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Estimating flood catastrophe bond prices using approximation method of the loss aggregate distribution: Evidence from Indonesia

Riza Andrian Ibrahim[a], Sukono[b], Herlina Napitupulu[b], Rose Irnawaty Ibrahim[c], Muhamad Deni Johansyah[b] and Jumadil Saputra[d*]

[a]Doctoral of Mathematics Study Program, Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Jatinangor, Indonesia
[b]Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Jatinangor, Indonesia
[c]Faculty of Science and Technology, Universiti Sains Islam Malaysia, Bandar Baru Nilai, Malaysia
[d]Faculty of Business, Economics, and Social Development, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu, Malaysia

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A B S T R A C T

Losses experienced by the Indonesian government due to floods are predicted. It is because of the significance of population growth, closure of water catchment areas, and climate change in many regions in Indonesia. The government has tried to reduce the risk but faces insufficient funds. Therefore, new innovative funding sources are essential to overcome these limitations. One way to obtain it is through issuing Flood Catastrophe Bonds (FCB). Unfortunately, Indonesia has had no FCB price estimate until now. On the basis of this problem, this study aims to estimate the FCB price in Indonesia. The primary method used is the approximation method of the aggregate loss distribution. This method can compute the aggregate flood loss cumulative distribution function value faster. The FCB fair price estimation results are cheap because the risk of the instrument is significant. This significant risk is also proportional to the large return. Finally, further analysis shows that in Indonesia, the attachment point of the FCB has a relationship that is in line with the price, while the term of FCB does not. This research is expected to assist the Indonesian government in determining the fair price of FCB in Indonesia. This research can assist the investors in choosing FCB based on expected return, attachment point, and the term they want.

1. Introduction

Flooding is an event in which the land where humans live is extremely inundated by water (Sholihah et al., 2020). Generally, it is caused by an imbalance between the water volume and holding capacity (Čepienė et al., 2022). In rivers or lakes, extreme increases in water volume can occur due to heavy rains or river flows from other higher land. This extreme increase in water volume is not accompanied by sufficient capacity (Tabari, 2020). The actual water holding capacity must be managed so that it is always at least constant. However, the water-holding capacity decreases over time (Watanabe et al., 2018). On the river, many people use its banks to build settlements. It will narrow the width of the river. Then, this is exacerbated by the increasingly high silt deposition as a shipment from rivers on higher land. It causes the river to become shallower. Indonesia is a tropical country includes the country with the highest flood frequency in the world. Floods in Indonesia always occur seasonally every year. The floods directly impacted various areas of people's lives and caused trillions of losses (Pauline, 2008). Floods damage different infrastructures, pollute water supplies, cause crop failure, reduce the number of tourists, and stop economic distribution activities. For example, the Wasior flood that occurred on 4 October 2010 in Wasior, West Papua, caused damage to the airfield, residents' houses, hospitals, bridges, and several churches

* Corresponding author.
E-mail address: jumadil.saputra@umt.edu.my (J. Saputra)

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(Dalimunthe, 2018). In detail, community communication links and the electricity network were cut off. Also, the community's economic activities were paralysed (Zain et al., 2021). The flood left 158 people dead and 145 people missing. Lastly, the 2007 floods in Jakarta caused a financial loss of 5.2 trillion IDR and left at least 190,000 sick people (Wicaksono & Herdiansyah, 2019). The flood submerged about 70% of the land in Jakarta, with an average water level of four meters.

The trillions of losses experienced by the Indonesian government annually require increased attention to overcome them (Wiyanti & Halimatussadiah, 2021). It is done so that the risk of such loss can be minimised. Moreover, the frequency of flood catastrophes in Indonesia is predicted to be higher in the future in line with the significance of population growth, closure of water catchment areas, and climate change in many regions of Indonesia. Unfortunately, so far, the Indonesian government has only relied on the state budget, regional budget, and assistance from the community. It causes the government's funding capacity for catastrophe prevention to be very limited. In more detail, this loss is often commensurate with the need for the necessary countermeasures (Gurnardi & Setiawan, 2015). Therefore, new and innovative funding sources must be sought for flood catastrophe management for the government. The Indonesian government is currently reviewing the issuance of flood catastrophe bonds (FCB) to obtain new sources of funds for flood mitigation (Hengky et al., 2021; Kurniawan et al., 2021). This financial instrument is new because no entity has issued FCB in Indonesia. The review includes the mechanism and the fair pricing of FCB. This study will focus on the fair pricing of FCB in Indonesia.
Several studies on estimating general catastrophic bond prices have been carried out. Siyamah et al. (2021) estimated catastrophe bond prices using a stochastic process without jump-diffusion and geometric Brownian motion. They used the Monte Carlo and quasi-Monte Carlo methods in the simulation step. In the simulation result, the quasi-Monte Carlo method had better accuracy than the other. Chao and Zou (2018) estimated the valuation of catastrophe bonds using the CIR-Copula-POT model. The model was then simulated using the Monte Carlo method.

