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Neutrosophic multivariate EWMA control chart

Wibawati[a], Muhammad Ahsan[a*] and Hidayatul Khusna[a]

[a]*Department of Statistics, Institut Teknologi Sepuluh Nopember, Surabaya 60111, Indonesia*

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A B S T R A C T

The MEWMA chart is one of the traditional multivariate charts which are widely employed in inspecting the quality of manufacturing and services. This chart is created through monitoring the small shifts of mean vectors of variable quality characteristics. Often in practice, the measurement of a quality characteristic produces uncertain, incomplete values, so that ambiguous numbers are obtained. In this condition, a neutrosophic-based control chart can overcome the problem resulting from the ambiguous data. The paper's objective is to construct a new multivariate monitoring scheme based on a neutrosophic chart, namely the neutrosophic Multivariate EWMA (NMEWMA). Furthermore, the performance of the new multivariate monitoring scheme is evaluated in detecting process shifts employing the Average Run Length (ARL) and Standard Deviation Run Length (SDRL). This control chart is an innovation in the quality monitoring of uncertain data. The research result obtained indicates that the NMEWMA chart performs better than the MEWMA in finding the small mean shifts as well as in the real case application.

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1. Introduction

One of the methods which are used to discover process shifts is the control chart. This technique is extensively utilized in industry to observe the manufacturing process. The multivariate charts that are commonly used in the manufacturing process include T^2 Hotelling (Chong et al., 2019; Tiryaki & Aydin, 2022), MCUSUM (Xie et al., 2021; Imran et al., 2022), and MEWMA (Ajadi et al., 2021). In the service industry, one of the multivariate charts that is extensively applied is the MEWMA chart, which is designed to investigate the process mean vector and is more efficient for detecting small shifts (Montgomery, 2020). The MEWMA control chart involves information on current and previous observations and is an expansion of the EWMA univariate chart. While the traditional MEWMA chart is appropriate when the observations are correct, certain, or precise, in practice, the results of observations or measurements of quality characteristics often produce ambiguous data. In this situation, the use of conventional charts is not appropriate. When there is ambiguity or randomness in the observed characteristics, the traditional charts are not appropriate to use. In case the observations or parameters studied are vague or have ambiguity, control charts based instead on fuzzy logic are a suitable choice for monitoring the process. Zadeh discusses how the fuzzy-logic method has been widely used in the unpredictability of a situation (Zadeh, 2005). Senturk and Erginel (2009) state that "Human judgments, evaluations, and decisions are included in observations, and a numerical random variable of a manufacturing process consists of the variability due to the measurement devices, environmental conditions, or human subjectivity". The variables cause ambiguity in the measurement system. As a result, fuzzy charts have generally been used in situations where there is uncertainty. According to Khademi and Amirzadeh, "fuzzy data lies in the present manufacturing and service process" (Khademi & Amirzadeh, 2014), so many researchers have concentrated on such control charts, such as (Khan et al., 2018; Rowlands & Wang, 2000). Several actual examples of the implementation of fuzzy control charts involve the manufacture of synthetic buttons by a clothing business in Turkey, the creation of piston rings for automobile engines through a forging process to ensure statistical control of the rings' interior diameter, and a food industry operation in Pakistan that fills cooking oil containers. Smarandache explained that the fuzzy approach is a part of neutrosophic analysis (Smarandache, 2010). Several fundamental works on neutrosophic statistics are

* Corresponding author.
E-mail address: muh.ahsan@its.ac.id (M. Ahsan)

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cited in (Smarandache, 2014). Neutrosophic statistics are applicable when the samples or parameters are fuzzy, inexact, indecisive, or vague, whereas the classical statistics method assumes that all data observations are determined, precise, and certain. For the analysis of imprecise data, neutrosophic statistics are more powerful than conventional statistics.

Several authors focused on the development of a control chart based on neutrosophic sets. They performed a rock study based on neutrosophic statistics (NS) and they present the efficacy of these statistics (Chen, Ye, & Du, 2017; Chen, Ye, Du, et al., 2017). Another researcher suggested the $\bar{X}$ chart based on the neutrosophic exponentially weighted moving average (NEWMA) to monitor quality under an uncertain environment (Aslam et al., 2019). Aslam et al. developed variable charts and neutrosophic-based attributes. Aslam and Khan (2019) suggested the $\bar{X}$ chart by using neutrosophic statistics. Using the NS, both variable and attribute charts were proposed by Aslam, employing resampling (Aslam, 2019). This chart contains two statistics and two pairs of control limits (LCL_L, UCL_L) and (LCL_U, UCL_U). However, so far, the development of neutrosophic control charts has been limited. Several researchers have developed such control charts based on univariate cases, such as NEWMA (Aslam et al., 2019), neutrosophic control charts S (Z. Khan, Gulistan, Chammam, et al., 2020), and neutrosophic mean deviation (Khan, Gulistan, Hashim, et al., 2020). Recently, for univariate control charts, researchers have suggested monitoring scheme with X-Bar chart by employing the Neutrosophic-Based Generalized Multiple Dependent State Sampling (Khan et al., 2022), Neutrosophic Maxwell Distribution based chart (Shah et al., 2023), and Moving average control chart with neutrosophic statistics (Aslam et al., 2023). Meanwhile, cases of multivariate quality characteristics require handling with multivariate neutrosophic control charts. When used to test the mean of two populations, neutrosophic T^2 Hotelling statistics are more sensitive compared to the conventional T^2 Hotelling statistics. A neutrosophic T^2 Hotelling chart has been developed based on the neutrosophic T^2 Hotelling statistics, and used in investigating abnormality in the glass manufacturing process (Wibawati et al., 2022), while other researchers also proposed the Multivariate T^2 Chart for Neutrosophic Data applied to the chemical sector (Saritha & Varadharajan, 2023). Neutrosophic control charts, on average, are more sensitive than conventional control charts. Meanwhile, Hotelling's T^2 control chart is ineffective for small process shifts, whereas the MEWMA control chart can handle this problem.

