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A q-rung orthopair fuzzy decision-making framework considering experts trust relationships and psychological behavior: An application to green supplier selection

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A B S T R A C T

Theselection of an optimal supplier is a critical and open challenge in supplychain management. While experts' assessments significantly influence thesupplier selection process, their subjective interactions can introduceunpredictable uncertainty. Existing methods have limitations in effectivelyrepresenting and handling this uncertainty. The paper aims to address thesechallenges by proposing a novel approach that leverages q-rung orthopair fuzzysets (q-ROFSs). The novelty of the proposed approach lies in its ability toaccurately capture experts' preferences through the use of q-ROFSs, which offermembership and non-membership degrees, providing a broader expression spacecompared to conventional fuzzy sets. Additionally, it incorporates social networkanalysis (SNA) to effectively consider the trust relationships among experts.The proposed approach is divided into three stages. The first stage presents anovel method to determine experts' weights by combining SNA, the Bayesianformula, and the maximum entropy principle. This approach allows for a moreprecise representation of varying levels of expertise and influence amongexperts, addressing the uncertainty arising from subjective interactions. Thesecond stage introduces a hybrid weight determination method to determinecriteria weights within the context of q-ROFSs. Entropy plays a crucial role incapturing fuzziness and uncertainty in q-ROFSs, while the projection measuresimultaneously provides information about the distance and angle between alternatives.By employing both objective weights estimated using entropy and normalizedprojection measure and subjective weights derived using an aggregation operatorand a score function, the presented approach achieves a comprehensiveassessment of criteria importance. To incorporate both subjective and objectiveweights effectively, game theory is applied which allows us to aligndecision-making with both quantitative and qualitative aspects, making themethod more versatile and applicable. The third stage redefines thetraditional Combined Compromise Solution (CoCoSo) method using Bonferroni meanoperators which captures interrelationships among arguments to be aggregated. Furthermore, in recognition of theimportance of an expert risk preferences and psychological behaviors, we applyregret theory, ensuring that the chosen solutions align more effectively withtheir underlying preferences and aspirations. The applicability andeffectiveness of the proposed approach are demonstrated through a numericalexample of green supplier selection. The comparative analysis illustrates thepracticality and real-world relevance while the sensitivity analysis confirmsthe stability and robustness of the proposed approach.

1. Introduction

The socioeconomic framework of any country is profoundly shaped by the progress of its industrial sectors (Digalwar et al., 2020). Given the growing environmental awareness and mounting pressure from governments, stakeholders, and competitors, industrial organizations must embrace the principles of green supply chain management (GSCM) in their

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organizational strategies (Ghosh et al., 2021). GSCM aims to minimize environmental impact and reduce waste at every stage of the supply chain, encompassing product sourcing, design, manufacturing, packaging, transportation, distribution, and post-consumption disposal (Ghosh et al., 2022). Previous research has revealed that implementing GSCM practices such as green purchasing, green design, green manufacturing, green transportation, and green marketing can yield substantial improvements in both financial and environmental performance (Rupa & Saif, 2022; Sahoo & Vijayvargy, 2021; Samad et al., 2021). The effectiveness of a company's GSCM is influenced by both internal green efforts and the green practices of its suppliers (Ghosh et al., 2022). Given that businesses frequently outsource a wide range of tasks, green supplier selection (GSS), which is a crucial component of GSCM, plays a crucial role in assisting businesses in maintaining their strategic positioning in the marketplace (Ghosh et al., 2023). It has been demonstrated that GSS may significantly lessen an organization's environmental costs in terms of trash output and carbon dioxide emissions (Coskun et al., 2022). Therefore, it is vital to evaluate and choose green providers.

The information available for an alternative throughout the green supplier selection process may be of a qualitative linguistic value, imprecise, or incomplete nature. (Zadeh 1965) introduced the idea of fuzzy set and used it to multiple decision-making applications by taking into account different viewpoints in order to handle the ambiguous and imprecise data. Later many variants, of fuzzy sets came into existence such as the intuitionistic fuzzy set (IFS) (Atanassov & Stoeva, 1986), hesitant fuzzy set (HFS) (Torra 2010), Pythagorean fuzzy set (PFS) (Yager 2013), etc., to better manage these uncertainties. Researchers have resolved supplier selection problems under various fuzzy sets (Bisht 2023; Bonab et al., 2023; Chai et al., 2023). A thorough examination of fuzzy decision-making is provided in (Kahraman et al., 2016; Liu & Liao, 2017), which unambiguously illustrates the need for a generalized fuzzy set that is better able to manage ambiguity and vagueness. Due to the boundary constraint, IFS and PFS have the major drawback of being unable to give a wider/flexible field for preference elicitation. (Yager 2016) developed a generalized fuzzy set called q-rung orthopair fuzzy set, which offers a broader field for preference elicitation, to deal with the problem and to minimize the shortcoming of IFS and PFS.

Existing decision models often overlook the objective trust relationships among experts due to their reliance on the assumption that decision-making experts are independent (Wu et al., 2017). However, with the advent of online social networks, this assumption no longer holds true in many cases. During group decision-making (GDM), personal opinions are often supported by family, friends, or like-minded individuals within the social network context (Liang et al., 2017; Perez et al., 2016). Social networks foster interactions that are built on trust, and the application of social network analysis (SNA) has been proven to simplify the complexity of GDM and improve decision-making quality (Chu et al., 2016).

Additionally, behavioral experiments demonstrate that under uncertainty and risk, decision-makers frequently exhibit bounded rationality (Camerer 1998). Therefore, when making decisions, decision-makers psychological tendencies should be taken into account. The regret theory was separately proposed by (Loomes & Sugden, 1982; Bell 1982), in which rejoice and regret aspects were incorporated into the utility values to more accurately portray intuitive judgments. The regret theory has been utilized in numerous disciplines. (Ali et al., 2022) employed q-rung model based on regret theory to stock selection problem. (Liu et al., 2022) developed a framework for collective decision-making that incorporates the regret theory.

The literature has introduced a plethora of MCDM techniques aimed at addressing the challenge of selecting green suppliers in a q-ROFSs environment. (Pinar et al., 2021) extended fuzzy TOPSIS to q-rung orthopair fuzzy environment for solving GSS problem. (Krishankumar & Ecer, 2023) employs q-rung orthopair fuzzy CRADIS approach for selection of IoT service provider. (Guneri & Deveci, 2023) evaluates the GSS problems in the defense industry using the extended EDAS approach in q-ROF environment. (Krishankumar et al., 2023) proposed an integrated approach for biomass location selection in q-ROF environment. But there is no study in the literature about an integrated framework that takes into account the relationship of trust between experts and psychological behavior. Additionally, no one has calculated the objective weights of criteria for the assessment of the green supplier selection problem using the concepts of entropy and projection measure. Here, we create a hybridized technique using q-ROFS, and we use it to evaluate the GSS problem using completely ambiguous knowledge regarding the standards and subject matter experts.

