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A planning model for repairable spare part supply chain considering stochastic demand and backorder: an empirical investigation

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A B S T R A C T

Today, improving machine availability is vital for industries to compete in the global market. Spare parts play an essential role in the maintenance and repair of equipment, but planning an extensive network in strategic industries with various spare parts can be very challenging due to the existence of different decision factors. The spare parts supply chain deals with inventory management issues, which necessitates considering the related decisions such as determining the stock level and order quantity. Moreover, demand uncertainty and long supply time make decision-making more complex. This paper presents a repair and supply planning model for repairable spare parts while considering a modified formulation of demand uncertainty to minimize costs. The model determines the optimal stock level, lateral transshipment, assignment of spare part orders to suppliers, equipment to repair centers, and the number of intervals over the planning horizon used in demand estimation. This research contributes to the literature by integrating recent decisions, using demand approximation by piecewise linearization, and considering backorder in warehouses evaluated by queuing models. A hybrid approach, including heuristic and genetic algorithms, is used to optimize the model using data from an oil company. The results show that using piecewise linearization and integrated repair and supply planning decisions optimizes costs and improves performance. Also, the availability is affected by the demand estimation, which necessitates precision prediction.

1. Introduction

Maintenance is a significant cost factor, accounting for about 25% of total costs (Hora, 1987). Also, it is a crucial area that is necessary to guarantee quality (Taji et al., 2022). Therefore, it is vital to maintain a significant stock level of spare parts to support maintenance systems and avoid downtime (Öner et al., 2007). Planning a complex network necessitates integrating various planning decisions to minimize total costs while maintaining service levels (Melo et al., 2009). (Kosanoglu et al., 2018) declare that the shortages of spare parts are the reason for more than 80% of downtimes. Besides the importance of availability, it is necessary to consider both the stock level and inventory costs (González-Varona et al., 2020; Wang et al., 2015) although uncertainty makes the estimations more complex (Yousefi-Babadi et al., 2021).

Spare parts in strategic industries are high-value inventories (KIM & PARK, 1985; Schulze & Weckenborg, 2012). Still, many challenges in supplying spare parts can be addressed, such as long lead time, shortage in warehouses, and low-quality spare parts (Gehret et al., 2020; Jiang et al., 2021). Sherbrooke is one of the pioneers who studied repair and supply planning using the METRIC[1] model to optimize the stock level of repairable spare parts in warehouses considering order queues (Sherbrooke, 1968); however, existing studies consider lead-time demand and formulate it by using a stand-alone probability distribution that does not give a good fit for low-demand[2] spare parts.

[1] Multi-Echelon Technique for Recoverable Item Control

[2] Low-demand is relative not absolute number

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The competition motivates industries to increase production, affecting the machines' lifetime (L. Liu & Cheung, 1997; Shastri et al., 2013). Managers in industries, especially in strategic industries, are concerned about the maintenance and repair operation costs due to limitations in budget and the way it affects the company's performance. Therefore, demand estimation is critical in inventory management, indeed, inventory subordinates the demand (Shaikh et al., 2020). Muckstadt (1973) addressed a two-echelon, multi-item, and multi-indenture model for spare part inventory management in which the demand follows the Poisson process. The spare parts are repairable, and unlimited repair capacity is considered. Following the basic models, Wong et al. (2005) presented an aircraft spare part inventory management model that is single-item, and pooling is considered as well as delayed lateral transshipment. (Wingerden et al., 2019) examined the effect of the emergency warehouse in the two-echelon network considering emergency shipment to extend the previous models. The results show that the emergency warehouse can reduce costs by up to 30%. Moini et al. (2021) developed a mathematical model for an integrated forward-reverse spare parts network that considers stochastic demand and assignment of equipment to disassembly centers. Indeed, they redesigned the basic supply chain network by adding inspection centers, which determine the repair or purchase policy by considering multiple factors.

Zammori et al. (2020) focus on risky missions that make repairing the spare parts essential, but the objective function is maximizing the probability of completing the operations by considering the limitation of space for the selection of spare parts. They used two distributions of exponential and Weibull to estimate the demand. The problem under the second distribution is solved by approximation, which results in better results. These researches focus on issues such as I) optimizing the decisions involving the stock level, emergency supply, lateral transshipment, and spare part order assignment to repair centers, II) deterministic and stochastic models are studied that the latter one analyses the uncertainty in demand over the lead-time, III) capacity and budget constraints are considered to embrace the real-world problems IV) These works address the capacity, performance, and multi-indenture models, but none of them considered the supply with other decisions such as repair. Also, most works just consider the lead-time demand in formulating the uncertainty. To examine the queuing models for performance evaluation, the following papers are reviewed.

He and Hu (2014) investigated the emergency supply formulated by a queuing model for minimizing the response time considering the M/M/1 model. To solve the developed model, a genetic algorithm is implemented. The repair cost is also considered in the model. Gholamian and Heydari (2017) examined a location-inventory problem that specifies the location of distribution centers. They also utilized the METRIC model considering (s-1, s) inventory replenishment policy. Guo et al. (2019) considered a repairable spare part network so that there is a correlation between the shortage of these parts. The spare parts in the single-echelon, multi-item supply network are repaired or replaced in case of breakdown. The Markov model is used to formulate the system. The inventory replenishment policy (-1, s+1) is used in this model. Liu et al. (2020) published a paper on the location-inventory problem considering the supply disruption. The supply network includes suppliers, depots, and retailers. The inventory level of depots is formulated using the continuous-time Markov process. The proposed mathematical model is solved using a hybrid genetic algorithm. Qin et al. (2021) considered a two-echelon spare part network under performance-based service. The model includes profit- and cost-centric objective functions. The greedy algorithm solves the model. Babaveisi et al. (2022) considered two models for integrating forecasting and planning decisions. They used a piecewise linearization technique which improves the forecasting accuracy and cost while back ordering and availability are not considered in their research.

Although the above researchers considered queuing models, they did not discuss capacity and possible backorders in warehouses. Also, demand is usually defined as a parameter while we consider it as a variable that improves the degree of decision integration. In this paper, the research gaps are as follows. Also, a brief comparison of the reviewed papers is presented in Table 1.

- The existing spare part planning models focus on separate decisions that may result in sub-optimality. An integrated planning model to consider inventory management and supply and repair decisions are not sufficiently investigated.
- The demand (i.e., equipment failure rate) is uncertain in real-world problems, but it is almost discussed by a predefined distribution that may not sufficiently fit the demand pattern for all problems. A piecewise linearization technique for approximating the demand in each interval over the planning horizon can enhance planning accuracy.
- Lateral transshipment is critical to reducing shortages in both central and local warehouses. The inventory management models for repairable spare parts, such as the METRIC model, usually focus on shortages in local warehouses while considering shortages in central warehouses is vital to formulate the lateral transshipment.
- The supply capacity, defects, and delivery time constraints are considered for the assignment of spare part orders to suppliers, while other research does not focus on supply decisions to be integrated with other planning decisions.