According to the background, this research focuses on developing a neutrosophic-based multivariate EWMA chart that can overcome the problem of ambiguous data. The proposed chart is named as NMEWMA and its ability to observe process shifts is investigated using the Average Run Length (ARL) and Standard Deviation Run Length (SDRL). The suggested chart is used to track data from the glass manufacturing process.

2. Materials and Methods

The Multivariate EWMA (MEWMA) chart was first suggested by Lowry et al. MEWMA charts perform well for monitoring a small shift in the mean process (Lowry et al., 1992). Let $\boldsymbol{X_i}, i = 1,2,3, \dots, n,$, be a $p \times 1$ random vector and follow a p-variate normal distribution, $\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \Sigma)$. The MEWMA control chart statistics are stated as follows:

$$\boldsymbol{Z}_i = \lambda \boldsymbol{X}_i + (1-\lambda)\boldsymbol{Z}_{i-1} \quad (1)$$

where $0 < \lambda \leq 1$,

$$T_i^2 = \boldsymbol{Z}_i' \Sigma_{\boldsymbol{Z}_i}^{-1} \boldsymbol{Z}_i \quad (2)$$

with $\boldsymbol{\Sigma}_{\boldsymbol{Z}_i} = \frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\boldsymbol{\Sigma}$.

The samples are stated as control if, for all i, $T_i^2 < H$; if at least one observation falls beyond the control limit, the process is called out-of-control. MEWMA control limits for some smoothing parameters (λ) and the number of quality characteristics (p) are presented in Table 1, which contains the ARL performance for MEWMA for various values of λ for quality characteristics (p) equal to 2, 4, 6, 10, and 15. The control limit (H) is estimated to produce an in-control ARL_0 = 200 (Montgomery, 2020).

Table 1
MEWMA control limits

p	λ							
	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.8
2	7.35	8.64	9.65	10.08	10.31	10.44	10.52	10.58
4	11.22	12.73	13.87	14.34	14.58	14.71	14.78	14.85
6	14.60	16.27	17.51	18.01	18.26	18.39	18.47	18.54
10	20.72	22.67	24.07	24.62	24.89	25.03	25.11	25.17
15	27.82	30.03	31.59	32.19	32.48	32.63	32.71	32.79

The design of the suggested chart is developed according to the traditional MEWMA chart through the following steps:

Let $\widetilde{X_N} \in [X_L, X_U]$ be a neutrosophic vector random variable with a sample size $n_N \in [n_L, n_U]$, where X_L and X_U are the minimum and maximum observations, respectively. The neutrosophic vector random variable can be written as (Aslam & Arif, 2020):

$$\tilde{X}_N \epsilon \left[\begin{bmatrix} x_{11L} & \cdots & x_{1kL} & \cdots & x_{1pL} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1L} & \cdots & x_{nkL} & \cdots & x_{npL} \end{bmatrix}, \begin{bmatrix} x_{11U} & \cdots & x_{1kU} & \cdots & x_{1pU} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1L} & \cdots & x_{nkU} & \cdots & x_{npU} \end{bmatrix}\right]$$

By adopting the traditional MEWMA, we define the vectors of NMEWMA:

$$\widetilde{\boldsymbol{Z}}_{Ni} = \lambda \widetilde{\boldsymbol{X}_{Ni}} + (1-\lambda)\widetilde{\boldsymbol{Z}}_{N(i-1)}, \tag{3}$$

where initially $\widetilde{Z_0} = \tilde{\mu}_0$, $\widetilde{\boldsymbol{Z}}_{Ni} \in [Z_{NL}, Z_{NU}]$ and $0 < \lambda \leq 1$ is the smoothing constant, and the NMEWMA statistics are defined as:

$$\tilde{T}^2_{Ni} = \widetilde{\boldsymbol{Z}}'_{Ni} \widetilde{\sum}^{-\boldsymbol{1}}_{N\widetilde{\boldsymbol{Z}}_{\boldsymbol{i}}} \widetilde{\boldsymbol{Z}}_{\boldsymbol{Ni}} \tag{4}$$

The process sends an out-of-control (OOC) if

$$\tilde{T}^2_{Ni} > \widetilde{UCL_N},$$

where $\widetilde{UCL}_N \in \left(\widetilde{UCL}_{NL}, \widetilde{UCL}_{NU}\right)$.