Following a review of the literature above, we note the following difficulties:

1. Inability to manage subjectivity and ambiguity.
2. Ineffective modeling of attribute relationships.
3. Weights of experts are a crucial element in the aggregation process that must be computed methodically to reduce subjective randomness caused by human interaction. Determining the weights of experts is a vital concern in the MCDM procedure. However, in the existing literature, the assignment of weights to each expert has often been explicitly determined by the authors (Garg & Chen, 2020; Arsu & Aycin, 2021; Kumar & Chen, 2022; Deveci et al., 2022), introducing potential subjective ambiguity.
4. Many researchers have used social network analysis (SNA) to control the GDM process. Due to their impact on experts' decision-making behavior, trust relationships in SNA have garnered more research attention in last years, hence the use of SNA to determine experts' weights in q-ROF environment is an important issue to be addressed.
5. Uncertainty and ambiguity in preference elicitation must be correctly taken into account in order to calculate attribute weights in a reasonable way under the q-ROFS framework. The direct values for criterion weights have been taken into consideration by the researchers in the past (Kumar & Chen, 2022; Deveci et al., 2022;

Bakir et al., 2021; Kakati & Rahman, 2022). Conversely, different weights of the criteria would result in different decisions. As a result, computing the criterion weights in actual MCDM issues is crucial.

6. Decision analysis should take into consideration the psychological behaviors of decision-makers since decisions and behaviors are frequently correlated.
7. Prioritizing alternatives by expanding well-known ranking techniques under q-ROFS is an intriguing problem that should be prioritized.
8. The validation of the proposed decision framework's application in practice and a discussion of its strengths and weaknesses in the context of q-ROFS are both very desirable.

The literature highlights the versatility and effectiveness of q-ROFS, a generalized fuzzy set, in handling ambiguity and uncertainty. To promote logical and systematic decision-making, it is critical to address the aforementioned issues and minimize direct human involvement in the decision-making process. The second challenge involves developing a decision-making approach that accurately models the relationships among the arguments to be integrated. This is crucial for capturing the complexities and interdependencies among different factors. The third and fourth challenges emphasize the need for a systematic method to determine the weights of experts. This enables the systematic collection of preferences from each DM, ensuring their contributions are appropriately considered. The fifth challenge require a systematic approach to determine the weights of attributes. The sixth and seventh challenges involves the proper establishment of ranking order of alternatives, taking into account the psychological behavior of experts. This ensures a comprehensive and well-informed decision-making process. Methodically assigning weights to attributes, encourages the sensible prioritization of alternatives, which reduces decision-making errors. Finally, the last challenge is crucial because it exhorts the proposed framework's actual application and suggests its stability and coherence by contrasting it with alternative approaches.

In response to these difficulties and in an effort to mitigate them, we make the following significant contributions:

- The utilization of q-ROFS (Yager 2016), is widely preferred due to its flexibility in enabling experts to express their preferences while reducing uncertainty and subjective randomness. q-ROFS provides a comprehensive framework for handling uncertainty by adjusting the parameter q, allowing for enhanced flexibility in preference articulation and modeling uncertainty across three dimensions: membership, non-membership, and hesitancy grade. The versatility of q-ROFS is evident from its definition, which accommodates various scenarios and allows for effective management of uncertainty.
- Additionally, past researches made it abundantly evident that the systematic computation of expert's weights lowers errors in the decision-making process and moderates subjective randomness from human interaction. The value of expert weights is rigorously assessed by taking into account their trust relationships with each other. Thus, instead of assigning each expert the same weight, an optimization model is built using maximum entropy principle and Bayesian principle to determine the expert weights in the setting of social networks.
- To mitigate the adverse impacts of subjective and objective factors, an integrated weighting model is developed. This model combines an objective weight determination process that utilizes projection and entropy measures with a subjective weighing process that employs aggregation operators and score functions. By incorporating q-ROF data, this approach effectively estimates the weights of criteria, effectively balancing both subjective and objective considerations.
- Bonferroni mean operators are employed to account for the interrelationships among arguments.
- As mentioned earlier, decision-makers often exhibit bounded rationality when faced with uncertainty and risk. Therefore, it is crucial to incorporate the psychological behaviors of decision-makers into decision analysis. In this article, the regret theory is presented within the decision-making system under the q-ROF framework. Subsequently, a novel approach to decision analysis is proposed.
- To make the fuzzy set more useful and suitable for MAGDM, it is necessary to expand common ranking techniques under q-ROFS. As a result, to prioritize alternatives in a logical way within the q-ROFS context, a popular and effective CoCoSo ranking approach is enhanced in response to its flexibility and application.
- To validate the viability of the proposed framework, a green supplier selection problem is employed as a practical test case. This evaluation aims to assess the strengths and weaknesses of the proposed approach in comparison to existing methodologies. By subjecting the framework to this real-world scenario, we can effectively analyze its performance, advantages, and potential limitations.

The paper is systematized as follows:

Section 2 provides a comprehensive overview of the fundamentals of IFS and q-ROFS. In Section 3, we present the proposed decision structure, which includes a detailed explanation of the systematic weight calculations for attributes and experts. Section 4 introduces the MAGDM technique based on the suggested model. To establish the applied application of the proposed framework, Section 5 presents a case study on green supplier selection. In Section 6, a comparative analysis is conducted to evaluate the advantages and disadvantages of the suggested framework in contrast to other existing approaches. The reliability of the proposed framework is further demonstrated in Section 7 through sensitivity analysis. Lastly, Section 8 concludes the paper by summarizing the findings and discussing future directions for research.

2. Preliminaries

An overview of key ideas utilized in the present study can be found in the section below.

2.1. q-Rung Orthopair fuzzy set

Definition 1. (Yager 2016) A q-rung orthopair fuzzy (q-ROF) set M in the universe of discourse X is categorized by three functions namely degree of membership, degree of non-membership and degree of indeterminacy and is represented as:

$$M = \{\langle x, u_M(x), v_M(x)\rangle | x \in X\}$$

where $u_M: X \rightarrow [0,1]$ denotes the membership degree, $v_M: X \rightarrow [0,1]$ denotes the non-membership degree, and satisfy the following condition $0 \leq u_M(x)^q + v_M(x)^q \leq 1$, $q \geq 1$ and the degree of indeterminacy is given by: $\pi_M(x) = (u_M(x)^q + v_M(x)^q - u_M(x)^q v_M(x)^q)^{\frac{1}{q}}$.

Definition 2. (Yager 2016) Let $a = (u_a, v_a)$ be a q-ROFN, the score function S of a is given as:

$$S(a) = u_a{}^q - v_a{}^q \tag{1}$$

The value of $S(a)$ lies between the closed interval $[-1,1]$.

Definition 3. (Yager 2016) Let $a = (u_a, v_a)$ be a q-ROFN, the accuracy function H of a is given as:

$$H(a) = u_a{}^q + v_a{}^q \tag{2}$$

The value of $H(a)$ lies between the closed interval $[0,1]$.

The following q-ROFNs comparison rules are Based on the score and accuracy functions.

Definition 4. (Yager 2016) Let $a = (u_a, v_a)$ and $b = (u_b, v_b)$ be two q-ROFNs, $S(a), S(b)$ and $H(a), H(b)$ be the score and accuracy functions for a and b, then:

1. If $S(a) > S(b)$, then $a > b$.
2. If $S(a) = S(b)$, then
   - If $H(a) > H(b)$ then $a > b$.
   - If $H(a) = H(b)$ then $a = b$.

2.2 The regret theory

Regret theory, initially introduced by (Zhang et al., 2016), captures the remorse decision-makers feel when they choose a suboptimal solution instead of the optimal one. Conversely, rejoice represents the elation experienced when the optimal alternative is selected.