Nowak and Romaniuk (2013) calculated the catastrophe bond prices using a risk-free spot interest rate model, assuming catastrophe events do not depend on financial market conditions. Then, they used the Monte Carlo method to simulate it. Shao et al. (2015) estimated the earthquake catastrophe bond prices using the equilibrium approach and extreme value theory. Simulation of the model was carried out using the Monte Carlo method. Jarrow (2010) used simple-robust methods to estimate catastrophe bond prices in his research. The model had a closed solution, so it did not require a simulation method to obtain the price. Then, Vaugirard (2003) evaluated catastrophe bond prices using a simple arbitrage approach. In detail, the Monte Carlo method was used to simulate the model. Nowak and Romaniuk (2018) estimated catastrophe bond prices using the stochastic jump-diffusion process and the multi-factor Cox-Ingersoll-Ross (CIR) model. Then, the model was simulated using the Monte Carlo method. Burnecki and Giuricich (2017) applied the weak convergence of the compound renewal process to estimate catastrophe bond prices. Then, the estimation was determined by simulation using the Monte Carlo method. Ma and Ma (2013) applied a non-homogeneous compound Poisson process to estimate catastrophe bond prices. The simulation process of their model was carried out using a mixed approximation method. Then, Härdle and Cabrera (2010) calculate the earthquake catastrophe bond in Mexico using the extreme value theory. Meanwhile, there is only one research on estimating catastrophe bond prices in Indonesia. It was done by Gunardi and Setiawan (2015). They estimated Indonesia's earthquake catastrophe bond prices using the generalised extreme value distribution and CIR model. In detail, the estimation was done through simulation using the Monte Carlo method. Other researchers have never estimated FCB prices in Indonesia. The gaps from previous studies are discussed in this paragraph. From the studies that have been described previously, there has been no research that examines the estimation of FCB prices in Indonesia. Then, almost all previous studies used the Monte Carlo method for the estimation simulation, even though this method requires an iterative process with long computations. It can be a loophole for catastrophe bond pricing efficiency.

Based on the research gaps that have been carried out, this study aims to estimate the price of FCB in Indonesia using a simulation method that is computationally faster and more efficient. The simulation method used is the approximation method of the aggregate loss distribution. The method has advantages over the Monte Carlo method. The approximation method of the aggregate loss distribution does not require iterative calculations, while the Monte Carlo method requires many iterations. In other words, the approximation method of the aggregate loss distribution causes the computation to be faster and more efficient than the Monte Carlo method. This research is expected to help the Indonesian government determine the price of FCB in their country. This research can also be a reference for investors choosing FCB based on expected return, attachment point, and the term they want.

2. Materials and Methods

2.1 Materials

The data used in this study is non-zero flood catastrophe loss data in Indonesia from 2010 to 2021. The 400-sized data was obtained from the National Catastrophe Management Agency of the Republic of Indonesia (https://gis.bnpb.go.id). To analyse the data, the approaches used are parametric and numerical. Finally, to help with data computing, we use the help of software R version 4.1.2 and Scilab version 6.1.1. The two software can be obtained as open source. The R software version 4.1.2 is used for computing the exact cumulative distribution function (CDF) value of the loss aggregate. In contrast, the Scilab software version 6.1.1 is used to compute the best loss aggregate CDF approximation method selection.

2.2 Methods

The steps for estimating FCB prices in Indonesia are visually presented in Figure 1. Briefly, the explanation is as follows:

1) Determining the probability distribution of a single flood catastrophic loss in Indonesia.
   This section fits the exact distribution of single flood catastrophe loss data with several theoretical probability distributions. This fitting was carried out formally using two different tests, namely the Kolmogorov-Smirnov and Anderson-Darling tests. The probability distribution with the smallest test statistic value from the two tests is chosen.
2) Determining the intensity of flood catastrophes in Indonesia.
   This section is the design of a mathematical model for flood loss aggregate using the compound Poisson process (CPP). The flood catastrophe intensity was calculated first using Eq. (6) to apply the CPP.
3) Determining the most suitable loss aggregate CDF approximation method.
   This section contains the selection of the most suitable approximation method to represent the flood loss aggregate's CDF. Four approximation methods are considered: the approximation distributions of normal, gamma, normal-power (N-P2), and gamma inverse Gaussian (GIG). The one with the lowest root-mean-squared error (RMSE) is selected.
4) Estimating the FCB price.
   FCB prices were estimated using the present value of the expected value of principal and coupon payments.