The ARL metrics are employed to assess the suggested chart's performance. The following are the detailed performance evaluation steps for this control chart:

1. Generating n data from a normal multivariate neutrosophic distribution, $\widetilde{\boldsymbol{X}}N_p(\widetilde{\boldsymbol{\mu}}, \tilde{\Sigma})$, $\widetilde{\boldsymbol{\mu}} \epsilon (\boldsymbol{\mu}_L, \boldsymbol{\mu}_U)$, $\widetilde{\boldsymbol{\Sigma}} \epsilon (\boldsymbol{\Sigma}_L, \boldsymbol{\Sigma}_U)$, where $\widetilde{\boldsymbol{\mu}} = \binom{0}{0}$; $\widetilde{\boldsymbol{\Sigma}} = \begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix}$; $r = 0.5,\ r = 0.25, r = 0.5,$ and $r = 0.9$.
2. Select the value of $\lambda = 0.1$, 0.2, and 0.3, and $\widetilde{UCL}_N$ depends on the number of $p = 2$, 3, and 4.
3. Calculate the statistics of NMEWMA ($\widetilde{\boldsymbol{Z}}_{Ni}$).
4. Calculate ARL_0 NMEWMA and $SDRRL_0$ NMEWMA for 10,000 replications.
5. Repeat step (1) for the level shift of the parameter $\widetilde{\boldsymbol{\mu}} = \widetilde{\boldsymbol{\mu}} + \widetilde{\boldsymbol{\delta}}$, for 10,000 replications.
   The level shift of the mean vector is $\widetilde{\boldsymbol{\mu}} = \widetilde{\boldsymbol{\mu}} + \widetilde{\boldsymbol{\delta}}$; $\widetilde{\boldsymbol{\delta}} = (0.1, 0{,}1)'; (0.2{,}0.2)'; \ldots, (2{,}2)'$.
6. Calculate ARL_1 NMEWMA and $SDRL_1$ NMEWMA.

3. Discussion

3.1. Performance of Neutrosophic MEWMA (NMEWMA) Control Chart

The neutrosophic MEWMA control chart's performance was investigated by employing the ARLs and SDRLs, and using various process shifts, parameter smoothing (λ), the number of characteristics (p), and correlation of the quality characteristics (r). The NMEWMA's performance is compared to that of the classical MEWMA. In this simulation, several scenarios are given for parameter smoothing (λ), including 0.1, 0.2 and 0.3. The suggested chart is made up of two charts, NMEWMA lower ($NMEWMA_L$) and NMEWMA upper ($NMEWMA_U$). The value of ARL and SDRL is obtained from the first out-of-control signal from $NMEWMA_L$ or $NMEWMA_U$ for each level shift as the run length (RL) for 10,000 replications. This section compares the proposed chart's performance with MEWMA chart. The comparison of various shifts $\left(\widetilde{\boldsymbol{\delta}} = (0.1, 0{,}1)'; (0.2{,}0.2)'; \ldots, (2{,}2)'\right)$ with $r = 0.25$; $p = 2$ and $\lambda = 0.3$ is shown in Fig. 1 and Table A (Appendix).

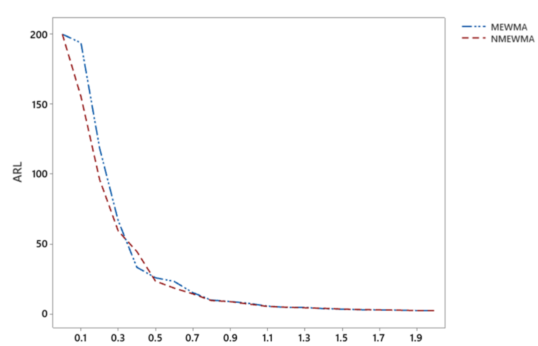
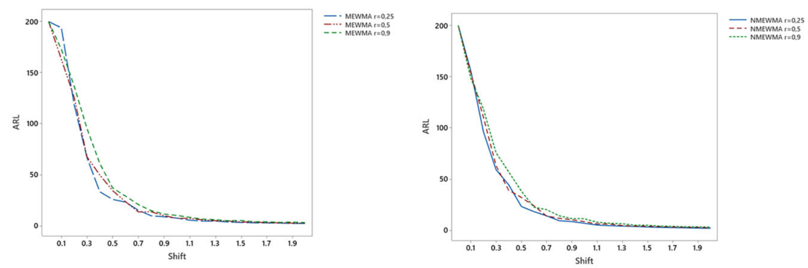


Fig. 1. ARL of NMEWMA and MEWMA with $r = 0.25$; $p = 2$, and $\lambda = 0.3$.