Definition 7. As per (Zhang et al., 2016), provides the utility function v(x) for attribute value x:

$$v(x) = x^\alpha, 0 < \alpha < 1 \tag{3}$$

where α is the risk aversion coefficient of decision makers and utility function satisfies, $v'(x) > 0, v''(x) < 0$.

Definition 8. (Zhang et al., 2016) The definition of the regret-rejoice function is as follows:

$$R(\Delta v) = 1 - e^{-\delta \Delta v}, \delta > 0 \tag{4}$$

where Δv denotes the difference in utility value of two alternatives and $R(\Delta v)$ denotes the regret-rejoice function with respect to Δv . δ is the regret aversion coefficient of decision makers and regret-rejoice $R(\Delta v)$ function satisfies, $R'\big((\Delta v)\big) > 0, R''\big((\Delta v)\big) < 0$. When $R(\Delta v) > 0$, it represent rejoice function otherwise it represents regret function.

Definition 9. (Zhang et al., 2016) By considering the evaluation values of X and Y as x and y respectively, the perceived utility value for an alternative X can be derived by combining the utility function of X with the regret-rejoice function.

$$U(X) = v(x) + R(\Delta v), \qquad \Delta v = v(x) - v(y) \tag{5}$$

2.3. Social Network Analysis

Social Network Analysis (SNA) is a valuable tool for studying the connections between enterprises and other social entities (Liu et al., 2019). It allows for the exploration of various aspects, including location attributes and structural balance (Perez et al., 2016), such as centrality, trust, and prestige. In traditional Group Decision Making (GDM) processes, decision-makers are often assumed to be independent of each other, overlooking the trust relationships that exist among experts. However, in real-life scenarios, experts' individual opinions can be influenced by the trust they place in others. Thus, trust relationships among experts play a crucial role in shaping the final decisions in GDM problems. A social network can be defined as a social structure consisting of a set of edges (L) and a set of nodes (E). The edges represent the trust relationships among experts, with each relationship represented by a node.

3. Proposed Framework

3.1. Method for expert weight determination in q-ROFNs

Social network matrix: For a set of experts $E = \{e_1, e_2 \dots e_m\}$ under the social network, the trust degree of expert e_i to expert e_j can be represented with linguistic terms to represent the ambiguous nature of experts. Therefore, we can construct a trust relationship matrix, $W_{m\times m} = (w_{ij})$ where w_{ij} refers to the trust degree of expert e_i to expert e_j. Thus, $W_{m\times m} = (w_{ij})$ can be regarded as a directed matrix.

$$W = \begin{bmatrix} w_{11} & \cdots & w_{1m} \\ \vdots & \ddots & \vdots \\ w_{m1} & \cdots & w_{mm} \end{bmatrix}$$

where w_{ij} represents the linguistic terms. Table 1, represents the linguistic terms and their corresponding q-ROFNs to measure the trust relationship among experts.

Table 1
Linguistic terms to q-ROFNs

Linguistic terms	q-ROFNs
Extremely high (EH)	(0.98,0.01)
Very high (VH)	(0.9,0.6)
High (H)	(0.75,0.6)
Moderate (M)	(0.55,0.45)
Low (L)	(0.7,0.8)
Very low (VL)	(0.6,0.9)
Extremely low (EL)	(0.01,0.98)

Considering the real-world problems, it is not always true that the trust degree of expert e_i to expert e_j is equal to the trust degree of expert e_j to expert e_i. Thus, for a $W_{m\times m} = (w_{ij}), w_{ij} = w_{ji}$ cannot generally be established.

Convert the trust degree of experts w_{ij} into crisp numbers using the score function.

Bayesian network, also referred to as a belief network, is a well-known concept in probability theory. It can be visualized as a Directed Acyclic Graph (DAG) consisting of nodes that represent variables, interconnected by directed edges. These edges symbolize the interdependence and influence between the nodes. The trust relationship among experts is expressed using conditional probability within the framework of a Bayesian network.

Let's assume that the decision-making process involves m experts. $E = \{e_1, e_2 \dots e_m\}$. The conditional probability $Prob\left(\frac{e_i}{e_j}\right)$ is used to represent the trust of expert e_j on expert e_i, and $P(e_i)$ is used to represents the weight of experts.

$$Prob(e_i) = Prob\left(\frac{e_i}{e_1}\right).Prob(e_1) + Prob\left(\frac{e_i}{e_2}\right).Prob(e_2) + \cdots Prob\left(\frac{e_i}{e_m}\right).Prob(e_m)$$

$$Prob(e_i) = \sum_{i=1, i\neq k}^{m} Prob\left(\frac{e_i}{e_k}\right).Prob(e_k)$$

Finally, we utilize the maximum entropy principle to obtain the weight of experts. The optimization model is constructed as:

$$\begin{cases} \max f = \sum_{a=1}^{m} -w(e_i)\ln(w(e_i)) \\ s.t.\ w(e_i) = \sum_{i=1, i\neq k}^{m} w\left(\frac{e_i}{e_k}\right).w(e_k) \\ 0 \leq w(e_i) \leq 1, i = 1,2, \dots . m \\ \sum_{i=1}^{m} w(e_i) = 1 \end{cases}$$

3.2. Integrated attribute weight determination model in q-ROFNs

3.2.1. Estimate the objective weights of attributes

Definition 10. (Liu et al., 2021) Let $a = (a_{1,}a_2 \dots . a_n)$ and $b = (b_{1,}b_2 \dots . b_n)$ be two q-ROFS. Then the normalized projection of a on b is given as:

$$Nproj_b(a) = \frac{ab}{ab + ||b|^2 - ab|} \tag{6}$$

where $ab = \sum_{i=1}^{n} \lambda_{a_i}^q \lambda_{b_i}^q + \eta_{a_i}^q \eta_{b_i}^q + \pi_{a_i}^q \pi_{b_i}^q$, $|b_i|^2 = (\lambda_{b_i}^q)^2 + (\eta_{b_i}^q)^2 + (\pi_{b_i}^q)^2$ and $|b| = \sqrt{\sum_{i=1}^{n} |b_i|^2}$

The objective weights of attribute are calculated by combining the normalized projection and entropy method. The entropy value of each attribute is determined as follows:

$$p_i = \frac{1 - h_i}{\sum_{i=1}^{m} 1 - h_i} \tag{7}$$

where h_i represents the entropy of j^{th} attribute and is calculated as:

$$h_j = \frac{-1}{\ln(n)} \sum_{i=1}^{n} \left[\frac{Nproj_{X_r}(X_i)}{\sum_{i=1}^{n} Nproj_{X_r}(X_i)} \ln \left(\frac{Nproj_{X_r}(X_i)}{\sum_{i=1}^{n} Nproj_{X_r}(X_i)} \right) \right] \tag{8}$$

here $Nproj_{X_r}(X_i)$ represents the normalized projection of X_i on positive ideal reference point X_r, and n represents the number of alternatives.