The state-of-the-art shows that spare part repair and supply planning models did not simultaneously consider planning decisions (stock level, spare part order assignment to suppliers, repair assignment, and lateral transshipment). Additionally, repair constraints such as expertise, capacity, and time are not considered in supply decisions such as capacity, defect rate, and supply time. Another issue is performance evaluation which other works did not focus on queuing model from the aspect of lateral transshipment planning in a multi-period model. Also, we consider stochastic demand which is formulated through piecewise linearization. We seek to find the answers to these questions: 1) what is the optimal number of intervals obtained by the piecewise linearization? 2) what are the optimal spare part stock levels? 3) what are the optimal flows between the facilities? 4) what and how many spare parts to order?

In this paper, a mathematical model is developed integrating the supply and repair decisions of spare parts with (s-1, s) policy, validated by a case study in an Iranian oil company. The model determines inventory management decisions, lateral transshipment, equipment assignment to repair centers, spare parts assignment to repair centers from central warehouses used in repairing the equipment, and supply decisions. A piecewise linearization technique determines the number of intervals over the planning horizon used in demand estimation. Also, a queuing model assesses the performance in warehouses and reflects the real performance of the system. Finally, sustainability analyses are performed. Since the inventory management model accounts for the np-hard ones, a hybrid approach is used to solve such a model.

This paper includes the following sections: First, the problem is explained and the model is presented followed by the case study, computations, and results. Next, sensitivity analyses are provided. Finally, conclusions and future research opportunities are expressed.

Table 1
Literature review brief

Authors	Decisions Types	Objective function				Solution method			Case study	Uncertainty	
		Min*. cost	Max**. profit	Min. shortage	Max. availability	Heuristic	Meta-heuristic	Exact		Constant mean	Variable mean
(Muckstadt, 1973)	IM			✓		✓				✓	
(Wong et al., 2005)	IM, A			✓				✓		✓	
(Wingerden et al., 2019)	IM, A	✓				✓				✓	
(Moini et al., 2021)	IM, A	✓						✓	✓	✓	
(He & Hu, 2014)	IM			✓			✓				
(Gholamian & Heydari, 2017)	IM	✓					✓	✓		✓	
(Guo et al., 2019)	IM				✓	✓					
(Y. Liu et al., 2020)	IM, A	✓					✓				
(Qin et al., 2021)	IM		✓			✓			✓	✓	✓
(Babaveisi et al., 2022)	IM	✓						✓	✓	✓	
Present paper	S, R, A, IM	✓				✓	✓		✓		✓

* Min. : Minimizing, ** Max. : Maximizing, **S: supply, R: repair, A: Assignment, IM: inventory management**

2. Problem statement

This study discusses the repairable spare part planning model which considers inventory management decisions. The queues of orders of spare parts in warehouses affect the system's performance since it impacts the availability, so we formulate the order queues to apply this effect. There are two flows in the network: equipment and spare parts—the failed equipment transfers from the operational bases where it is installed. Defective equipment is inspected in the inspection center where the experts are gathered. The equipment is technically examined to determine if it is repairable or not. This decision highly depends on the company's policy, spare part lifetime, and the tradeoff between repair and purchase costs that are considered as the repairability probability. A general perspective of the network is shown in Fig. 1.

Two terms are defined: equipment (LRU[3]) and spare parts (SRU[4]). Each LRU can be an expensive component of a machine composed of SRUs. The LRU is assigned to either inner or outer-company repair centers. The company owns Inner-company repair centers, so the company supplies the resources such as materials, workers, and investments. Outer-company repair centers are independent of the company resources, such as tools, and physical assets. The failed LRU requires SRU in the repair operation that is supplied from central warehouses. The SRU in central warehouses is supplied by the suppliers based on the capacity, defect rate, and delivery time. The repaired equipment is assigned to local warehouses, central warehouses, and operational bases. Both central and local warehouses may confront shortages that can be enhanced by planning the lateral transshipment.

[3] Line-Replaceable Unit
[4] Shop-Replaceable Unit

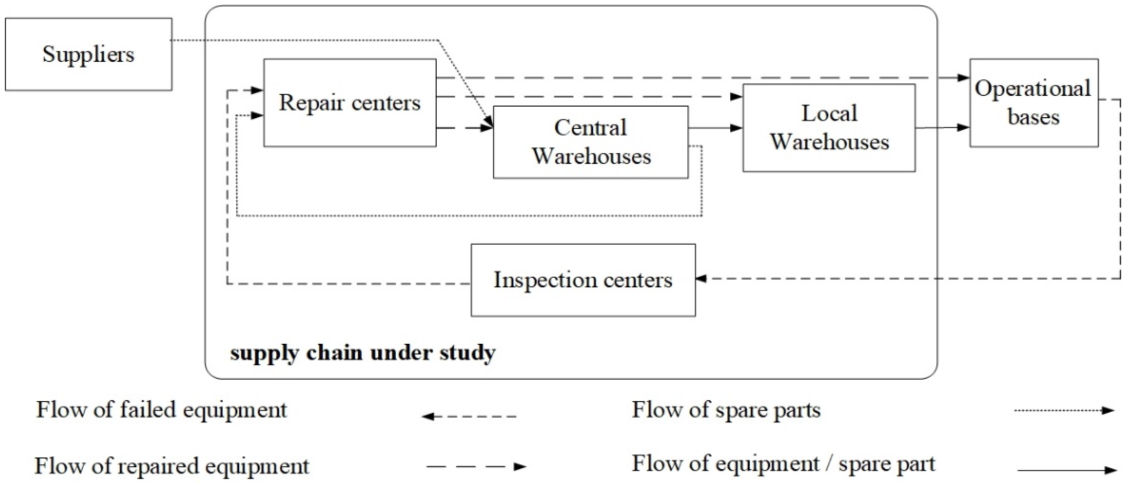


Fig. 1. Repairable spare part inventory system

Spare parts examined in this research follow (S-1, S) replenishment policy. METRIC model is used to handle the spare parts inventory management decisions such as stock level in local and central warehouses and order quantity. The model considers repair and supply planning decisions and inventory management to make optimal decisions such as: determining stock level in central and local warehouses and equipment assignment to repair centers based on repair time, expertise, and capacity. Also, we specify the inventory planning of spare parts for inner-system repair centers and the number of time intervals over the planning horizon for demand estimation by piecewise linearization.

3. Model

The notations including sets and indices, parameters, and variables are illustrated in the following. Then, objective functions, constraints, and formulations are presented.