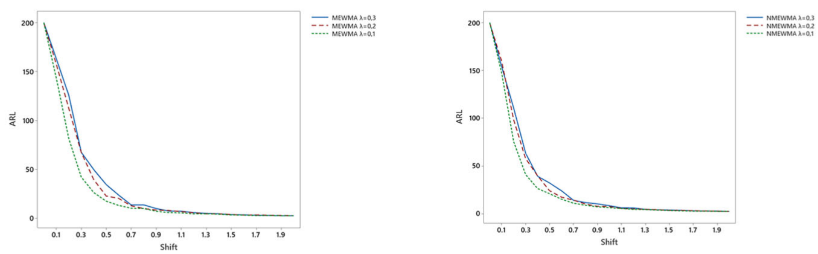
Fig. 1 shows that the ARLs produced by NMEWMA are smaller than MEWMA for small shifts (0.1–0.6). The value of ARL_0 equals to 200, and the value of $SDRL_0$ NMEWMA is close to 200 for high correlation ($r = 0.9$). This chart indicates that, for larger shifts, NMEWMA is quicker to detect an out-of-control process. The other schemes of quality characteristics correlation (r) are shown in Fig. 2. There are three schemes of correlation, $r = 0.25, r = 0.5,$ and $r = 0.9$, representing small, medium, and high correlation, respectively.



(a) (b)

Fig. 2. (a) ARL MEWMA; (b) ARL NMEWMA (p = 2 and λ = 0.3).

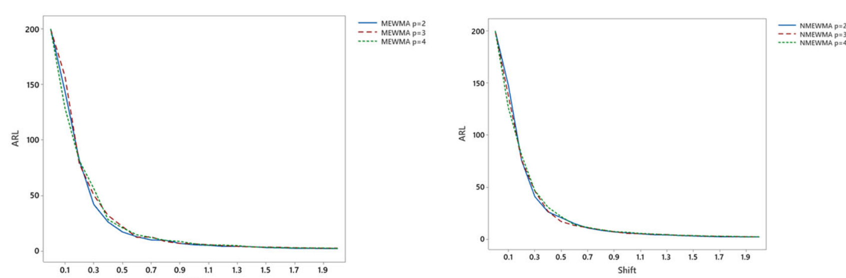
Based on Fig. 2, Table A, and Table D, among the scheme of correlations, the performance of NMEWMA is more sensitive than the MEWMA charts. For both NMEWMA and MEWMA, the higher the correlation in the chart, the more sensitive it is. The suggested chart's ability in detecting shift is also compared with the MEWMA chart for various values of parameter smoothing (λ) and the number of quality characteristics (p), as shown in Fig. 3 and Fig. 4.



(a) (b)

Fig. 3. (a) ARL MEWMA; (b) NMEWMA (r = 0.5 and p = 2).

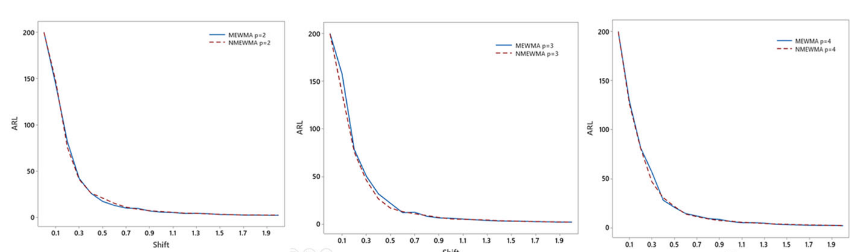
The efficiency of the NMEWMA and MEWMA control charts is similar, as illustrated in Fig. 3, Table B, and Table E, where the smaller λ, the more sensitive is the performance of NMEWMA and MEWMA for small-to-medium process mean shifts. This is because the control chart's ARL shrinks at small smoothing parameters/weights. For the larger shifts, both of these charts have the same performance.



(a) (b)

Fig. 4. (a) ARL MEWMA; NMEWMA (for p = 2,3,4 ; r = 0.5; λ = 0.1).

According to Fig. 4, Fig. 5, and Table C, it is visible that MEWMA and NMEWMA have the same performance for the various numbers of quality characteristics.



(a) (b) (c)

Fig. 5. ARL MEWMA and ARL NMEWMA; r = 0.5 and λ = 0.1. (a) p = 2; (b) p = 3; (c) p = 4.

