it is clear from the correlation values that the considered criteria are totally uncorrelated or independent in nature. Now, an attempt can be made to show the effects of those selection criteria on the final ranking of the candidate material alternatives, especially on the best- and worst-ranked materials. In this material selection problem for cryogenic storage tank, three criteria, i.e. YM, TC and SH have equal priority or relative importance value of 0.05. So, any one of them can be first discarded from consideration in the subsequent calculations. Here, criterion SH is selected arbitrarily to be discarded first and this is treated as case 2, whereas, case 1 is the situation when all the seven criteria are taken into account for determination of the rank orderings of the candidate materials using VIKOR, TOPSIS and PROMETHEE methods.

Gradually, the other least important criteria are omitted from further considerations and the followings situations arise as case 1: all criteria included; case 2: SH excluded; case 3: SH and TC excluded; case 4: SH, TC and YM excluded; case 5: SH, TC, YM and YS excluded; case 6: SH, TC, YM, YS and TE excluded; and case 7: only TI included. Thus, case 7 is that situation when the candidate alternatives for the cryogenic storage tank material selection problem are evaluated and ranked based on the single criterion (TI) having the maximum initial weight of 0.28. Tables 3, 4 and 5 respectively show the rankings of the seven alternative materials for the seven possible cases as obtained using VIKOR, TOPSIS and PROMETHEE methods. From these tables, it is quite interesting to observe that in all the cases for the three MCDM methods, the position of the top-ranked material alternative, i.e. SS 301-FH remains unaltered and it exactly corroborates with the observation of Manshadi et al. (2007). Al 5052-O and Al 2024-T6 were identified as the two worst-chosen materials for this application, and the rankings of the alternative materials derived using VIKOR, TOPSIS and PROMETHEE methods also reveal the same fact. It is also worthwhile to note that in case 7 (when only TI criterion is considered for ranking of the materials), all the three MCDM methods provide the same ranking of the materials.

Table 3
Rankings of cryogenic storage tank materials using VIKOR method for seven cases

Material	Manshadi et al. (2007)	Case						
		1	2	3	4	5	6	7
Al 2024-T6	6	7	7	7	7	7	4	7
Al 5052-O	7	6	6	6	6	6	3	6
SS 301-FH	1	1	1	1	1	1	1	1
SS 310-3AH	3	3	3	4	4	4	5	4
Ti-6Al-4V	2	2	2	2	2	2	2	5
Inconel 718	4	4	4	3	3	3	7	3
70Cu-30Zn	5	5	5	5	5	5	6	2

Table 6 gives the Spearman's rank correlation coefficient values between the rankings of the candidate materials achieved using the three MCDM methods for all the seven cases and those derived by Manshadi et al. (2007) applying a modified digital logic method. From these results, it is quite clear that the rankings of the best- and worst-preferred alternatives solely depend on criterion TI, having the maximum priority value/importance. Selection of SS 301-FH as the most appropriate material for cryogenic storage tank is well justifiable because in the decision matrix of Table 1, it has the maximum toughness index value of 770 (toughness index is a beneficial attribute).

Table 4
Rankings of cryogenic storage tank materials using TOPSIS method for seven cases

Material	Manshadi et al. (2007)	Case						
		1	2	3	4	5	6	7
Al 2024-T6	6	5	7	5	5	5	3	7
Al 5052-O	7	6	6	6	6	4	2	6
SS 301-FH	1	1	1	1	1	1	1	1
SS 310-3AH	3	4	4	4	4	7	7	4
Ti-6Al-4V	2	2	5	2	2	2	5	5
Inconel 718	4	3	2	3	3	3	6	3
70Cu-30Zn	5	7	3	7	7	6	4	2

Table 5
Rankings of cryogenic storage tank materials using PROMETHEE method

Material	Manshadi et al. (2007)	Case						
		1	2	3	4	5	6	7
Al 2024-T6	6	7	7	7	7	7	7	7
Al 5052-O	7	6	6	6	6	6	2	6
SS 301-FH	1	1	1	1	1	1	1	1
SS 310-3AH	3	4	4	4	4	4	4	4
Ti-6Al-4V	2	3	2	3	2	2	3	5
Inconel 718	4	2	3	2	3	3	6	3
70Cu-30Zn	5	5	5	5	5	5	5	2