3.2.2. Estimate the subjective weights of attributes

Each expert gives the weight of attributes according to their knowledge and understanding, which are represented in the form of matrix as:

$$\begin{bmatrix} \vartheta^1 \\ \vartheta^2 \\ \vartheta^3 \\ \vdots \\ \vartheta^s \end{bmatrix} = \begin{bmatrix} w_1^1 & w_2^1 & w_3^1 & \dots & w_m^1 \\ w_1^2 & w_2^2 & w_3^2 & \dots & w_m^2 \\ w_1^3 & w_2^3 & w_3^3 & \dots & w_m^3 \\ \vdots & \vdots & \ddots & \ddots & \dots & \vdots \\ w_1^s & w_2^s & w_3^s & \dots & w_m^s \end{bmatrix}$$

Combine the attribute weights by each expert using the q-ROFWA.

$$z_i = q - ROFWA(w_i^1, w_i^2 \dots \dots . w_i^s) \tag{9}$$

Finally, determine the subjective weights of attributes using score function as:

$$\omega_i = \frac{S(z_i)}{\sum_{i=1}^{m} S(z_i)} \tag{10}$$

3.2.3. Estimate the combined weights of attributes

Objective weighing methods neglects the decision makers experience and knowledge and, on the contrast, the subjective weighing methods includes high subjectivity and hence hybrid weighing methods are preferred over objective and subjective weighing methods and provides reasonable and effective results. Therefore, the final attribute weights are determined by integrating the objective and subjective weights through the application of game theory.

In order to address multiple conflicts or entities, game theory is employed to find an optimal solution that achieves equilibrium. By considering different weighting methods as individual players, the comprehensive weight is determined using the concept of Nash equilibrium. The calculations steps of the procedure are:

Consider there are k attribute weights using k different weighing methods. A weight set of k vectors can be formed using the linear combination expressed as:

$$W = \sum_{i=1}^{k} \alpha_i w_i^T, \quad \alpha_i > 0 \tag{11}$$

In line with the principles of game theory, the attainment of an optimal equilibrium weight vector occurs when k entities reach a consensus. This consensus can be viewed as the fine-tuning of the weight coefficient α_i. The objective behind this optimization process is to minimize the disparity between W and w_i, which can be achieved by employing the following formula:

$$min \left\| \sum_{i=1}^{k} \alpha_i w_i^T - w_i^T \right\|^2 \quad (i = 1,2,3 \dots . k) \tag{12}$$

Based on the differentiation property of the matrix the requirement for the optimal first-order derivative of the equation is established.

$$\sum_{i=1}^{k} \alpha_i \times w_i \times w_i^T = w_i \times w_i^T \tag{13}$$

Thus, we have

$$\begin{bmatrix} w_1.w_1^T & w_1.w_2^T & \dots & w_1.w_k^T \\ w_2.w_1^T & w_2.w_2^T & \dots & w_2.w_k^T \\ \vdots & \vdots & \vdots & \vdots \\ w_k.w_1^T & w_k.w_2^T & \dots & w_k.w_k^T \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_k \end{bmatrix} = \begin{bmatrix} w_1 \times w_1^T \\ w_2 \times w_2^T \\ \vdots \\ w_k \times w_k^T \end{bmatrix}$$

Solving these k equations in k variables we get the values of α_i. Finally, normalized the values of α_i

$$\alpha_i^* = \frac{\alpha_i}{\sum_{i=1}^{k} \alpha_i} \tag{14}$$

such that $\sum_{i=1}^{k} \alpha_i^* = 1$.

The optimal attribute weight is determined using eq. (15).

$$W^* = \sum_{i=1}^{k} \alpha_i^* w_i^T \tag{15}$$

3.3. Regret theory under q-ROFNs

Based on section (2.2) of regret theory, for calculating perceived utility function we need to select an ideal reference possible point with maximum outcome to calculate the regret values of other alternatives, hence we select the positive ideal alternative as the reference point. The difference between the K_{iH} value as obtained by the CoCoSo method of the alternative and reference point is considered as the utility function, the less the difference the less in the regret of selecting the alternative over the optimal one.

For a MCDM problem with n alternatives and m attributes the weighted distance of alternative from the ideal reference point is defined as:

$$d(A_i, A_r) = K_{rH} - K_{iH} \tag{16}$$

The utility value of alternative A_i is calculated as:

$$v(A_i) = \left(d(A_i, A_r)\right)^{\alpha} \tag{17}$$

Variation of the utility function with different value of α is shown in Fig.1. From Fig.1. It is evident that the utility function exhibits a monotonically increasing trend as the value of α increases.

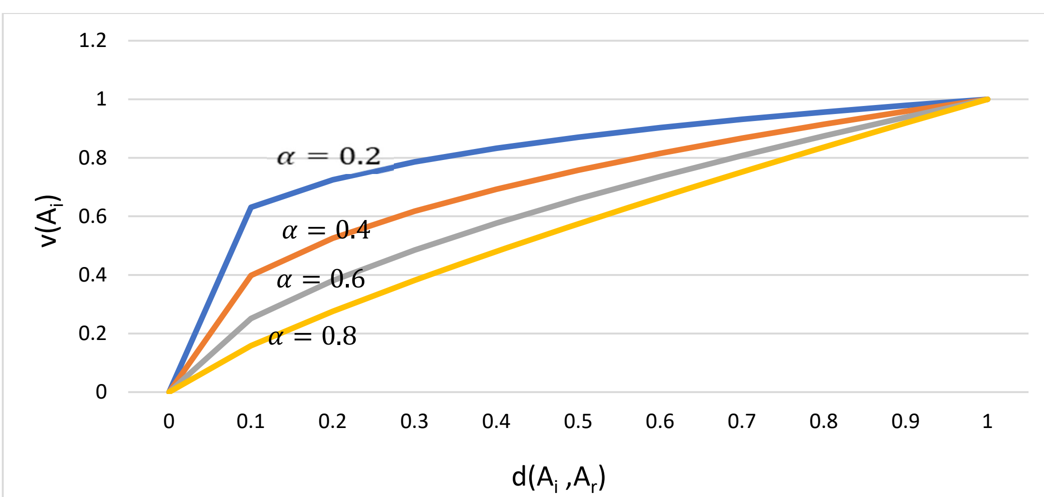

Fig.1. Utility function

The regret-rejoice function of alternative A_i is obtained as:

$$R(A_i, A_r) = 1 - e^{-\delta \Delta v}, \delta > 0 \tag{18}$$

where $\Delta v = \left(d(A_i, A_r)\right)^\alpha - \left(d(A_r, A_r)\right)^\alpha \geq 0$, since $d(A_r, A_r) = 0$, $\Delta v = \left(d(A_i, A_r)\right)^\alpha$

The defined regret-rejoice function reduces computing cost by avoiding pair-wise comparison of all options. Remorse is less likely to happen if the alternative is closer to the reference point, which is shown by a smaller value of $d(A_i, A_r)$. Fig. 2 depicts how the regret rejoice function changes in relation to δ.