3.2. Application of Neutrosophic MEWMA (NMEWMA) Control Chart

The NMEWMA chart was used on simulated data to assess how well the control chart detects out-of-control observations on modest and large process mean shifts. The data was generated by following a normal multivariate distribution of 50 samples with a number of quality characteristics equal to 4. In this simulation, the data is separated into two parts. The first 80% of the initial sample had a normal multivariate distribution with a neutrosophic mean vector $\widetilde{\boldsymbol{\mu}}_{N0}$ and a covariance matrix $\widetilde{\boldsymbol{\Sigma}}_{N0}$. The next 20% of samples had a normal multivariate distribution with a mean vector $\widetilde{\boldsymbol{\mu}}_{N1} = \widetilde{\boldsymbol{\mu}}_{N0} + \delta\widetilde{\boldsymbol{\Sigma}}_{N0}$, where $\delta \neq 0$. These samples were classified as either in-control or out-of-control. In this simulation, the in-control sample is generated from a normal multivariate distribution with parameter $\widetilde{\boldsymbol{\mu}}_{N0} = \widetilde{\mathbf{0}}$. Meanwhile, the out-of-control sample is generated from a normal multivariate distribution with parameter $\widetilde{\boldsymbol{\mu}}_{N,}$ using $\delta = 0.75$ for small process shifts and $\delta = 3$ for large process shifts, respectively. The smoothing parameter/weight λ is equal to 0.1. Based on the synthetic data, the results of the NMEWMA chart for small process shifts are depicted in Fig. 6.

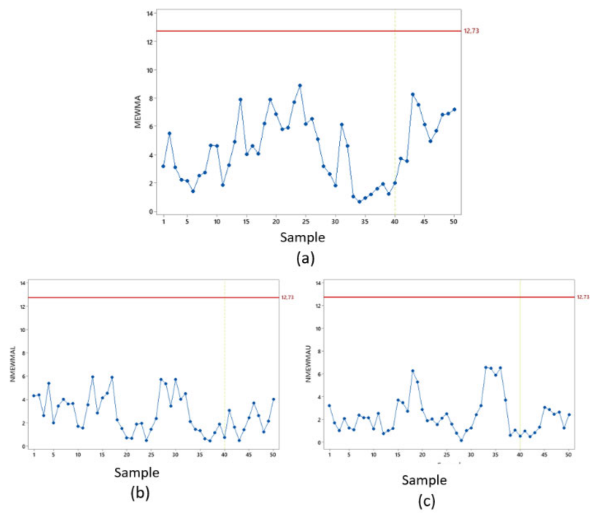


Fig. 6. MEWMA (a) NMEWMA lower, (b) upper, (c) for small shifts

Fig. 6 shows that both the MEWMA and the NMEWMA control charts indicate that the mean process has been controlled. This is because, in the resulting control chart, there are no out-of-control points. Therefore, based on these simulation data, both the MEWMA control chart and the NMEWMA control chart are insensitive in identifying small process mean changes. The results of utilizing the NMEWMA chart for the simulated observation for large process shifts are shown in Fig. 7. The neutrosophic MEWMA (NMEWMA) control chart application uses data from the Quality Assurance division for glass production quality characteristics in 2022. Four quality characteristics are used: zebra left ($\tilde{X}_{1N}$), zebra right ($\tilde{X}_{2N}$), cutter line ($\tilde{X}_{3N}$), and edge distortion (X_{4N}). The zebra's left and right targets are 60 mm, the cutter line is 115 mm, and the edge distortion is 40 mm. Table 2 displays data from the four quality characteristics.

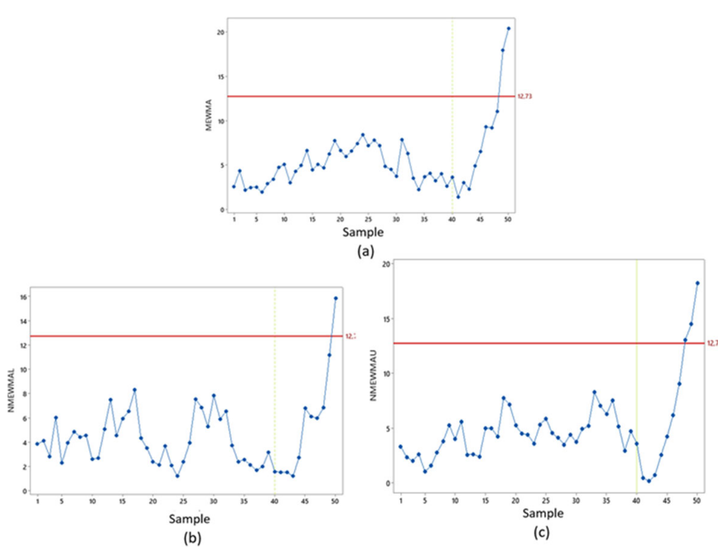


Fig. 7. MEWMA (a), NMEWMA lower (b), and upper (c) for large shifts.

Table 2
Neutrosophic data of the four quality characteristics.