Table 6
Spearman's rank correlation coefficient values for Example 1

Method	Spearman's rank correlation coefficient						
	Case						
	1	2	3	4	5	6	7
VIKOR	0.9643	0.9643	0.9286	0.9286	0.9286	0.3929	0.6071
TOPSIS	0.8571	0.6429	0.8571	0.8571	0.5000	0.1428	0.6071
PROMETHEE	0.8571	0.9286	0.8928	0.9286	0.8571	0.1786	0.6071

3.2 Example 2: high speed naval craft material

This problem deals with the selection of the best suited material for a high speed naval craft and the related decision matrix, as given in Table 7, consists of six alternative materials, i.e. grade A steel (M_1), single skin aluminium (A5086-H34) (M_2), aluminium sandwitch (honeycomb core) (M_3), LASCOR steel (M_4), composite (CFRP) carbon w/vinyl ester resin (M_5) and DUCTAL (UHP2C) (M_6), and nine criteria, e.g. yield strength (YS) (MPa), Young's modulus (YM) (GPa), fire resistance (FR), repairability (RY), resistance to corrosion (RC), fabrication cost (FC), risk (R), density (D) (kg/m^3) and overall potential for weight savings (WS). Among these nine criteria, yield strength, Young's modulus, fire resistance, repairability, resistance to corrosion and overall potential for weight savings are beneficial attributes, whereas, fabrication cost, risk and density are non-beneficial attributes. Table 8 shows the relative importance or weights assigned to various criteria in this problem. Rao and Patel (2010) derived a comparative ranking of the considered alternatives as 6-4-3-5-1-2 by applying a subjective and objective integrated MCDM method. On the other hand, Torrez

(Rao & Patel, 2010) and Fayazbakhsh et al. (2009) solved the same high speed naval craft material selection problem using a modified digital logic technique and Z-transformation statistics method, and obtained the rankings of the alternatives as 6-4-3-5-1-2 and 5-3-4-6-1-2 respectively. Applying utility concept and desirability function methods, Karande et al. (2013) derived the rank orderings of the alternative materials as 6-4-3-5-1-2 and 6-5-2-3-1-4 respectively.

The correlation matrix of Table 9 confirms that the considered nine criteria for this high speed naval craft material selection problem are independent to each other, and it is also observed from Table 8 that resistance to corrosion (RC) and overall potential for weight savings (WS) are respectively the least and the most important criteria for this problem. Hence, RC is first omitted from subsequent consideration and the rankings of the candidate materials are obtained applying VIKOR, TOPSIS and PROMETHEE methods. This situation is treated as case 2. After consecutively discarding the other less important criteria according to their priority values, the following cases arise as case 1: all criteria included; case 2: RC excluded; case 3: RC and RY excluded; case 4: RC, RY and R excluded; case 5: RC, RY, R and FR excluded; case 6: RC, RY, R, FR and YM excluded; case 7: RC, RY, R, FR, YM and FC excluded; case 8: RC, RY, R, FR, YM, FC and YS excluded; and case 9: only WS included. Here, case 9 corresponds to the situation where the material alternatives are evaluated and ranked based only on the most important criterion, i.e. WS. The rank orderings for all the nine cases using VIKOR, TOPSIS and PROMETHEE methods are provided in Tables 10-12 respectively.

Table 7

Decision matrix for high speed naval craft material selection problem (Rao & Patel, 2010)

Alternative	YS	YM	FR	RY	RC	FC	R	D	WS
M_1	234.4	204.1	0.6818	0.7727	0.3182	0.5000	0.3182	7800	0.0001
M_2	137.9	67	0.3182	0.6818	0.6818	0.3182	0.5000	2700	0.6818
M_3	268.9	67	0.5000	0.5000	0.6818	0.5000	0.5000	1800	0.7727
M_4	379.2	204.1	0.6818	0.5000	0.6818	0.7727	0.6818	5200	0.6818
M_5	1496.2	227.5	0.3182	0.5000	0.7727	0.7727	0.5000	1800	0.7727
M_6	220.6	53.9	0.7727	0.7727	0.7727	0.3182	0.7727	2500	0.6818