Based on Eq. (1) and Eq. (2), the perceived utility function for alternative Ai is calculated as:

$$U(X) = \left(d(A_i, A_r)\right)^\alpha + 1 - e^{-\delta(d(A_i, A_r))} \tag{19}$$

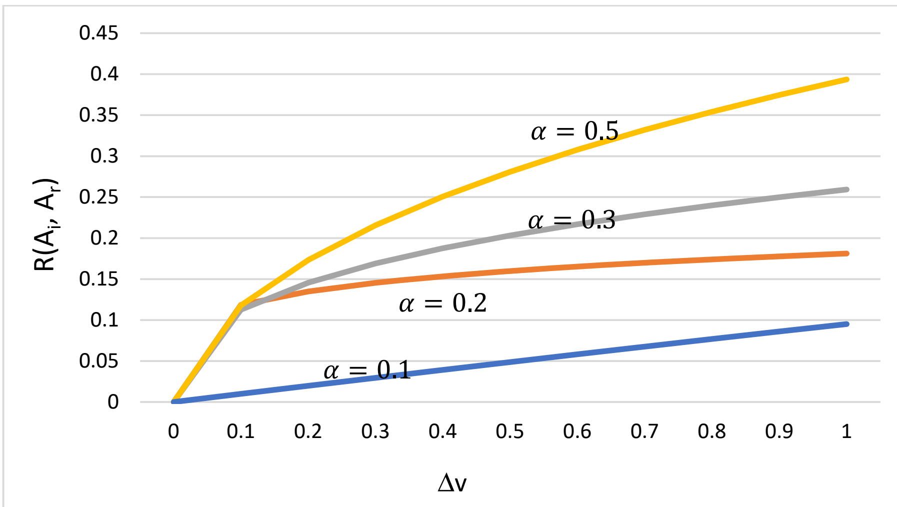

Fig. 2. Regret-rejoice function

4. MAGDM method based on proposed approach

This segment illustrates the decision-making problem in PHF environment with detailed steps to solve the problem.

4.1 Problem description

Let P = $\{P_1, P_2 \ldots . P_n\}$ be the set of *n* alternatives, C = $\{C_1, C_2 \ldots . C_m\}$ be the set of *m* attributes and D = $\{D_1, D_2 \ldots . D_k\}$ be the set of *k* decision-makers. The decision-makers evaluate the alternative in the form of PHFS. The attribute weights are indicated by $(w_1, w_2 \cdot \cdots w_m)$ such that $\sum_{i=1}^{m} w_i = 1$, and the weights of decision-makers are indicated by $(\omega_1, \omega_2 \cdot \cdots \omega_k)$ such that $\sum_{i=1}^{k} \omega_i = 1$.

4.2 Steps of proposed approach

To solve the MAGDM in q-ROF environment with unknown expert and attribute weights, the following steps are used.

Step.1. Formulate the decision-making matrix based on the evaluation of alternatives by individual expert in terms of q-ROFNs.

Step.2. Normalize the decision matrix given by each expert.

Step.3. Compute the expert weights using the procedure defined in section 3.1.

Step.4. Aggregate the decision matrix into a single matrix by means of q-ROFWA operator.

$$q - ROFWA(a_1, a_2, \ldots . a_n) = \left(1 - \left(\Pi_{i=1}^{n}\left(1 - u_i^q\right)^{w_i}\right)^{\frac{1}{q}}, \Pi_{i=1}^{n} v_i^{w_i}\right)$$

where w_i represents the weight of experts.

Step.5. Determine the attribute weights using the procedure defined in section 3.2.

Step.6. Calculate the weighted sequences SH_i and PH_i using q-ROFWBA and q-ROFWGBM operators to fully capture the correlations among decision attributes.

$$SH_i = \left(\frac{1}{m(m-1)} \sum_{i,j=1, i\neq j}^{m} (w_i\, a_i) \otimes (w_j a_j) \right)^{\frac{1}{s+t}}$$

$$= \left(\left(1 - \left(\Pi_{i,j=1,i\neq j}^{m} \left(1 - (1-(1-u_i^q)^{w_i})^s (1-(1-u_j^q)^{w_j})^t \right) \right)^{\frac{1}{m(m-1)}} \right)^{\frac{1}{q(s+t)}}, \left(1 - \left(1 - \left(\Pi_{i,j=1,i\neq j}^{m} (2 - \left(1 - v_i^{qw_i}\right)^s - \left(1 - v_j^{qw_j}\right)^t - \left(1 - \left(1 - v_i^{qw_i}\right)^s\right) \left(1 - \left(1 - v_j^{qw_j}\right)^t\right) \right)^{\frac{q}{m(m-1)}} \right)^{\frac{1}{s+t}} \right)^{\frac{1}{q}} \right)$$

$$PH_i = \frac{1}{s+t} \otimes_{i,j=1,i\neq j}^{m} \left(s a_i^{w_i} \oplus t a_j^{w_j} \right)^{\frac{1}{m(m-1)}}$$

$$= \left(\left(1 - \left(1 - \left(\Pi_{i,j=1,i\neq j}^{m} (2 - \left(1 - u_i^{qw_i}\right)^s - \left(1 - u_j^{qw_j}\right)^t - \left(1 - \left(1 - u_i^{qw_i}\right)^s\right) \left(1 - \left(1 - u_j^{qw_j}\right)^t\right) \right)^{\frac{q}{m(m-1)}} \right)^{\frac{1}{s+t}} \right)^{\frac{1}{q}}, \left(1 - \left(\Pi_{i,j=1,i\neq j}^{m} \left(1 - (1-(1-v_i^q)^{w_i})^s (1-(1-v_j^q)^{w_j})^t \right) \right)^{\frac{1}{m(m-1)}} \right)^{\frac{1}{q(s+t)}} \right)$$

Step.6. Calculate the relative significance of three pooling strategies using the following equations.

$$K_{iHa} = \frac{SH_i + PH_i}{\sum_{i=1}^{n} SH_i + PH_i} \tag{20}$$

$$K_{iHb} = \frac{SH_i}{min_i SH_i} + \frac{PH_i}{min_i PH_i} \tag{21}$$

$$K_{iHc} = \frac{\lambda SH_i + (1-\lambda) PH_i}{\lambda max_i SH_i + (1-\lambda) max_i SH_i} \tag{22}$$

Step.7. Combining all three strategies, a common value is calculated.

$$K_{iH} = \frac{(K_{iHa} + K_{iHb} + K_{iHc})}{3} + (K_{iHa} \times K_{iHb} \times K_{iHc})^{\frac{1}{3}} \tag{23}$$

Step.8. Compute the utility, regret-rejoice, perceived utility value for alternatives using eq. (16) - eq. (19) as defined in section.

Step.9. Finally rank the alternatives in ascending order of perceived utility value.

5. An illustrative example

This section verifies the proposed framework by applying it into case study of green supplier selection adopted from (Krishankumar et al., 2020). A group of three decision-makers are invited to rank four green suppliers based on five criteria: speed to deliver the product, the use of green design practices, the quality and service of the product, the overall cost, and the consumption of energy and resources. The latter two attributes are of the cost kind, while the first three are of the benefit type. To get at the solution, the suggested MAGDM approach's steps are used.

The evaluations of green suppliers by different experts in terms of q-ROFNs are represented by the decision matrices as shown in Table 2, Table 3 and Table 4.

Table 2
Evaluation by expert 1

	Att_1	Att_2	Att_3	Att_4	Att_5
I_1	(0.25,0.12)	(0.70,0.12)	(0.24,0.38)	(0.58,0.17)	(0.41,0.89)
I_2	(0.19,0.21)	(0.64,0.27)	(0.65,0.78)	(0.17,0.20)	(0.40,0.32)
I_3	(0.19,0.67)	(0.18,0.26)	(0.56,0.51)	(0.54,0.34)	(0.70,0.28)
I_4	(0.26,0.31)	(0.55,0.13)	(0.19,0.56)	(0.69,0.84)	(0.49,0.76)

Table 3
Evaluation by expert 2

	Att_1	Att_2	Att_3	Att_4	Att_5
I_1	(0.54,0.48)	(0.57,0.13)	(0.81,0.75)	(0.89,0.33)	(0.29,0.88)
I_2	(0.62,0.82)	(0.24,0.42)	(0.48,0.27)	(0.83,0.38)	(0.18,0.64)
I_3	(0.68,0.55)	(0.45,0.62)	(0.55,0.54)	(0.16,0.25)	(0.76,0.36)
I_4	(0.31,0.84)	(0.41,0.43)	(0.25,0.54)	(0.22,0.57)	(0.85,0.49)

Table 4
Evaluation by expert 3

	Att_1	Att_2	Att_3	Att_4	Att_5
I_1	(0.83,0.49)	(0.76,0.27)	(0.81,0.75)	(0.68,0.32)	(0.32,0.24)
I_2	(0.38,0.74)	(0.57,0.57)	(0.36,0.72)	(0.83,0.61)	(0.83,0.46)
I_3	(0.89,0.41)	(0.40,0.16)	(0.43,0.51)	(0.76,0.74)	(0.89,0.25)
I_4	(0.68,0.23)	(0.89,0.27)	(0.18,0.90)	(0.36,0.83)	(0.34,0.70)

1. Normalize the decision matrices by each expert. The normalized decision matrices as shown in Table 5, Table 6 and Table 7.