3.1. Sets and indices

$s \in S$ where $s_1, s_2 \subseteq s$	Equipment (LRU) and spare parts (SRU)
$k \in N$	Number of time intervals
$j, j' \in J$ where $w, r, i, c, s' \subseteq j, j'$	All nodes
$w, w' \in W$	Warehouses
$w_1 \subseteq w$	Central warehouses
$w_2 \subseteq w$	Local warehouses
$r \in R$	Repair centers
$r_1 \in r$	Inner-company repair centers
$r_2 \in r$	Outer-company repair centers
$i \in I$	Inspection center
$c \in C$	Operational bases
$s' \in S'$	Suppliers

3.2. Parameters

$tc_{sjj'}$	Transportation cost between node j and j'
$c_{ss'w_1}$	Ordering cost in central warehouse w_1 to supplier s'
$c_{sww'}^{lat}$	Lateral transshipment cost for spare part s between warehouse w and w'
rt_{sr}	Equipment s repair time in repair center r
cap_r	Repair center r capacity (Man-Hour)
cp_{rs}	1, if repair center r has the repair expertise for equipment s, else 0
$sc_{ss'}$	Capacity of supplier s' to supply spare part s
G_{si}	Repairability probability of equipment s in inspection center i
$pr_{ss'}$	Spare part s purchase cost from supplier s'
$pu_{s_1 s_2}$	Probability of demand for spare part s_1 in equipment $s_2 \in s$
h_{sj}	Spare part s holding cost in facility j (warehouse w or repair center r)
$def_{ss'}$	Spare part s defect rate of supplier s'

$del_{ss'}$	Spare part *s* delivery time from supplier s'
$mdef_s$	Maximum mean defect of spare part *s*
$mdel_s$	Minimum acceptable mean delivery time of spare part *s*
d_{sc}	Demand rate of spare part s in operational base *c*
I_{sw}^{0}	Spare part *s* initial inventory of warehouse *w*
I_{sr}^{R0}	Spare part *s* initial inventory in repair center *r*
$\tau_{sw_1w_2}$	Travel time of LRU *s* from central warehouse w_1 to local warehouse w_2
π'_s	Spare part s backorder cost
$\mu_{ss'w_1}$	Spare part *s* supply time from supplier s' to central warehouse w_1
$\tau_{sw_1} = \sum_{s'} \mu_{ss'w_1}$	Supplier s' cumulative travel time for Spare part s to central warehouse w_1
rc_{sr}	Repair cost of LRU *s* is repair center *r*

3.3. Decision variables

x'_{scik}	Number of LRU *s* from operational base *c* to inspection center *i* in interval *k*
y'_{sirk}	Number of LRU *s* from inspection center *i* to repair center *r* in interval *k*
$x^{(1)}{}_{ss'w_1k}$	Number of SRU *s* from supplier s' to central warehouse w_1 in interval *k*
$x^{(2)}{}_{srw_1k}$	Number of LRU *s* from repair center *r* to central warehouse w_1 in interval *k*
$y^{(1)}{}_{sw_1w_2k}$	Number of LRU *s* from central warehouse w_1 to local warehouse w_2 in interval *k*
$y^{(2)}{}_{srw_2k}$	Number of LRU *s* from repair center *r* to local warehouse w_2 in interval *k*
$z^{(1)}{}_{sw_2ck}$	Number of LRU *s* from local warehouse w_2 to operational base *c* in interval *k*
$z^{(2)}{}_{srck}$	Number of LRU *s* from repair center *r* to operational base *c* in interval *k*
A_{sck}	Availability of SRU *s* in interval *k*
I_{swk}^{+}	Expected on-hand inventory of LRU/SRU *s* in warehouse *w* in interval *k*
I_{swk}^{-}	Expected shortage of LRU/SRU *s* in warehouse *w* in interval *k*
st_{sjk}	SRU *s* stock level in facility *j* (warehouse *w* or repair center *r*) in interval *k*
$wa_{sw_{1k}}$	SRU *s* waiting time in the central warehouse w_1 in interval *k*
$lat_{sww'k}$	Number of LRU/SRU *s* from central(local) warehouse w to central(local) warehouse w' in interval *k*
ws_{sw_1rk}	Number of SRU s from central warehouse w_1 to repair center *r* in interval k

3.4. Model Assumptions

- Each equipment (LRU) has several spare parts (SRU), but each SRU only exists in one LRU.
- The demand for equipment (LRUs) depends on spare parts (SRUs),
- (S-1, S) replenishment policy is used.

3.5. Model formulation

A two-echelon repairable spare part network is discussed in this paper for inventory management decisions. The equipment failure rate follows the renewal process which is adopted from (Jin & Tian, 2012). It is assumed that there are $N_{sc}(t)$ random failures of spare part s for operational base c in the period [0, t]. The events are addressed as independent and identically distributed arrivals since the repaired equipment is technically the same as the new one. Considering $M_{sc}(t)$ as the average number of arrivals up to time t > 0, that is, the number of failures where the first arrival occurs at time s (s ≤ t). Also, $Z_{sc}(t)$ and K_{scj} denote the total failures and its occurrence time. $Z_{sc}(t)$ is calculated in Eq. (1).

$$Z_{sc}(t) = M_{sc}(t) + \sum_{j=1}^{N_{sc}(t)} M_{sc}(t - K_{scj}) \tag{1}$$

Time-varying demands of spare parts make the planning complicated due to significant fluctuations in various operational bases. The planning horizon is divided into intervals to simplify the complexity since the estimation would be more exact for shorter ranges. In this way, a stepwise form is used to approximate the expected stationary demand. So, the length of each interval and the planning horizon is respectively *L* and *T*. Let consider t_k as the length of interval *k* where $\mathbf{k} \in \{1, \dots, \mathrm{N}\}$. Assuming the exponentially distributed time-to-failure, the Poisson process estimates the average demand distribution in each interval. The time between the renewals follows the exponential distribution with the parameter α_s. Finally, the average demand in [0, t] is computed by Eq. (2). The expected demand $(\bar{d}_{sck})$ of spare part *s* in operational base *c* in interval *k* is calculated by Eq. (3).

$$E(Z_{sc}(t)) = \alpha_s t + \frac{\alpha_s d_{sc} t^2}{2} \tag{2}$$

$$\bar{d}_{sck} = \frac{E(Z_{sc}(t_k) - Z_{sc}(t_{k-1}))}{t_k - t_{k-1}} = \alpha + \frac{\alpha\lambda(t_k + t_{k-1})}{2}, \quad k = 1, \dots, n \quad where \quad n = \frac{t}{L_k} \tag{3}$$

The objective function $Cost_k$ is defined as minimizing the total costs for each interval k. Eqs. (1 – 7) relates to transportation costs. Eq. (1) shows the transportation cost between the central warehouses and suppliers. Eq. (2) presents the transportation cost of repair centers and other facilities. Eq. (3) is the related cost among the warehouses. Eq. (4) calculates the transportation cost from local warehouses to operational bases. Eq. (5) shows the transportation cost of operational bases to inspection centers. The transportation cost from inspection centers to repair centers can be seen in Eq. (6), and the transportation cost from central warehouses to repair centers is defined in Eq. (7). The ordering cost and purchase costs are shown in Eq. (8). Holding costs in warehouse and repair centers are respectively expressed in Eq. (9) and Eq. (10). Shortage and repair costs are computed by Eq. (11) and Eq. (12). Shortage cost is the equipment shutdown cost when the spare part is not available. The lateral transshipment costs are shown in Eq. (13).