Table 8

Criteria weights for high speed naval craft material selection problem

Criteria	YS	YM	FR	RY	RC	FC	R	D	WS
Weight	0.132	0.118	0.0833	0.0763	0.0694	0.125	0.0763	0.153	0.167

Table 9

Correlation matrix for Example 2

Criteria	YS	YM	FR	RY	RC	FC	R	D	WS
YS	1.000	0.608	-0.492	-0.518	0.352	0.689	-0.092	-0.331	0.299
YM	0.608	1.000	-0.025	-0.307	-0.340	0.840	-0.345	0.514	-0.358
FR	-0.492	-0.025	1.000	0.429	-0.297	-0.123	0.393	0.524	-0.403
RY	-0.518	-0.307	0.429	1.000	-0.452	-0.748	-0.075	0.401	-0.618
RC	0.352	-0.340	-0.297	-0.452	1.000	0.080	0.732	-0.880	0.960
FC	0.689	0.840	-0.123	-0.748	0.080	1.000	-0.050	0.124	0.132
R	-0.092	-0.345	0.393	-0.075	0.732	-0.050	1.000	-0.408	0.618
D	-0.331	0.514	0.524	0.401	-0.880	0.124	-0.408	1.000	-0.894
WS	0.299	-0.358	-0.403	-0.618	0.960	0.132	0.618	-0.894	1.000

Table 13 exhibits the values of the Spearman's rank correlation coefficients when the rankings of the six alternative materials as derived using VIKOR, TOPSIS and PROMETHEE methods are compared with those obtained by Rao and Patel (2010) while adopting a subjective and objective integrated MCDM approach. It is revealed from Tables 10-12 that in all the three MCDM methods, for case 9, the choices for the best (composite carbon w/vinyl ester resin) and the worst (grade A steel) chosen materials almost match with the findings of Rao and Patel (2010). It reveals that in this material selection problem too, the choices for the best and the worst preferred materials depend solely on criterion WS, having the maximum priority weight. It is also quite clear from Table 7 that material

M_5 and material M_1 have the maximum and the minimum WS values respectively (WS being a beneficial attribute), which confirms the dependency of rank orderings of the material alternatives predominantly on criterion WS.

Table 10
Rankings of high speed naval craft materials using VIKOR method

Material	Rao and Patel (2010)	Case 1	2	3	4	5	6	7	8	9
M_1	6	6	6	6	6	6	6	6	6	6
M_2	4	4	4	5	5	4	4	5	4	4
M_3	3	3	3	2	3	2	2	2	2	2
M_4	5	5	5	4	4	5	5	4	5	5
M_5	1	1	1	1	1	1	1	1	1	1
M_6	2	2	2	3	2	3	3	3	3	3

Table 11
Rankings of high speed naval craft materials using TOPSIS method

Material	Rao and Patel (2010)	Case 1	2	3	4	5	6	7	8	9
M_1	6	6	6	6	6	6	6	6	6	6
M_2	4	4	4	4	4	4	4	4	4	4
M_3	3	2	2	2	2	2	2	2	2	2
M_4	5	5	5	5	5	5	5	5	5	5
M_5	1	1	1	1	1	1	1	1	1	1
M_6	2	3	3	3	3	3	3	3	3	3

Table 12
Rankings of high speed naval craft materials using PROMETHEE method

Material	Rao and Patel (2010)	Case 1	2	3	4	5	6	7	8	9
M_1	6	6	4	5	6	6	6	6	6	6
M_2	4	5	6	6	5	5	4	5	4	3
M_3	3	2	2	2	2	2	2	2	1	1
M_4	5	4	5	4	4	3	5	3	5	4
M_5	1	1	1	1	1	1	1	1	2	2
M_6	2	3	3	3	3	4	3	4	3	5

Table 13
Spearman's rank correlation coefficient values for Example 2

Method	Spearman's rank correlation coefficient, Case 1	2	3	4	5	6	7	8	9
VIKOR	1.0000	1.0000	0.8857	0.9428	0.9428	0.9428	0.9428	0.9428	0.9428
TOPSIS	0.9428	0.9428	0.9428	0.9428	0.9428	0.9428	0.9428	0.9428	0.9428
PROMETHEE	0.8857	0.7143	0.7714	0.8857	0.7143	0.9428	0.7143	0.8286	0.5429