Table 5
Normalized decision matrix by expert 1

	Att_1	Att_2	Att_3	Att_4	Att_5
I_1	(0.25,0.12)	(0.70,0.12)	(0.24,0.38)	(0.17, 0.58)	(0.89, 0.41)
I_2	(0.19,0.21)	(0.64,0.27)	(0.65,0.78)	(0.20, 0.17)	(0.32, 0.40)
I_3	(0.19,0.67)	(0.18,0.26)	(0.56,0.51)	(0.34, 0.54)	(0.28, 0.70)
I_4	(0.26,0.31)	(0.55,0.13)	(0.19,0.56)	(0.84, 0.69)	(0.76, 0.49)

Table 6
Normalized decision matrix by expert 2

	Att_1	Att_2	Att_3	Att_4	Att_5
I_1	(0.54,0.48)	(0.57,0.13)	(0.81,0.75)	(0.89, 0.33)	(0.88, 0.29)
I_2	(0.62,0.82)	(0.24,0.42)	(0.48,0.27)	(0.38, 0.83)	(0.64, 0.18)
I_3	(0.68,0.55)	(0.45,0.62)	(0.55,0.54)	(0.25, 0.16)	(0.36, 0.76)
I_4	(0.31,0.84)	(0.41,0.43)	(0.25,0.54)	(0.57, 0.22)	(0.49, 0.76)

Table 7
Normalized decision matrix by expert 3

	Att_1	Att_2	Att_3	Att_4	Att_5
I_1	(0.83,0.49)	(0.76,0.27)	(0.81,0.75)	(0.32, 0.68)	(0.24, 0.32)
I_2	(0.38,0.74)	(0.57,0.57)	(0.36,0.72)	(0.61, 0.83)	(0.46, 0.83)
I_3	(0.89,0.41)	(0.40,0.16)	(0.43,0.51)	(0.74, 0.76)	(0.25, 0.89)
I_4	(0.68,0.23)	(0.89,0.27)	(0.18,0.90)	(0.83, 0.36)	(0.70, 0.34)

2. The weight of experts are evaluated using the proposed method in section 3.1. The weights of experts are obtained as: $\omega_{e_1} = 0.30418, \omega_{e_2} = 0.41776, \omega_{e_3} = 0.27806$
3. Aggregate the decision matrices given by each expert into a single decision matrix using the q-ROFWA as shown in Table 8.

Table 8
Aggregated decision matrix

	Att_1	Att_2	Att_3	Att_4	Att_5
I_1	(0.644,0.316)	(0.678,0.155)	(0.744,0.609)	(0.295, 0.724)	(0.831, 0.331)
I_2	(0.497,0.526)	(0.523,0.399)	(0.528,0.489)	(0.453, 0.512)	(0.534, 0.351)
I_3	(0.732,0.538)	(0.387,0.326)	(0.526,0.522)	(0.532, 0.357)	(0.312, 0.774)
I_4	(0.487,0.432)	(0.702,0.262)	(0.217,0.629)	(0.765, 0.357)	(0.663, 0.557)

4. For subjective weights, the evaluation of attributes by different experts are given in Table 9. Based on the evaluations given in Table 9 and using q-ROFWA and score function the subjective weights of attributes are obtained as: (0.19437, 0.09946, 0.27244, 0.24875, 0.18498).

Table 9
Evaluation of attributes

	Att_1	Att_2	Att_3	Att_4	Att_5
e_1	(0.9,0.1)	(0.75,0.15)	(0.95,0.05)	(0.91,0.12)	(0.8,0.2)
e_2	(0.67,0.32)	(0.65,0.55)	(0.89,0.2)	(0.84,0.2)	(0.6,0.1)
e_3	(0.8,0.3)	(0.54,0.3)	(0.79,0.3)	(0.87,0.35)	(0.9,0.3)

5. The objective weights are evaluated using the proposed method in section 3.2. The attribute weights are obtained as: (0.02384, 0.01308, 0.01029, 0.27977, 0.67302).
6. Using eq. (15) the combined attribute weights are obtained as: (0.0426, 0.02258, 0.03912, 0.27636, 0.61934).
7. The weighted sequences SH_i and PH_i have been obtained using q-ROFWBA and q-ROFWGBM for the values of the parameters s = t = 1. Using s = t = 1 not only make calculation easier, but also fully capture the correlations among the specified decision criteria. The values for alternatives are shown in Table 10. To convert the negative score values of SH_i into positive, it has been modified as: Crisp SH_i = score (SH_i)+1.

Table 10
SH_i and PH_i values for alternatives

	SH_i	PH_i	Crisp SH_i	Crisp PH_i
I_1	(0.390, 0.785)	(0.851, 0.301)	0.57535	0.59006
I_2	(0.307, 0.759)	(0.778, 0.258)	0.59097	0.45456
I_3	(0.273, 0.814)	(0.772, 0.330)	0.48119	0.42491
I_4	(0.426, 0.751)	(0.845, 0.284)	0.65294	0.58123

8. Relative significance of alternatives is obtained using eq. (20), eq. (21) and eq. (22) with the value of λ as 0.5 and is shown in Table 11.

Table 11
Relative significance of alternatives

	K_{iHa}	K_{iHb}	K_{iHc}	K_{iH}
I_1	0.267837	2.584349	0.937581	2.12904
I_2	0.240283	2.297891	0.841127	1.90084
I_3	0.208242	2	0.728965	1.65117
I_4	0.283638	2.724806	0.992896	2.2493

9. For the final ranking of alternatives, the utility, regret-rejoice and perceived utility values for alternatives are calculated and shown in Table 12.

Table 12
Ranking of alternatives

	$v(I_k)$	$R(I_k, I_r)$	$U(I_k)$	Rank
I_1	0.8485	0.3457	1.194	2
I_2	0.9737	0.3854	1.359	3
I_3	1.0944	0.4214	1.515	4
I_4	0.7744	0.3210	1.095	1

6. Comparative analysis and discussions

This section presented a comparison from both a numerical and theoretical standpoint. The proposed structure is compared to a number of innovative approaches. The techniques (Liu & Liu, 2018), (Pinar et al., 2021), (Krishankumar & Ecer, 2023), (Guneri & Deveci, 2023) are used for comparison with the suggested framework in order to ensure uniformity in comparison. The methodology for all of these techniques uses q-ROFNs. Table 13 lists the proposed and state-of-the-art methodologies' prioritization orders. Although there is a minor variation in the ranking order, the best option is the same for all approaches. The primary steps involved in the different approaches may result in these fine distinctions.