Subgroup	Zebra Left		Zebra Right		Cutter line		Edge distortion	
	X_{1NL}	X_{1NU}	X_{2NL}	X_{2NU}	X_{3NL}	X_{3NU}	X_{4NL}	X_{4NU}
1	59	61	60	62	140	158	25	30
2	60	62	59	60	170	198	72	73
3	60	60	60	62	139	145	28	30
4	60	61	60	64	122	166	50	52
5	58	62	60	63	115	145	18	21
6	60	61	60	65	112	152	15	37
7	60	60	60	60	136	143	30	37
8	60	61	60	61	141	149	34	38
9	60	62	60	61	120	155	21	26
10	60	60	60	60	136	143	30	37
11	59	60	60	61	128	144	34	40
12	64	64	62	63	117	145	20	30
13	60	60	60	60	136	143	30	37
14	58	60	61	61	118	160	38	48
15	60	60	58	60	106	155	28	38
16	60	60	60	62	116	155	23	30
17	60	61	60	61	141	149	34	38
18	60	60	60	62	139	145	28	30
19	60	60	60	60	136	143	30	37
20	61	62	60	63	137	161	32	39
21	57	60	60	62	131	146	42	55
22	61	62	60	65	130	151	31	49
23	60	61	60	61	141	149	34	38
24	61	62	59	64	139	151	33	42
25	60	60	58	59	133	162	36	38
26	61	66	61	64	123	138	34	37
27	60	60	60	62	139	145	28	30
28	59	60	57	59	143	166	17	30
29	60	61	60	61	141	149	34	38
30	56	59	57	59	140	142	20	28
31	59	60	60	60	136	137	29	37
32	58	61	61	61	141	149	24	30
33	60	60	60	62	139	145	28	30

The statistical value of the neutrosophic MEWMA (NMEWMA) is obtained (shown in Table 3), and the control limit $UCL_{NMEWMA} = 12.73$. Based on the performance evaluation, when p equals 4, the NMEWMA is more sensitive for the parameter smoothing. In this case, λ equals 0.1, so in this case study, we use $\lambda = 0.1$. Fig. 8 shows the findings of the neutrosophic MEWMA (NMEWMA) control chart.

Table 3
Statistics of neutrosophic MEWMA (NMEWMA) lower and upper.

Subgroup	T^2_{NL}	T^2_{NU}	Subgroup	T^2_{NU}	T^2_{NL}
1	1.83	2.66	18	0.52	2.65
2	9.36	12.87	19	0.84	1.94
3	6.34	5.65	20	0.26	1.83
4	6.73	8.69	21	1.14	1.90
5	2.99	6.29	22	3.02	1.77
6	1.37	9.58	23	2.36	1.24
7	0.72	4.33	24	3.54	1.07
8	0.50	2.45	25	1.45	2.14
9	0.38	2.68	26	4.14	2.82
10	0.24	0.74	27	2.50	1.85
11	0.63	0.12	28	0.59	2.55
12	3.02	1.02	29	0.62	2.12
13	2.62	0.65	30	0.62	3.97
14	3.96	0.21	31	1.88	3.09
15	5.26	0.41	32	1.93	3.30
16	6.74	0.41	33	1.60	3.42
17	4.31	0.28			

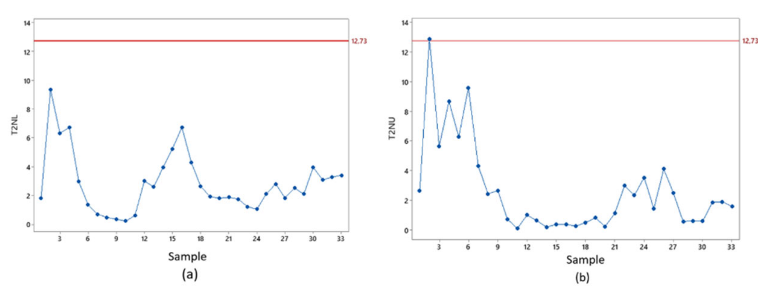


Fig. 8. (a) Neutrosophic MEWMA lower; (b) Neutrosophic MEWMA upper.

Fig. 8 shows the MEWMA neutrosophic control chart (NMEWMA), indicating that the mean of the glass production process has not been controlled. The NMEWMA control chart produces 1 point that is out of the UCL_{NU} control limit, namely the upper NMEWMA control chart. The comparison of the neutrosophic MEWMA control chart with the conventional MEWMA control chart is shown in Fig. 9. By using NMEWMA, the second observation is detected as an out-of-control observation, while, by using MEWMA, the second observation is still detected as in-control. This is because NMEWMA involves two control charts with two control limits, meaning that this chart is more sensitive.

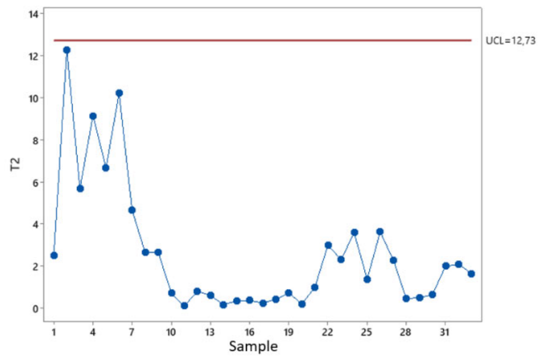


Fig. 9. MEWMA control chart.