3.3 Example 3: light load wagon wall material

While selecting the most appropriate material for designing light load wagon walls, Findik and Turan (2012) considered aluminium, aluminium alloys, low carbon steel, titanium alloys, nickel alloys, zinc alloys and copper alloys as the candidate alternatives whose performances were evaluated based on five criteria, i.e. density (D), specific stiffness (SS), corrosion resistance (CR), wear resistance (WR) and material cost (MC). Among these, density and material cost are non-beneficial, while the remaining three are beneficial criteria. The original decision matrix for this material selection problem is given in Table 14. Findik and Turan (2012) determined the values of different criteria weights as $w_D = 0.4$, $w_{SS} = 0.1$, $w_{CR} = 0.1$, $w_{WR} = 0.1$ and $w_{MC} = 0.3$, and applied weighted property index method to rank the considered materials as 2-1-3-4-7-6-5. Aluminium alloys were the best choice for designing light load wagon walls, followed by aluminium. Nickel alloys were evolved out as the least preferred choice of material. For this example, Karande et al. (2013) achieved the rankings of the considered materials as 2-1-3-4-7-6-5 and 2-1-4-5-6-3-7 respectively, while applying utility concept and desirability function approaches.

The correlation coefficient values, as provided in Table 15, prove the independency relations between the considered criteria. Amongst the five criteria, SS, CR and WR have the minimum criteria weight value of 0.1, so anyone from them can be arbitrarily taken as the least important criteria to be discarded from further calculations. Here, SS is omitted first and the same material selection problem for light load wagon walls is again solved using VIKOR, TOPSIS and PROMETHEE methods, giving the rank orderings of the candidate materials as respectively given in Tables 16-18. Subsequently, the other least important criteria are deleted from considerations, giving the following cases as case 1: all criteria included; case 2: SS excluded; case 3: SS and CR excluded; case 4: SS, CR and WR excluded; and case 5: only D included. Thus, in case 5, the seven material alternatives are evaluated and ranked based on the single criterion (i.e. D). Tables 16-18 also exhibit the rankings of the candidate materials for all the five cases using the three considered MCDM techniques. Table 19 provides the Spearman's rank correlation coefficients for these cases when the rankings are compared with those as already derived by Findik and Turan (2012). It is quite interesting to observe from Tables 16-18 that the positions of the best and the worst ranked materials remain unaltered in case 5, i.e. when the materials are evaluated using the single criterion value of D. As density is a non-beneficial attribute, its lower value is always preferable and aluminium alloys have the lowest density value of 2700 kg/m^3, which causes it to retain the top position in case 5.

Table 14
Data for load wagon wall material selection problem (Findik & Turan, 2012)

Alternative	Material	Density	Specific	Corrosion	Wear resistance	Cost
M_1	Aluminium	2700	0.03	4	3	2.32
M_2	Aluminium alloys	2700	0.03	4	3	2.30
M_3	Steel (low carbon)	7900	0.03	3	5	0.85
M_4	Titanium alloys	4500	0.02	5	3	13.00
M_5	Nickel alloys	8900	0.02	5	4	6.00
M_6	Zinc alloys	6000	0.02	3	1	2.20
M_7	Copper alloys	8930	0.02	5	5	2.30

Table 15
Correlation coefficient values for Example 3

Criteria	D	SS	CR	WR	MC
D	1.000	-0.519	0.169	0.556	-0.108
SS	-0.519	1.000	-0.495	0.159	-0.514
CR	0.169	-0.495	1.000	0.341	0.620
WR	0.556	0.159	0.341	1.000	-0.128
MCt	-0.108	-0.514	0.620	-0.128	1.000

Table 16
Rankings of light load wagon wall materials using VIKOR method

Material	Findik and Turan (2012)	Case				
		1	2	3	4	5
M_1	2	2	2	2	2	2
M_2	1	1	1	1	1	1
M_3	3	4	5	4	4	5
M_4	4	5	4	5	5	3
M_5	7	7	7	7	7	7
M_6	6	3	3	3	3	4
M_7	5	6	6	6	6	6