Table 13
Ranking from different methods

Methods	Ranking	Optimal alternative
Proposed method	$I_4 > I_1 > I_2 > I_3$	I_4
q-ROFBWA operator (Liu & Liu, 2018)	$I_4 > I_2 > I_1 > I_3$	I_4
q-ROFBWG operator (Liu & Liu, 2018)	$I_4 > I_1 > I_2 > I_3$	I_4
q-ROF-TOPSIS (Pinar et al., 2021)	$I_4 > I_2 > I_1 > I_3$	I_4
Method by (Krishankumar & Ecer, 2023)	$I_4 > I_1 > I_2 > I_3$	I_4
Method by (Guneri & Deveci, 2023)	$I_4 > I_2 > I_1 > I_3$	I_4

Table 14 clearly outlines the benefits of the proposed approach. We may conclude the following from the analysis:

1. q-ROF preference information as the data representation approach is efficient and provides a wide range of opportunities to elicit preferences.
2. The proposed framework offers ways for analytically estimating attribute weights and DM weights, in contrast to existing approaches used in the q-ROFS context. These methods efficiently capture reluctance and ambiguity in the preference information and minimize errors in the decision-making process. (Koksalmis & Kabak 2019) asserted that weights of DMs must be computed methodically in order to decrease subjective randomness from social involvement

and mistakes in the decision-making process, while (Kao 2010) offered explicit reasons regarding the significance of characteristics' weight computation.

3. Trust ties between experts are crucial in problems involving collective decision-making in the real world. Thus, the expert weights are determined considering the trust relationship among them in the form of social trust network.
4. By accurately capturing the correlation between attributes, the q-ROFBWM operator successfully aggregates q-ROFNs, whereas the BM operator is more flexible than the HM (Heronian mean) operator. The DMs' weights, a crucial aggregation parameter, are computed systematically in contrast to state-of-the-art aggregation operators in the q-ROFS context.
5. Furthermore, for logical prioritization of green providers, the well-known CoCoSo technique is expanded to the q-ROFS environment based on the regret theory, which hasn't been discussed in the existing literature. The proposed approach considers how other options will turn out and steers clear of making a regrettable decision. A logical justification for the prioritizing order is provided by the CoCoSo method's ability to prioritize alternatives while taking into account various compromise solution forms and ranking criterion categories, which encourages the research's current focus.
6. Based on the sensitivity analysis of both equal and unequal weights of attributes, it is evident that the proposed decision framework is stable and can efficiently capture the competitiveness that exists among green providers. The proposed framework is also clearly shown to be stable in Fig. 6 even after the weights of the attributes have been correctly modified.
7. It is apparent from the Spearman correlation coefficient values (see Table 15) that the proposed decision framework is compatible with previous approaches in the q-ROF environment.

Table 14
Comparative analysis of different methods

Factors	Proposed method	Method by (Liu & Liu, 2018)	Method by (Pinar et al., 2021)	Method by (Krishankumar & Ecer, 2023)	Method by (Guneri & Deveci, 2023)
Attribute weight	Projection + Entropy + Score function	No	AHP	Cronbach	AHP
Expert weight	SNA	No	No	CRITIC	No
Trust relationships among experts	Yes	No	No	No	No
Attributes interactions	Yes	Yes	No	No	No
Ranking method	CoCoSo	No	TOPSIS	CRADIS	EDAS
Psychological behavior of experts	Yes	No	No	No	No
Compromise solution	Yes	No	No	No	No

The ranking generated through the new approach and existing approaches are compared in order to calculate the Spearman's correlation coefficient, which helps to create a more compelling case for the viability of the proposed strategy, and is shown in Table 15. SSC greater than 0.75 between two ranking implies that the correlation between two models is significant and it can be easily observed from Table 15 that the SSCs of the ranking by proposed model and the ranking by other models is greater than 0.75, which shows the ranking by proposed model is highly correlated with the ranking by other models. Thus, the proposed model can provide effective and reliable results to MADM problems.

Table 15
The SCCs between the ranking results

	Proposed method	q-ROFBWA operator	q-ROFBWG operator	(Pinar et al., 2021)	(Krishankumar & Ecer, 2023)	(Guneri & Deveci, 2023)
Proposed method	1	0.8	1	0.8	1	0.8
q-ROFBWA operator	-	1	0.8	1	0.8	1
q-ROFBWG operator	-	-	1	0.8	1	0.8
(Pinar et al., 2021)	-	-	-	1	0.8	1
(Krishankumar & Ecer, 2023)	-	-	-	-	1	0.8
(Guneri & Deveci, 2023)	-	-	-	-	-	1

7. Sensitivity analysis

This section explores the effect of various parameters on the final ranking of alternatives. For this we study the ranking order for different values of α, δ, λ.

7.1 Sensitivity analysis of λ

The value of the parameter λ must be defined in order to calculate the integrated score functions in the q-ROF Bonferroni CoCoSo model. The value $\lambda = 0.5$ was used for the initial solution calculation. This made it possible for weighted Bonferroni functions to define different trade-offs in an equal manner. In order to replicate the change in the parameter $0 \leq \lambda \leq 1$, many scenarios were created. Fig.3 illustrates how changing the parameter can affect the utility value. Table 16 shows the ranking results, which demonstrates that altering parameter $0 \leq \lambda \leq 1$ does not affect the initial ranking of alternatives and that the initial ranking $I_4 > I_1 > I_2 > I_3$ is credible. But it's interesting to observe that the optimal alterative never changes, proving the isotonicity of the proposed method. DMs are able to choose a suitable value for λ depending on their preferences.

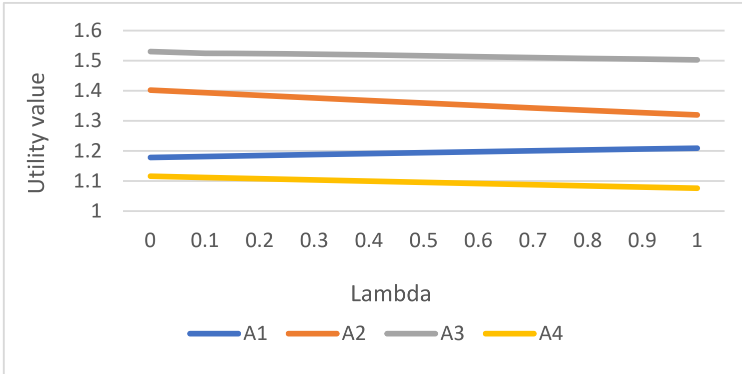

Fig. 3. Utility value for different λ

Table 16
Ranking with different λ

λ	Ranking	Optimal alternative
0	$I_4 > I_1 > I_2 > I_3$	I_4
0.1	$I_4 > I_1 > I_2 > I_3$	I_4
0.2	$I_4 > I_1 > I_2 > I_3$	I_4
0.3	$I_4 > I_1 > I_2 > I_3$	I_4
0.4	$I_4 > I_1 > I_2 > I_3$	I_4
0.5	$I_4 > I_1 > I_2 > I_3$	I_4
0.6	$I_4 > I_1 > I_2 > I_3$	I_4
0.7	$I_4 > I_1 > I_2 > I_3$	I_4
0.8	$I_4 > I_1 > I_2 > I_3$	I_4
0.9	$I_4 > I_1 > I_2 > I_3$	I_4
1	$I_4 > I_1 > I_2 > I_3$	I_4

7.2 Sensitivity analysis of α

As seen in Fig. 4, the ranking order of alternatives is persistent for all values of α. The utility function increases monotonically with varying values of α, which is the cause. As a result, the ranking order of utility function for alternatives is constant for different values of α, $v(A_4) < v(A_1) < v(A_2) < v(A_3)$. The regret-rejoice function is independent of α, hence the perceived utility value of alternatives is ranked in a consistent manner $U(A_4) < U(A_1) < U(A_2) < U(A_3)$.