$$Min\ Cost_k = \sum_s \sum_{s'} \sum_{w_1} tc_{ss'w_1} x^{(1)}{}_{ss'w_1k} \tag{4}$$

$$+ \sum_s \sum_r \sum_{w_1} tc_{srw_1} x^{(2)}{}_{srw_1k} + \sum_s \sum_r \sum_{w_2} tc_{srw_2} y^{(2)}{}_{srw_2k} + \sum_s \sum_r \sum_c tc_{src} z^{(2)}_{srck} \tag{5}$$

$$+ \sum_s \sum_{w_1} \sum_{w_2} tc_{sw_1w_2} y^{(1)}{}_{sw_1w_{2k}} \tag{6}$$

$$+ \sum_s \sum_{w_2} \sum_c tc_{sw_2c}\, z^{(1)}_{sw_2ck} \tag{7}$$

$$+ \sum_s \sum_c \sum_i tc_{sci}\, x'_{scik} \tag{8}$$

$$+ \sum_s \sum_i \sum_r tc_{sir} y'_{sirk} \tag{9}$$

$$+ \sum_s \sum_{w_1} \sum_{r_1} tc_{sw_1r_1}\, ws_{sw_1r_{1k}} \tag{10}$$

$$+ \sum_s \sum_{w_1} \sum_{s'} (c_{ss'w_1} + pr_{ss'}) x^{(1)}{}_{ss'w_{1k}} \tag{11}$$

$$+ \sum_s \sum_w h_{sw} I^{+}_{swk} \tag{12}$$

$$+ \sum_s \sum_r h_{sr} st_{srk} \tag{13}$$

$$+ \sum_s \sum_w \pi'_s I^{-}_{swk} \tag{14}$$

$$+ \sum_s \sum_r \sum_i rc_{sr} y'_{sirk} \tag{15}$$

$$+c^{lat}_{sw_1w'_1}\left(\sum_s\sum_{w_1}\sum_{w'_1} lat_{sw_1w'_1k}\right)+c^{lat}_{sw_2w'_2}\left(\sum_s\sum_{w_2}\sum_{w'_2} lat_{sw_2w'_2k}\right) \tag{16}$$

The flow of failed equipment from operational bases to inspection centers is computed using Eq. (14) which the demand in each interval is calculated by Eq. (3) is used to compute the total demand in each interval. Eq. (15) ensures the equality of reverse flow and the number of failures.

$$\sum_i x'_{scik} = L_k.\bar{d}_{sck} \qquad \forall s,c,k \tag{17}$$

$$\sum_{w_2} z^{(1)}{}_{sw_2ck} + \sum_r z^{(2)}{}_{srck} = L_k.\bar{d}_{sck} \qquad \forall s,c,k \tag{18}$$

Since the warehouses do not have an infinite capacity for storing spare parts, the performance of the warehouse is formulated through a queuing model. Given the queuing model of the Erlang loss system as *M*/*G*/*S*/*S*, the stock level is specified by S. The expected shortage and fill rate in local warehouses are as follows, respectively shown by L and β which is reproduced from the previous models (Karush, 1957). The variable ρ_{swk} accounts for the utilization rate.

$$L(st_{swk},\rho_{swk}) = \sum_{i=-st_{swk}}^{-1}(-i)\times p(X=i) = \frac{\frac{\rho_{swk}^{st_{swk}}}{st_{swk}!}}{\sum_{i=0}^{st_{swk}}\frac{\rho_{swk}^{i}}{i!}} \tag{19}$$

$$\beta(st_{swk},\rho_{swk}) = 1-\frac{\frac{\rho_{swk}^{st_{swk}}}{st_{swk}!}}{\sum_{i=0}^{st_{swk}}\frac{\rho_{swk}^{i}}{i!}} \tag{20}$$

It is assumed that the state variable is defined as $x_{ss'w_1k} \leq st_{swk}$ which presents the inventory level and the maximum shortage $\overline{st}_{sk} = \sum_w st_{swk}$ as the upper bound. The demand for local warehouses from central and other local warehouses is obtained using the fill rate in local warehouses. Assuming λ_{sk} as the demand for spare parts in the central warehouse in interval k, when no shortage exists, the demand is calculated in Eq. (18). Also, the demand in central warehouses (λ'_{sk}) in case of shortage is presented in Eq. 19, which is the demand of local warehouses when no stock is on-hand. Demand in each local warehouse is calculated by $\gamma_{sw_2k} = \sum_c z^{(1)}{}_{sw_2ck} + \sum_{w'_2} lat_{sw_2w'_2k}$ i.e. the total demand (including the operational bases and other local warehouses). The steady-state equations (Özkan et al., 2015), shown in Eq. (20), then we adapt it to our model.

$$\lambda_{sk} = \sum_{w_2}\gamma_{sw_2k} \qquad \forall s,k \tag{21}$$

$$\lambda'_{sk} = \sum_{w'_2}\beta_{sw_2k}\cdot\gamma_{sw_2k} \qquad \forall s,k \tag{22}$$

$$\pi_{(x_{ss'w_1k})} = \begin{cases} \dfrac{\lambda'_{sk}}{(st_{sw_1k}-x_{ss'w_1k}).(1/\mu_{ss'w_1})}\pi_{(x_{ss'w_1k}+1)}, & -\overline{st}_{sw_1k} \leq x_{ss'w_1k} < 0 \\ \dfrac{\lambda_{sk}}{(st_{sw_1k}-x_{ss'w_1k}).(1/\mu_{ss'w_1})}\pi_{(x_{ss'w_1k}+1)}, & 0 \leq x_{ss'w_1k} < st_{sw_1k} \end{cases} \tag{23}$$

Eqs. 21-26 are the METRIC model formulations used for computing the expected on-hand inventory, backorder, and waiting time. This model was proposed by (Sherbrooke, 1968) and is developed more by other researchers (Axsäter, 1993; Rappold & Van Roo, 2009). The expected on-hand inventory is presented in Eqs. (21) and Eq. (25), and the average shortage is calculated in Eq. (22) and Eq. (26). The average waiting time is calculated by Eq. (23) using the little law; then, the replenishment time in local warehouses is computed by adding the travel time to the waiting time which is presented in Eq. (24).

$$I^{+}_{sw_1k} = \sum_{j=1}^{st_{sw_1k}} j.P(X=j) = \sum_{j=1}^{st_{sw_1k}} j\frac{e^{-\lambda_{sk}\tau_{sw_1}}(\lambda_{sk}\tau_{sw_1})^{st_{sw_1k}-j}}{(st_{sw_1k}-j)!} \tag{24}$$

$$\forall s,w_1,k$$

$$I^{-}_{sw_1k} = \sum_{s'=1}^{S'}\sum_{-st_{sw_1k}}^{-1} -j.P(X=j) = \sum_{s'=1}^{S'}\sum_{-st_{sw_1k}}^{-1} -x_{ss'w_1k}\pi_{x_{ss'w_1k}} \tag{25}$$

$$wa_{sw_1k} = \frac{I^-_{sw_1k}}{\lambda_{sk}}, \lambda_{sk} \neq 0 \quad (26)$$

$$\bar{\tau}_{sw_2k} = \sum_{w_1} (\tau_{sw_1w_2} + wa_{sw_1k}) \quad (27)$$

$$\forall s, w_2, k$$

$$I^+_{sw_2k} = \sum_{j_s=1}^{st_{sw_2k}} j_s \times \frac{e^{-\gamma_{sw_2k}\bar{\tau}_{sw_2}} (\gamma_{sw_2k}\bar{\tau}_{sw_2k})^{st_{sw_2k}-j_s}}{(st_{sw_2k} - j_s)!} \quad (28)$$

$$st_{sw_2k} \geq I^+_{sw_2k} - I^-_{sw_2k} + \gamma_{sw_2k}\bar{\tau}_{sw_2k} \quad (29)$$