Table 17
Rankings of light load wagon wall materials using TOPSIS method

Material	Findik and Turan (2012)	Case				
		1	2	3	4	5
M_1	2	2	2	2	2	2
M_2	1	1	1	1	1	1
M_3	3	4	4	4	4	5
M_4	4	7	7	7	7	3
M_5	7	6	6	6	6	7
M_6	6	3	3	3	3	4
M_7	5	5	5	5	5	6

Table 18
Rankings of light load wagon wall materials using PROMETHEE method

Material	Findik and Turan (2012)	Case				
		1	2	3	4	5
M_1	2	2	2	3	2	2
M_2	1	1	1	1	1	1
M_3	3	3	3	2	4	5
M_4	4	5	5	5	5	3
M_5	7	7	7	7	7	7
M_6	6	4	4	4	3	4
M_7	5	6	6	6	6	6

Table 19
Spearman's rank correlation coefficients for Example 3

Method	Spearman's rank correlation coefficient				
	Case				
	1	2	3	4	5
VIKOR	0.7857	0.7500	0.7857	0.7857	0.8214
TOPSIS	0.6429	0.6429	0.6429	0.6429	0.8214
PROMETHEE	0.8929	0.8929	0.8571	0.7857	0.8214

3.4 Example 4: material for a product operating in a high-temperature environment

This example of selecting the most appropriate material for a product designed for operating in a high-temperature environment was solved by Rao (2008) using traditional VIKOR method and the ranking of the candidate materials was obtained as 3-2-5-4-1. Jahan et al. (2011) also considered the same problem and solved it employing comprehensive VIKOR method, deriving the same rank ordering of the materials. The performances of five alternatives were evaluated based on four criteria, i.e. tensile strength (TS), Young's modulus (YM), density (D) and corrosion resistance (CR), as given in Table 20. The corresponding values of the criteria weights were determined as $w_{TS} = 0.1343$, $w_{YM} = 0.2687$, $w_D = 0.5287$ and $w_{CR} = 0.0683$ which are taken here for subsequent analyses. In this problem, tensile strength, Young's modulus, and corrosion resistance are beneficial attributes, whereas, density is the solitary non-beneficial attribute. Table 21 displays the related correlation matrix showing the independency between the four criteria. Now, the corrosion resistance criterion having the minimum priority weight of 0.0683 is discarded first and this situation is treated as case 2. Subsequent omission of the remaining least important criteria leads to the following cases as case 1: all criteria included; case 2: CR excluded; case 3: CR and TS excluded; and case 4: CR, TS and YM excluded. On solving this material selection problem for the four cases using VIKOR, TOPSIS and PROMETHEE methods, different ranking patterns are observed. For case 4, the rank orderings of the candidate materials as derived by VIKOR, TOPSIS and PROMETHEE methods are the same, i.e. 3-2-5-4-1. The Spearman's rank correlation coefficients between the rankings obtained for the considered four cases using the three MCDM methods and those of Rao (2008) are given in Table 22.

In all the cases and for all the MCDM methods, the best suited material for this selection problem remains as material 5. Again, it is interesting to note here that the choice of the best material is solely dictated by the density criterion with the maximum priority weight. This choice is justified, as in Table 20, material alternative 5 has the minimum density value of 1.71 gm/cm^3, density being a non-beneficial attribute.

Table 20
Decision matrix for a product operating in a high-temperature environment (Rao, 2008)

Alternative	Tensile strength	Young's modulus	Density (gm/cm^3)	Corrosion resistance
1	1650	58.5	2.3	0.5
2	1000	45.4	2.1	0.335
3	350	21.7	2.6	0.335
4	2150	64.3	2.4	0.5
5	700	23	1.71	0.59

Table 21
Correlation matrix for Example 4

Criteria	TS	YM	D	CR
TS	1.000	0.956	0.166	0.369
YM	0.956	1.000	0.259	0.137
D	0.166	0.259	1.000	-0.578
CR	0.369	0.137	-0.578	1.000

Table 22
Spearman's rank correlation coefficients for Example 4

Method	Spearman's rank correlation coefficient			
	Case			
	1	2	3	4
VIKOR	1.0000	1.0000	1.0000	1.0000
TOPSIS	0.9000	0.7000	0.7000	1.0000
PROMETHEE	0.9000	1.0000	1.0000	1.0000