5. Conclusions

This work develops a new Neutrosophic Multivariate EWMA (NMEWMA) control chart. The performance of the proposed NMEWMA chart was assessed using the ARL and SDRL. The performance of the proposed NMEWMA chart is measured using various process shifts (δ), parameter smoothing correlation of quality characteristics (r). Based on ARL and SDRL, NMEWMA is more sensitive than MEWMA. Control charts are used for both simulation and real data. Based on the real-life application and simulation studies, the proposed NMEWMA chart shows better performance when compared to the conventional chart. The Neutrosophic MEWMA for the subgroup will be investigated in other performance analyses and also for different uncertainty levels for future research.

Conflicts of Interest

The authors of the article declare that they have no potential conflicts of interest concerning the research, authorship, and/or publication of this article.

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References

Ajadi, J. O., Zwetsloot, I. M., & Tsui, K.-L. (2021). A new robust multivariate EWMA dispersion control chart for individual observations. *Mathematics*, *9*(9), 1038.

Aslam, M. (2019). Control chart for variance using repetitive sampling under neutrosophic statistical interval system. *IEEE Access*, *7*, 25253–25262.

Aslam, M., AL-Marshadi, A. H., & Khan, N. (2019). A new X-bar control chart for using neutrosophic exponentially weighted moving average. *Mathematics*, *7*(10), 957.

Aslam, M., & Arif, O. H. (2020). Multivariate analysis under indeterminacy: An application to chemical content data. *Journal of Analytical Methods in Chemistry*, *2020*.

Aslam, M., Khan, K., Albassam, M., & Ahmad, L. (2023). Moving average control chart under neutrosophic statistics. *AIMS Mathematics*, *8*(3), 7083–7096.

Aslam, M., & Khan, N. (2019). A new variable control chart using neutrosophic interval method-an application to automobile industry. *Journal of Intelligent & Fuzzy Systems*, *36*(3), 2615–2623.

Chen, J., Ye, J., & Du, S. (2017). Scale effect and anisotropy analyzed for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics. *Symmetry*, *9*(10), 208.

Chen, J., Ye, J., Du, S., & Yong, R. (2017). Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers. *Symmetry*, *9*(7), 123.

Chong, N. L., Khoo, M. B., Haq, A., & Castagliola, P. (2019). Hotelling's T2 control charts with fixed and variable sample sizes for monitoring short production runs. *Quality and Reliability Engineering International*, *35*(1), 14–29.

Imran, M., Sun, J., Zaidi, F. S., Abbas, Z., & Nazir, H. Z. (2022). Multivariate cumulative sum control chart for compositional data with known and estimated process parameters. *Quality and Reliability Engineering International*, *38*(5), 2691–2714.
Khademi, M., & Amirzadeh, V. (2014). Fuzzy rules for fuzzy $ overline {X} $ and $ R $ control charts. *Iranian Journal of Fuzzy Systems*, *11*(5), 55–66.
Khan, M. Z., Khan, M. F., Aslam, M., Niaki, S. T. A., & Mughal, A. R. (2018). A fuzzy EWMA attribute control chart to monitor process mean. *Information*, *9*(12), 312.
Khan, N., Ahmad, L., & Aslam, M. (2022). Monitoring Using X-Bar Control Chart Using Neutrosophic-Based Generalized Multiple Dependent State Sampling with Application. *International Journal of Computational Intelligence Systems*, *15*(1), 73.
Khan, Z., Gulistan, M., Chammam, W., Kadry, S., & Nam, Y. (2020). A new dispersion control chart for handling the neutrosophic data. *IEEE Access*, *8*, 96006–96015.
Khan, Z., Gulistan, M., Hashim, R., Yaqoob, N., & Chammam, W. (2020). Design of S-control chart for neutrosophic data: An application to manufacturing industry. *Journal of Intelligent & Fuzzy Systems*, *Preprint*, 1–9.
Lowry, C. A., Woodall, W. H., Champ, C. W., & Rigdon, S. E. (1992). A multivariate exponentially weighted moving average control chart. *Technometrics*, *34*(1), 46–53.
Montgomery, D. C. (2020). *Introduction to statistical quality control*. John Wiley & Sons.
Rowlands, H., & Wang, L. R. (2000). An approach of fuzzy logic evaluation and control in SPC. *Quality and Reliability Engineering International*, *16*(2), 91–98.
Saritha, M., & Varadharajan, R. (2023). *Multivariate Hotelling-T 2 Control Chart for Neutrosophic Data*.
Senturk, S., & Erginel, N. (2009). Development of fuzzy X∼-R∼ and X∼-S∼ control charts using α-cuts. *Information Sciences*, *179*(10), 1542–1551.
Shah, F., Aslam, M., & Khan, Z. (2023). New Control Chart Based On Neutrosophic Maxwell Distribution with Decision Making Applications. *Neutrosophic Sets and Systems*, *53*(1), 18.
Smarandache, F. (n.d.). Introduction to Neutrosophic Statistics. 2014. *Craiova: Sitech & Education Publishing*.
Smarandache, F. (2010). Neutrosophic logic-a generalization of the intuitionistic fuzzy logic. *Multispace & Multistructure. Neutrosophic Transdisciplinarity (100 Collected Papers of Science)*, *4*, 396.
Smarandache, F. (2014). Introduction to Neutrosophic Statistics. *ArXiv*, *abs/1406.2000*.
Tiryaki, S., & Aydin, A. (2022). Multivariate Hotelling T2 Control Chart for Monitoring Some Quality Characteristics in Medium Density Fiberboard Manufacturing Process. *Drvna Industrija*, *73*(1), 35–46.
Wibawati, W., Ahsan, M., Khusna, H., Qori'atunnadyah, M., & Udiatami, W. (2022). Multivariate Control Chart based on Neutrosophic Hotelling T2 Statistics and Its Application. *JTAM*, *Vol 6, No 1*. https://doi.org/10.31764/jtam.v6i1.5442
Xie, F., Sun, J., Castagliola, P., Hu, X., & Tang, A. (2021). A multivariate CUSUM control chart for monitoring Gumbel's bivariate exponential data. *Quality and Reliability Engineering International*, *37*(1), 10–33.
Zadeh, L. A. (2005). Toward a generalized theory of uncertainty (GTU)—an outline. *Information Sciences*, *172*(1–2), 1–40.