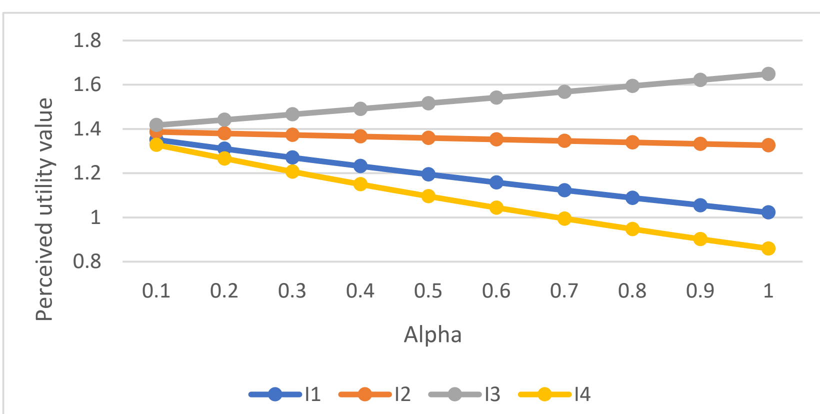

Fig. 4. Perceived utility value for different α

7.3 Sensitivity analysis of δ

As shown in Fig. 5, the ranking order of alternatives is constant for all values of δ. The regret-rejoice function monotonically increases with varying values of, which is the cause. Thus the ranking of regret- rejoice function for options is consistent for different values of δ, $R(A_4, A_r) < R(A_1, A_r) < R(A_2, A_r) < R(A_3, A_r)$. The utility function is independent of δ, hence the perceived utility value of alternatives is ranked in a consistent manner $U(A_4) < U(A_1) < U(A_2) < U(A_3)$. This research illustrates that the proposed approach produces reasonable findings and offers suitable outputs to aid DM in decision-making, in accordance with the sensitivity analysis.

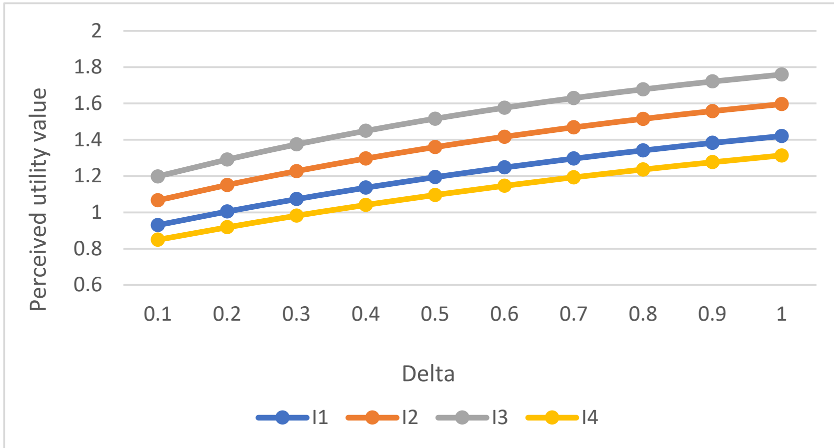

Fig. 5. Perceived utility value for different δ

7.4 Sensitivity analysis for attribute weights

The sensitivity assessment is carried out in this part to validate that the suggested weight determination approach produces rankings that are stable. Table 17 and Fig.6. shows the ranking results for different attribute weights obtained by subjective method, objective method, equal weights and proposed approach. For subjective weight and equal weight, the ranking order is $I_4 > I_1 > I_3 > I_2$ and the optimal alternative is I_4, while for the objective weight and proposed approach the ranking order is $I_4 > I_1 > I_2 > I_3$ and the optimal alternative is I_4. Although the rankings produced by the various methods change slightly, the best option is always the same, hence the proposed approach is stable in terms of optimal selection. For the better visualization of the results the correlation coefficients of the weights obtained by the proposed approach and other approaches are depicted in Table 18.

Table 17

Ranking of alternatives for different attribute weights

	Combined weights	Subjective weights	Objective weights	Equal weights
I_1	2	2	2	2
I_2	3	4	3	4
I_3	4	3	4	3
I_4	1	1	1	1

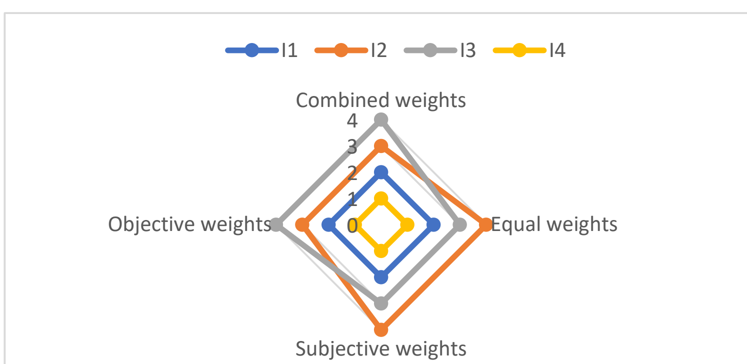

Fig.6. Sensitivity analysis for attribute weights

Table 18

Spearman correlation coefficients between ranking results with varying attribute weights

	Combined weights	Subjective weights	Objective weights	Equal weights
Combined weights	1	0.8	1	0.8
Subjective weights	-	1	0.8	1
Objective weights	-	-	1	0.8
Equal weights	-	-	-	1

8. Conclusions

Green supplier selection is becoming a crucial topic for discussion as a result of growing environmental concerns and the subjectivity of human thought. In the context of q-ROFS, a novel decision-making framework is proposed for this aim. In comparison to other regular orthopair fuzzy sets, q-ROFSs give decision makers more choice in how they express their ideas because they support a wider variety of membership and non-membership grades. The projection and entropy measure were used in this technique to obtain the objective weights of the criterion, while the experts' knowledge and the score function were used to estimate the subjective weights of the criteria. A combined weighting game theory-based model is then used to establish the final weights of the criteria. Additionally, the framework offers a way to determine expert weights considering the social trust network among them. By accurately recording the interrelationship between attributes, preferences are compiled. The CoCoSo technique is broadened in the framework of q-ROFS based on the regret theory, which effectively takes expert psychology into account, for the logical prioritization of alternatives. By varying the attribute weights and strategy values, the sensitivity analysis is carried out. Finally, the framework's viability is proved via the use of green supplier selection, and its advantages and disadvantages are evaluated through comparison to existing approaches. The findings demonstrate that, in comparison to the many existing approaches, the proposed framework is more reliable and consistent.

For the future work, the proposed framework can be combined with different theories like prospect theory, rough set theory etc. to get effective results, also it can be extended to the new decision framework which can handle the partial expert and attribute information. It would be fascinating to use the proposed approach to address a variety of further real-world decision-making issues.

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