Balance equations in inspection and repair centers are presented in Eq. (27) and Eq. (28).

$$\sum_r y'_{sirk} = \sum_c G_{si} \times x'_{scik} \qquad \forall s, i, k \quad (30)$$

$$\sum_i y'_{sirk} = \sum_{w_1} x^{(2)}{}_{srw_1k} + \sum_{w_2} y^{(2)}{}_{srw_2k} + \sum_c z^{(2)}{}_{srck} \qquad \forall s, r, k \quad (31)$$

The repair of equipment in repair centers is divided into two types. The first type comes in inner-company repair centers, which use company resources, but the second type is the outer-company repair centers that operate independently, often through a contract. Eq. 29 shows the spare parts which are needed for repairing each equipment in the inner-company repair center. The repair expertise and capacity constraint are also expressed in Eq. (30) and Eq. (31).

$$I^{R0}_{sr_1} + \sum_{w_1} ws_{sw_1r_1k} \geq pu_{ss_1} \times \sum_i y'_{s_1ir_1k} + st_{s_1r_1k} \qquad \forall s, r_1, k \quad (32)$$

$$\sum_i rt_{sr} \times y'_{sirk} \leq cap_r \qquad \forall s, r \quad (33)$$

$$\sum_i y'_{sirk} \leq M \times cp_{rs} \qquad \forall s, r, k \quad (34)$$

The supply capacity from suppliers, maximum allowed defect, and delivery time constraints are listed in Eqs. (32-34).

$$\sum_{w_1} x^{(1)}{}_{ss'w_1k} \leq sup_{ss'} \qquad \forall s, s', k \quad (35)$$

$$\sum_{s'} \sum_{w_1} def_{ss'} x^{(1)}{}_{ss'w_1k} \leq mdef_s \sum_{w_1} \sum_{s'} x^{(1)}{}_{ss'w_1k} \qquad \forall s, k \quad (36)$$

$$\sum_{w_1} \sum_{s'} del_{ss'} x^{(1)}{}_{ss'w_1k} \leq mdel_s \sum_{w_1} \sum_{s'} x^{(1)}{}_{ss'w_1k} \qquad \forall s, k \quad (37)$$

Eqs. 35 and 36 present the balance equations in local and central warehouses.

$$I^0_{sw_2} + \sum_{w_1} y^{(1)}{}_{sw_1w_2k} + \sum_r y^{(2)}{}_{srw_2k} + \sum_{w'_2 \neq w_2} lat_{sw'_2w_2k} = st_{sw_2k} + \sum_c z^{(1)}{}_{sw_2ck} + \sum_{w'_2 \neq w_2} lat_{sw_2w'_2k} \qquad \forall s, w_2, k \quad (38)$$

$$I^0_{sw_1} + \sum_{s'} x^{(1)}{}_{ss'w_1k} + \sum_r x^{(2)}{}_{srw_1k} + \sum_{w'_1 \neq w_1} lat_{sw'_1w_1k} = st_{sw_1k} + \sum_{w_2} y^{(1)}{}_{sw_1w_2k} + \sum_r ws_{sw_1rk} + \sum_{w'_1 \neq w_1} lat_{sw_1w'_1k} \qquad \forall s, w_1, k \quad (39)$$

The availability in each time interval is calculated by Eq. (37). Finally, the domains of variables are shown.

$$A_{sck} = 1 - \frac{\sum_w I^-{}_{swk}}{d_{sc} \sum_k L_k} \qquad \forall s, k \quad (40)$$

$$x'_{scik},\ y'_{sirk},\ x^{(1)}_{ss'w_1k}, x^{(2)}_{srw_1k},\ y^{(1)}_{sw_1w_2k},\ y^{(2)}_{srw_2k},\ z^{(1)}_{sw_2ck},\ z^{(2)}_{srck},\ st_{swk}\ \mathbb{Z}^+;\ \ I^+_{swk},\ I^-_{swk} \in \mathbb{R}^+$$

4. Computations and solution procedure

The case study and solution approach is presented in this section; then, the results are obtained. Iran, one of the biggest countries in the world, possesses a prominent share of global reserves. The National Iranian South Oilfields Company (NISOC) is a major Iranian oil company including Aghajari, Marun, Kranj, Bibi Hakimeh, Rag Sefid, Ahwaz, and Gachsaran. Three central warehouses and six local warehouses exist in this area. The inner-company repair centers are assumed to be close to operational bases, while the outer-company repair centers are far away, so they are charged with prominent transportation costs. Three repair centers include two inner-company repair centers, and the other one is outer-company—two equipment with their spare parts, listed below.

- Rotary pump
  - Idlers
  - Retainer
  - Seal
  - Power Rotor
  - Bearing Snap Rings
- Ball valve
  - Bearing plate
  - Thrust bearing
  - Stem bearing
  - Gland packing
  - Seat seal

The minimum mean delivery time and maximum mean defect are 100 and 40 days for the rotary pump and ball valve. The inner-company and outer-company repair centers have a maximum repair capacity of 2000 man-hours. Additionally, the repair time for repairing the rotary pump and ball valve is 70 and 50 man-hours. The rotary pump and ball valve repair costs are 500 and 200. Table 2 involves the failure rate of the equipment and spare parts in operational bases (operational bases). The rotary pump and ball valve repairability probability are 0.85 and 0.9, respectively. This value determines the input of the repair centers, and the rest are disposed out of the network.

Table 2
Demands of operational bases

Equipment/ Spare parts		**Rotary pump**	Idlers	Retainer	Seal	Power Rotor	Bearing Snap Rings	**Ball valve**	Bearing plate	Thrust bearing	Stem bearing	Gland packing	Seat seal
Operational bases	1	**7**	2	2	1	1	1	**3**	1	0	0	1	1
	2	**3**	3	1	1	1	0	**5**	2	1	1	0	1
	3	**5**	1	1	1	0	2	**7**	2	2	0	1	2
	4	**5**	5	3	1	1	0	**1**	0	0	1	0	0
	5	**2**	1	0	1	0	0	**2**	0	1	0	1	0

4.1. Solution approach

To solve the non-linear mathematical model, the heuristic is shown below—the main complexity of the model comes from the queuing formulation. Figure 2 illustrates the heuristic approach used for solving the inventory model.