3.5 Example 5: flywheel material

Jee and Kang (2000) selected the best suited material for design of a flywheel, taking into account ten alternative materials (four metals and six unidirectional fiber-reinforced-epoxy composites) whose performances were evaluated based on four criteria, i.e. fatigue limit (σ_{limit}/ρ) (FL), fracture toughness (K_{IC}/ρ) (FT), price per unit mass (PM) and fragmentability (F). The decision matrix for this material selection problem is exhibited in Table 23. Among these four criteria, fatigue limit, fracture toughness and fragmentability are beneficial attributes, and price/mass is a non-beneficial attribute. Jee and Kang (2000) considered four cases of subjective weights for the criteria among which the first case, where w_{FL} = 0.4, w_{FT} = 0.3, w_{PM} = 0.2 and w_F = 0.1 is used here for the subsequent analyses. Using TOPSIS method, a ranking of the material alternatives was achieved as 5-9-7-6-8-3-4-2-1-10 by Jee and Kang (2000). On the other hand, Chatterjee et al. (2009) solved the same problem using VIKOR and ELECTRE II methods, and obtained the corresponding rankings of the alternatives as 9-10-8-6-7-5-2-4-1-3 and 10-9-8-6-7-3-2-4-1-5 respectively. For the same material selection problem, Rao and Patel (2010) and Jahan et al. (2012) derived the rank orderings of the materials as 9-10-7-6-8-3-5-2-4-1 and 6-9-7-5-8-4-2-3-1-10 respectively while employing a subjective and objective integrated multi-attribute decision-making method and an extended TOPSIS method. Table 24 provides the correlation coefficient values between the four considered criteria for this material selection problem for designing a flywheel. It is clear from the correlation matrix that the four criteria are entirely independent to each other.

Discarding sequentially the least important criteria in order of their importance (fragmentability first with the minimum priority weight of 0.1) gives rise to the following cases as case 1: all criteria included; case 2: F excluded; case 3: F and PM excluded; and case 4: F, PM and FT excluded. For case 4, i.e. when the ten alternative materials are ranked based on only fatigue limit criterion, VIKOR, TOPSIS and PROMETHEE methods provide the same material ranking as 7-10-8-6-9-5-3-4-1-2. Table 25 gives the Spearman's rank correlation coefficients for all the cases when the ranking patterns of the candidate materials as achieved employing the three MCDM techniques are compared with those of Jee and Kang (2000). It is observed that for the extreme case of selecting the material for flywheel design based on a single criterion with the maximum weight of 0.4 (i.e. case 4), the most preferred material is Kevlar 49-epoxy FRP for all the three MCDM methods. From Table 23, it is clear that Kevlar 49-epoxy FRP has the maximum fatigue limit value of 616.4384 which justifies its selection as the first choice.

Table 23
Quantitative data for flywheel material selection problem (Jee & Kang, 2000)

Sl. No.	Materials	σ_{limit}/ρ	K_{IC}/ρ	Price/Mass	Fragmentability
1.	300M	100	8.6125	4200	Poor (3)
2.	2024-T3	49.6454	13.4752	2100	Poor (3)
3.	7050-T73651	78.0142	12.5532	2100	Poor (3)
4.	Ti-6Al-4V	108.8795	26.0042	10500	Poor (3)
5.	E glass-epoxy FRP	70	10	2735	Excellent (9)
6.	S glass-epoxy FRP	165	25	4095	Excellent (9)
7.	Carbon-epoxy FRP	440.2516	22.0126	35470	Fairly good (7)
8.	Kevlar 29-epoxy FRP	242.8571	28.5714	11000	Fairly good (7)
9.	Kevlar 49-epoxy FRP	616.4384	34.2466	25000	Fairly good (7)
10.	Boron-epoxy FRP	500	23	315000	Good (5)

Table 24
Correlation coefficient values for Example 5

Criteria	FL	FT	PM	F
FL	1.000	0.693	0.535	0.311
FT	0.693	1.000	0.172	0.339
PM	0.535	0.172	1.000	-0.053
F	0.311	0.339	-0.053	1.000

Table 25
Spearman's rank correlation coefficients for Example 5

Method	Spearman's rank correlation coefficient			
	Case			
	1	2	3	4
VIKOR	0.5151	0.5515	0.4788	0.5151
TOPSIS	0.7818	0.7939	0.5030	0.5151
PROMETHEE	0.6242	0.6242	0.6000	0.5151