Appendix A

Table A
The ARLs of MEWMA and NMEWMA when $p = 2$ and $\lambda = 0.3$ ($ARL_0 = 200$)

Shift	$r = 0.25$		$r = 0.5$		$r = 0.9$	
(δ)	MEWMA	NMEWMA	MEWMA	NMEWMA	MEWMA	NMEWMA
0.1	193.8	155.3	163	153.7	172.8	148.4
0.2	118.5	96	125.6	110.4	136.6	118.4
0.3	66.4	59.1	67.3	63.1	95	75.7
0.4	33.1	44.4	49.5	39	60.7	57
0.5	25.6	23.2	34.2	32	36.8	38.3
0.6	23	18.2	23.4	23.8	28.9	22.4
0.7	15	14.1	13.4	13.8	20.6	20
0.8	9.6	9.3	13.5	11.5	14.6	14
0.9	8.6	8.5	9.7	10.1	11.3	11.2
1	7.3	6.8	7.2	8.2	9.9	11.1
1.1	5.3	5.1	7	6	8.1	7.7
1.2	4.4	4.5	5.6	5.9	6.3	6.7
1.3	4.4	4.1	4.8	4.4	5.8	6.2
1.4	3.5	3.7	4.4	3.9	4.7	4.8
1.5	3.1	3.2	3.5	3.7	5	4.7
1.6	2.7	2.8	3.3	3.4	3.7	3.9
1.7	2.6	2.6	2.9	2.9	3.7	3.7
1.8	2.4	2.4	2.7	2.7	3.1	3.2
1.9	2.1	2.1	2.4	2.6	3.4	3.2
2	2	2	2.4	2.2	2.9	2.8

Appendix B

Table B
The ARLs of MEWMA and NMEWMA when $p = 2$ and $r = 0.5$.

Shift	λ=0.1		λ=0.2		λ=0.3	
(δ)	MEWMA	NMEWMA	MEWMA	NMEWMA	MEWMA	NMEWMA
0	200	200	200	200	200	200
0.1	143.5	147.3	157.7	159.6	163	153.7
0.2	81.4	75.4	112.4	98.3	125.6	110.4
0.3	41.9	40.8	67.2	57.6	67.3	63.1
0.4	26	26.1	39.3	39.5	49.5	39
0.5	17.1	20.6	22.4	23.8	34.2	32
0.6	12.8	14.9	20	17	23.4	23.8
0.7	9.9	10.7	12.4	13.9	13.4	13.8
0.8	9.6	8.5	9.6	9.8	13.5	11.5
0.9	6.8	6.9	8	7.4	9.7	10.1
1	5.5	6	7.6	7.2	7.2	8.2
1.1	5.1	5.1	6.4	5.1	7	6
1.2	4.1	4.2	5.2	4.6	5.6	5.9
1.3	4.2	4	4.2	4.3	4.8	4.4
1.4	3.8	3.4	4.1	3.8	4.4	3.9
1.5	3	3	3.5	3.1	3.5	3.7
1.6	2.8	2.6	2.8	3.1	3.3	3.4
1.7	2.4	2.3	3	2.8	2.9	2.9
1.8	2.4	2.3	2.7	2.5	2.7	2.7
1.9	2.2	2.1	2.5	2.3	2.4	2.6
2	2.1	2.1	2.2	2	2.4	2.2

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