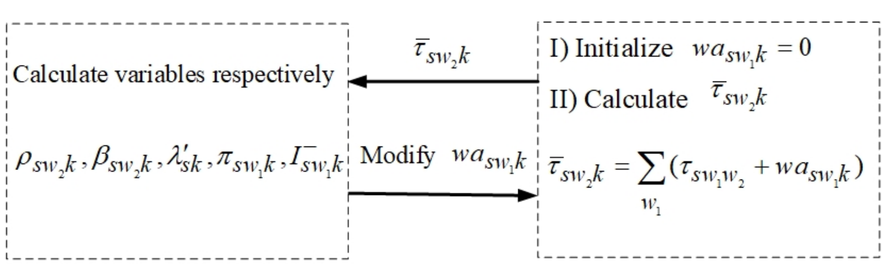


Fig. 2. A heuristic approach for the inventory model

The pseudo-code is presented as follows. The heuristic involves four stages that are described below. The two-year planning horizon includes five intervals. It is initially assumed that there is no shortage in local warehouses so no lateral transshipment exists. The procedure continues until the waiting time does not change more than epsilon (small amount). When the results of the heuristic is obtained, the Genetic Algorithm (GA) is used to solve the rest of the model by MATLAB R2019a with Intel(R) Core (TM) i5-9400F CPU @ 2.90GHz and 16 GB Ram Personal Computer.

***While* k ≤ N**

<u>Stage one:</u> Initialisation

$$wa_{sw_1k} = 0 \qquad \forall s \in S, \forall w_1 \in W_1, \forall k \in K$$

$$\bar{st}_{sk} = \sum_{w} st_{swk} \qquad \forall s \in S, \forall k \in K$$

$$\lambda_{sk} = \sum_{w_2} \lambda'_{sw_2k} \qquad \forall s \in S, \forall k \in K$$

$$\bar{\tau}_{sw_2k} = \sum_{w_1} (\tau_{sw_1w_2} + wa_{sw_1k}) \qquad \forall s \in S, \forall w_2 \in W_2$$

$$lat_{sw_2'w_2k} = 0 \qquad \forall s \in S, \forall w, w' \in W, w \neq w'$$

$$\beta_{sw_2k} = 1 - L(st_{sw_2k}\bar{\tau}_{sw_2k}.\gamma_{sw_2k}) \qquad \forall s \in S, \forall w_2 \in W_2, \forall k \in K$$

<u>Stage two:</u> Local warehouse performance evaluation

$$lat_{sw_2''w_2k} = (1 - \beta_{sw_2'k})\, lat_{sw_2'w_2k} \qquad \forall s \in S, \forall w_2 \in W_2, \forall k \in K, 1 \leq w_2,\ w_2' \leq W_2'$$

$$\gamma_{sw_2k} = \sum_{w_2'} lat_{sw_2w_2'k} \qquad \forall s \in S, \forall w_2 \in W_2, \forall k \in K$$

$$\beta_{sw_2k} = 1 - L(st_{sw_2k}, \bar{\tau}_{sw_2k}.\gamma_{sw_2k}) \qquad \forall s \in S, \forall w_2 \in W_2, \forall k \in K$$

<u>Stage three</u>: Central warehouse performance evaluation

$$\lambda'_{sk} = \sum_{w_2'} \beta_{sw_2k} \cdot \gamma_{sw_2k}$$

$$\pi_{(x_{ss'w_1k})} = \begin{cases} \dfrac{\lambda'_{sw_1k}}{(st_{sw_1k} - x_{ss'w_1k}).(1/\mu_{ss'w_1})}\pi_{(x_{ss'w_1k}+1)}, & -\bar{st}_{sw_1k} \leq x_{ss'w_1k} < 0 \\ \dfrac{\lambda_{sw_1k}}{(st_{sw_1k} - x_{ss'w_1k}).(1/\mu_{ss'w_1})}\pi_{(x_{ss'w_1k}+1)}, & 0 \leq x_{ss'w_1k} < st_{sw_1k} \end{cases}$$

$$I^-_{sw_1k} = \sum_{s'=1}^{S'} \sum_{-st_{sw_1k}}^{-1} -j.P(X=j) = \sum_{s'=1}^{S'} \sum_{-st_{sw_1k}}^{-1} -x_{ss'w_1k}\pi_{x_{ss'w_1k}}$$

$$wa_{sw_1k} = \frac{I^-_{sw_1k}}{\lambda_{sk}}, \lambda_{sk} \neq 0$$

<u>Stage four</u>: Iterative procedure

Repeat the stage two and three until wa_{sw_1k} does not change significantly Set k = k+1

End

Considering M and N as the number of sources and the depots, the chromosome is filled with a random permutation of (M+N). A representation is considered for each stage of the network. The crossover operators is used to generate new solutions. Due to the chromosome structure, order crossover (OX) is used. One or two cut points can be used to generate new offspring. In the case of mutation, two random genes are replaced (Paydar et al., 2017). According to priority-based encoding representation, a chromosome is encoded, then each generated solution by the priority-based decoding algorithm justifies the feasibility. The search in the chromosome starts from the maximum priority value. Sources and depots are limited by the capacity or demand, which determines the flow between the nodes. The pseudo-code of the priority-based algorithm is shown below.

Inputs:

S: Set of sources, D: set of depots,
tc: Transportation cost from sources to depots,
cap: Capacity of $sources$,
de: Demand of $depots$
chromosome(S*D)

Outputs:
X: The flow between sources and depots
$While\ \ all(\text{chromosome}(:)) \neq 0$
Step1.Chromosome generation: $ran \leftarrow argmax\{ch(u), \text{u} \in (|D| + |S|)\}$;
Step2. Selecting node: $k^* = \left\lceil \frac{ran}{|D|+|S|)} \right\rceil$,
Step3.determining sources and depots
$v^* = argmin\{tc| \text{ ch} \neq 0\}$,
selecting a resource and depots with minimum transportation cost
Step4. calculate the flow between the resource and depot
$X = min\{ cap, de\}$,
Step5. Updating capacities
$cap = cap - X,\ \ de = de - X,$
End

5. Numerical study and analyses

Using the above solution procedure, the model is solved by the procedure. The case study is used to validate the model. The accuracy of the evaluation heuristic for the inventory management model is declared by (Wingerden et al., 2019). Various instances are also provided in Appendix A. The results are presented in the below tables to shed light on the results. The cost per interval is shown in Table 3.

Table 3
Cost per interval

Time interval	Cost
1	1.3×10^6
2	1.27×10^6
3	1.25×10^6
4	1.26×10^6
5	1.28×10^6

We present sensitivity analyses to investigate the effect of changing the parameters on decision variables.

5.1. The multi-period planning

Fig. 3 illustrates the changes in the number of intervals and the relation with cost. By increasing the number of intervals, the total cost first reduces to a minimum value; then, the costs start to rise with more than three intervals. The fluctuations come from the practical and theoretical context. Practically, this behavior could be justified by risk-sharing and inventory management costs since the supply will be continued through the planning horizon and the average shortage decreases.