4. Discussions

It is observed from Figure 1 that for all the five cited examples, the ranking performance of VIKOR method is superior to that of TOPSIS and PROMETHEE methods. VIKOR method is mainly based on the particular measure of closeness to the ideal solution and it focuses on selecting the best choice from a set of feasible alternatives in presence of mutually conflicting criteria by determining a compromise solution. It provides a maximum group utility for the majority, and a minimum of individual regret for the opponent. As compared to VIKOR method, TOPSIS is based on the concept that the chosen best alternative should have the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution. TOPSIS method introduces two reference points using vector normalization procedure, but does not consider the relative importance of the distances from those two points. It suggests that the best alternative in TOPSIS method may not always mean that it is the closest to the ideal solution. The difference in VIKOR and TOPSIS methods also appears in the normalization techniques adopted. VIKOR method uses linear normalization and the normalized value does not depend on the evaluation unit of a criterion, while TOPSIS method employs vector normalization and the normalized value can be different for different evaluation units of a particular criterion. Also, VIKOR method checks whether the top-ranked alternative can be considered better enough than the others by testing acceptability advantage and acceptable stability conditions. If any of these two conditions is not satisfied, then VIKOR proposes a set of compromise solutions based on maximum group utility of majority and minimum individual regret of opponent. But, TOPSIS method does not include such acceptability checks for the obtained solutions. PROMETHEE method, which uses six different types of preference functions, is actually based on maximum group utility, whereas, VIKOR method integrates maximum group utility and minimal individual regret simultaneously. It has been found that the ranking results provided by PROMETHEE may be the same in comparison to VIKOR, if PROMETHEE uses linear preference function. So, the main advantage of VIKOR method as compared to TOPSIS and PROMETHEE is that the final ranking result in VIKOR method is an aggregation of all criteria, their relative importance, and a balance between total and individual satisfaction. The compromise solution provided by VIKOR can be the groundwork for negotiations, involving the decision makers' preferences of criteria weights. Based on several advantages of VIKOR over TOPSIS and PROMETHEE, it is quite obvious that it would be a better performer, as validated in Fig. 1.

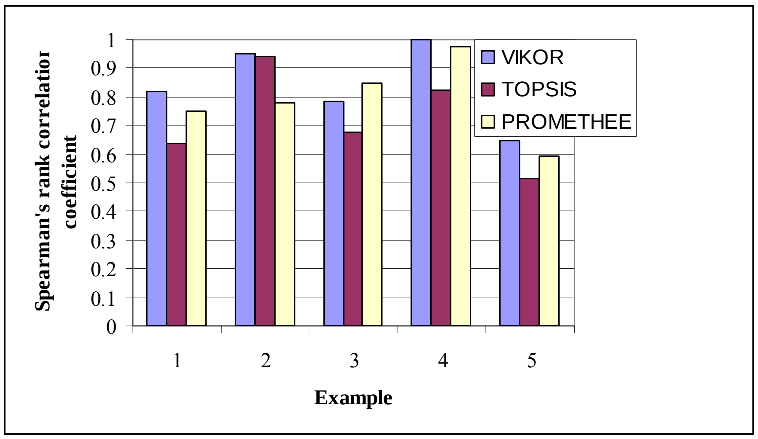

Fig. 1. Comparison between MCDM methods for five examples

5. Conclusions

In this paper, five material selection problems from diverse domains of applications are solved to show the effect of the number of criteria on the ranking performance of VIKOR, TOPSIS and PROMETHEE methods. It is interesting to note that the choices of the best and the worst materials solely depend on the most important criterion having the maximum priority weight. It can be claimed that the designers may now need not to construct the exhaustive material selection decision matrices and may only stress on identifying the most important criterion dictating the entire selection process. Here, the methodology by which the criteria weights are determined may also play an important role. This mathematical approach will substantially reduce the complexity involved in the decision-making process, as the best suited material may now be chosen based on a single criterion. It is also revealed that among the three considered MCDM methods, VIKOR outperforms the others due to its undeniable advantages. The validation of the interesting finding of this paper for other decision-making problems may be the future scope to be investigated by the researchers.

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