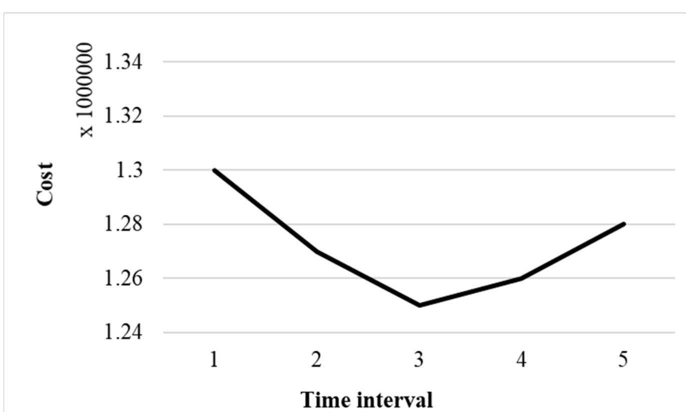
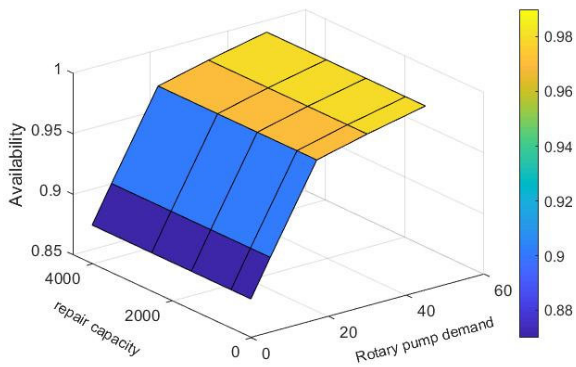


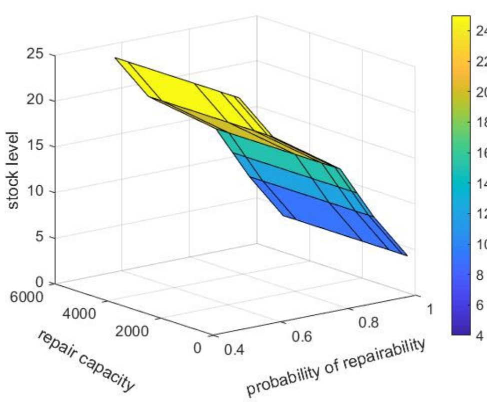
Fig. 3. Cost as the number of time intervals changes

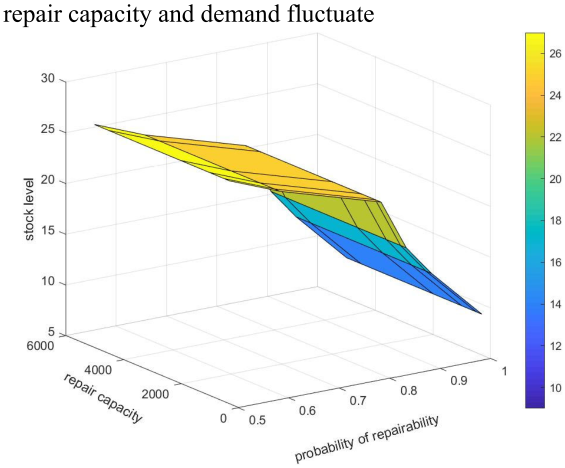
5.2. The relation of availability and stock level with repair and demand rate

The relation between repair capacity and demand with availability is illustrated in Fig. (4-a). The availability rises when the repair capacity enhances by increasing the failure rate. The performance-based servicing causes an increase in availability since it guarantees the lifetime of the equipment. The increase in repair capacity continues until it reaches the peak, then minor changes in the availability occur. Fig. (4-b) shows the relation between the stock levels and these parameters for interval one. The stock level increases as the probability of repairability (PR) falls, resulting from a rise in supply. In other words, as the output of the repair centers decreases, the system starts to compensate for the shortage in operational bases, especially in the case of stochastic demand. The rotary pump is too sensitive to PR compared with the ball valve, which could be interpreted as a limitation in supply, such as a long lead time or high purchase cost.



a) Equipment availability as the repair capacity and demand fluctuate





I) Rotary pump II) Ball valve

b) Central warehouse stock level

Fig. 4. Relation of availability (a) and stock level (b) with repair capacity and probability of repairability

The demand rate, i.e., the failure rate of the equipment, affects the optimal number of intervals. The more the demand rate is, the higher the number of planning intervals will be.

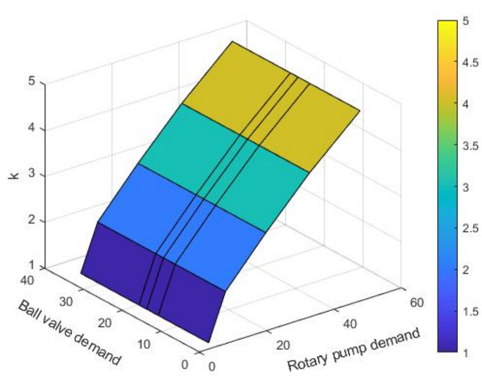


Fig. 5: The number of time intervals as the demand changes

Fig. 5 illustrates the relation between the optimal number of time intervals and the demand. We can also observe when the demand for the ball valve is fixed and the demand for the rotary pump increases, the number of intervals rises. Conversely, the positive change in ball valve demand for a fixed rotary pump demand does not significantly affect the optimal number of intervals. This behavior can be interpreted from the point of view of the ordering cost. The increase in rotary valve demand sharply affects the number of optimal intervals, while the number of intervals is not significantly affected by the demand of the ball valve.

5.3. *The performance analysis in warehouses*

The queuing model in the central warehouses is formulated as *M*/*G*/*S*/*S* to evaluate the performances and reduce shortages and inventory costs. In this section, the arrival and service rates are analyzed to observe the effects of changes on the average waiting time and costs over the planning horizon. The computations are in all the operational bases for each time interval. The results show the deviation from the optimized solution due to demand and supply time changes. The stock level highly correlates with lead time and demand, so as demand and supply time increase, the average waiting time and the shortages rise. Additionally, the costs are strongly affected in this situation due to unexpected shutdown and inventory costs. The less the demand is, the less the stock level and holding costs will be. In some cases, the waiting time is not significant, close to zero. It is noteworthy that some spare parts may not respond to waiting time as the other spare parts due to different demand patterns. In this case, it is vital to consider a family group for similar patterns that can be useful in inventory management.

Table 4
The effect of changes in queuing model parameters

Time interval (k)	*parameters*						*variables*			
	λ_{sk}		τ_{sw1}		wa_{sw_1k}		cost		Δcost (%)	
	s=1	s=7	s=1	s=7	s=1	s=7	s=1	s=7	s=1	s=7
1	20	10	100	60	0.03	0.00	9.200E+05	5.000E+04	-0.26	-0.96
	40	25	120	75	0.13	0.05	6.453E+06	2.438E+06	4.16	0.95
2	12	12	90	50	0.10	0.00	2.189E+06	5.500E+05	0.75	-0.56
	35	30	125	80	0.26	0.05	1.024E+07	2.610E+06	7.19	1.09
3	22	18	80	45	0.11	0.00	3.749E+06	1.279E+06	2.00	0.02
	20	35	130	75	0.22	0.11	5.566E+06	4.962E+06	3.45	2.97
4	14	12	95	60	0.08	0.00	1.937E+06	6.500E+05	0.55	-0.48
	30	40	135	75	0.21	0.06	7.503E+06	3.813E+06	5.00	2.05
5	17	14	90	60	0.10	0.00	2.421E+06	4.500E+05	0.94	-0.64
	25	26	135	100	0.19	0.12	5.933E+06	4.411E+06	3.75	2.53

Additionally, Fig. 6 illustrates changes in waiting time over the planning horizon. It can be seen that as the demand increases, the waiting time and total costs increase since the stock level in central warehouses cannot cover the demand of local warehouses.

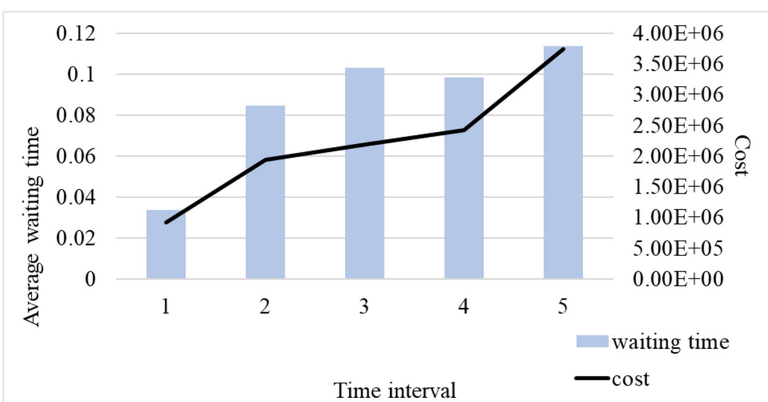
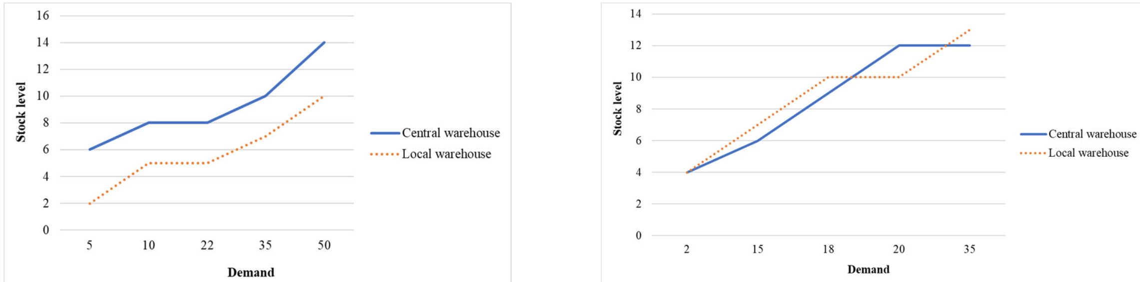


Fig. 6. The average waiting time in the central warehouse and cost vs. time interval

The multi-period planning impacts the stock levels since the supply is affected. The fluctuations are shown for the rotary pump and ball valve in Fig. 7 in interval one. Given the rotary pump, the stock level increases when the demand rises. Consequently, the total cost roughly increases with the demand, which is crucial to guarantee the minimum shortage. The stock level at the central and local warehouses rises as the demand, i.e., failure rate, increases, but this effect is stronger for the rotary pump. Another reason for rising the stock level by demand can be justified for obtaining a stable performance whole the chain. The repair time may be higher for achieving an optimal quality that can affect the stock levels.



(a)Rotary pump (b)Ball valve

Fig.7: Stock levels in warehouses

In addition to previous discussions, some managerial insights are presented below:

- The demand fluctuation, i.e,. the failure rate, affects the optimal number of intervals. More changes cause a higher number of planning intervals to smooth the demand for more accurate approximation.
- Industries usually use the repair costs based on a predetermined value, but using an effective repair cost by a performance-based policy can enhance the repair quality and optimize the repair costs; e.g., the payment can be proper to repair performance.
- Repair and supply time are two significant factors that affect the stock level. The higher value of these two factors leads to a higher probability of shortages which increases costs.
- Stock levels highly affect inventory costs and shortages, so the integrated supply and repair decisions improve the number of orders and optimize the total cost. An optimized tradeoff between the stock level and cost is crucial.
- Each spare part has a specific characteristic; therefore, the inventory management policies should be compatible with the demand pattern, supply limitation, and working conditions.
- Notably, processing the data is mandatory before feeding the model since some parts of the data may include noisy observations which cause errors in results.

6. Conclusion and future research opportunity

This research develops a model for planning a repairable spare part supply chain network considering the performance evaluation in warehouses which is formulated by queuing model. A stepwise linearisation approach is used to approximate the uncertain demand. Due to the complexity of the inventory management model (METRIC), a heuristic algorithm is used which solves the model in four stages by initializing the parameters and evaluating the performance of the inventory model in each stage so that the iteration continues until it meets the termination condition. A genetic algorithm is used to optimize the solution in which a priority-based algorithm is implemented to deal with the edge and node network of the supply chain. Indeed, this method assigns the nodes to each other by minimizing the transportation cost between them. The developed model determines the optimal number of intervals to obtain the almost linearized demand over each time interval, the optimal stock level in warehouses, spare part order assignment to suppliers, and the assignment of failed equipment to repair centers. The case study of the oil industry is used to validate the model's applicability. The model considers the repair constraints such as repair expertise, capacity, and repair time. Also, back ordering is formulated through the queuing model which obtains a criterion for the performance evaluation.

The findings are described in the following: 1) the number of intervals over the planning horizon affects the costs significantly due to demand estimation in each interval, which differs by fluctuating the length of intervals, 2) demand fluctuations impact costs, so demand forecasting prominently enhances the planning accuracy and optimize the stock level. 3) repair probability and capacity strongly correlate with order planning and stock level; moreover, shortages and unexpected shutdowns are affected, 4) the performance-based evaluation enhances the processes and optimizes the costs; e.g., the repair cost based on the availability enables the companies to achieve the maximum lifetime from the repaired equipment besides optimizing payment to repair centers. 5) Filtering the data is crucial when discussing the model since it affects the solutions. It should be pointed out that the weight of under-study items should be significant so that the optimization makes sense. The weight can be either value-based or volume-based. Future research can focus on various replenishment policies, formulating the performance in repair centers, considering the network design decisions, and developing the solution procedures through artificial intelligence or data-driven methods. Moreover, it is precious to discuss the reasoning process which can apply the effects of existing attributes and make the model more practical.

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Appendix

#NO	Operational bases	Warehouse		Repair centers	Demand		Cost	Optimal time interval	Stock level	
		Central	local		Rotary pump	Ball valve			Rotary pump	Ball valve
1	10	7	5	3	10	5	1.03E+05	1	2	1
2	15	2	4	3	12	7	1.80E+05	2	5	2
3	20	2	5	3	15	9	2.06E+05	3	4	2
4	22	3	7	5	20	10	3.23E+05	3	10	3
5	25	3	10	5	22	10	4.35E+05	3	12	4
6	27	3	12	5	25	12	2.00E+06	3	15	3
7	30	5	15	7	25	15	2.13E+06	5	15	5
8	32	5	17	7	25	17	2.52E+06	5	10	5
9	35	5	20	7	25	20	2.70E+06	5	10	3
10	40	7	22	10	25	25	2.71E+06	5	10	2
11	45	7	23	10	30	25	3.13E+06	6	14	4
12	50	7	25	10	35	27	3.34E+06	6	15	4
13	52	10	28	12	38	30	4.01E+06	6	15	8
14	54	10	30	12	40	32	4.52E+06	6	18	6
15	57	10	32	12	40	35	4.80E+06	7	22	10