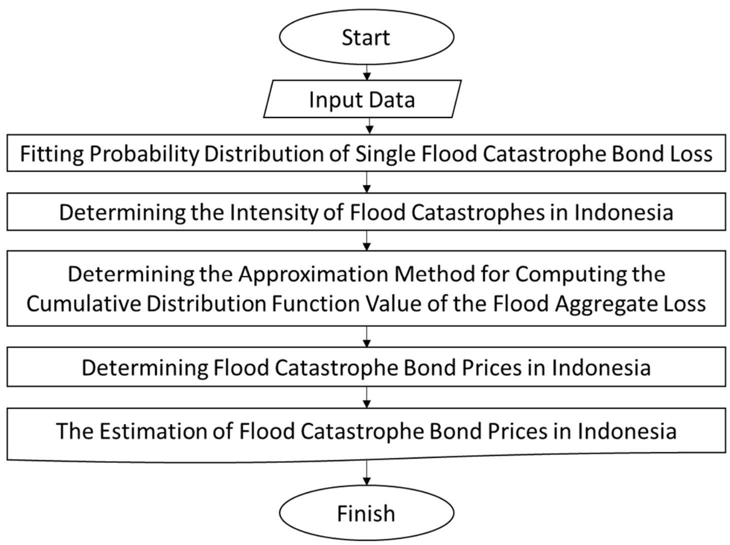


Fig. 1. The FCB Price Estimation Steps in Indonesia

2.3 The FCB Schema

Three entities fill the FCB scheme. The first entity is the sponsor. The sponsor is an agency that contains the state government, insurers, and reinsurers (Carayannopoulos & Perez, 2015). The role of the sponsor is as an insured for part or all the risk of flood catastrophe. The sponsor pays special-purpose vehicles (SPV) premium in exchange for the flood catastrophe risk. Then the second entity is the SPV. The SPV is an agency created by the sponsor to issue FCB and invest the proceeds from the sale and the premium received into short-term low-risk securities. The investment returns are then converted into (LIBOR) to immunise sponsors from risks of interest rate and default (Cummins & Weiss, 2009; Cummins, 2008). In addition, the SPV is an entity that will or will not pay claims to sponsors (Härdle & Cabrera, 2010). Finally, the third entity is the investor. Investors here act as insurers of part or all the flood catastrophe risk from sponsors. If the event that triggers the claim of FCB does not occur, investors will receive the entire principal and coupons (Pollner, 2001; Loubergé et al., 1999). However, if the opposite is true, the investor will lose the principal and coupons.

2.4 Fitting Distribution of Single Flood Catastrophe Loss

Fitting is conducted to determine the most suitable probability distribution to represent the exact probability distribution of single flood catastrophe loss data. Catastrophic losses generally have a probability distribution with a fat right tail (Ma & Ma, 2013). It is because there is a catastrophic flood event with rare extreme losses on the fat right tail. Therefore, in this study, fitting the exact probability distribution of single flood catastrophe loss data is approached on distributions with fat right tails such as exponential, log-logistic, log-normal, Pareto, and Weibull distributions. Suppose that X is a continuous random variable representing a single flood catastrophe loss. Then, suppose that $f_X(x)$ and $F_X(x)$ represent the probability density function (PDF) and the CDF (CDF) of X, respectively. The PDF and CDF of X represented by exponential, log-logistic, log-normal, Pareto, and Weibull random variables are briefly presented in Table 1.

Table 1
PDF and CDF of Exponential, Log-Logistic, Log-Normal, Pareto, and Weibull Random Variables

Random Variable	$f_X(x)$	$F_X(x)$	Parameter
Exponential	$\lambda e^{-\lambda x}$	$1-e^{-\lambda x}$	$\lambda > 0$
Log-Logistic	$\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\left\{1+\left(\frac{x}{\beta}\right)^{\alpha}\right\}^{-2}$	$\left\{1+\left(\frac{\beta}{x}\right)^{\alpha}\right\}^{-1}$	$\alpha > 0$, $\beta > 0$
Log-Normal	$\frac{e^{-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2}}{x\sigma\sqrt{2\pi}}$	$\Phi\left(\frac{\ln(x)-\mu}{\sigma}\right)^*$	$\sigma > 0$, $\mu \in \mathbb{R}$
Pareto	$\frac{\alpha\beta^{\alpha}}{(x+\beta)^{\alpha+1}}$	$1-\left(1+\frac{x}{\beta}\right)^{-\alpha}$	$\alpha > 0$, $\beta > 0$
Weibull	$\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$	$1-e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$	$\alpha > 0$, $\beta > 0$

Note: $^*\Phi(\cdot)$ represents the CDF of the standard normal random variable

Choosing which the most suitable probability distribution represents the exact probability distribution of X can be formally conducted using the Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) tests. The steps of the KS test and AD test are as follows:

1) Set the significance level α.
2) Determining the null and the alternative hypothesis. The KS and AD test null hypothesis states that the exact probability distribution of the data fits the theoretical probability distribution, while the alternative hypothesis states the opposite.
3) Determine the value of the KS and AD test statistics. The statistical values of the KS and AD tests can be determined by the following equation (Law & Kelton, 2000; Anderson & Darling, 1952; Stephens, 1974):

$$D = \sup_{x}\{|F_e(x) - F_X(x)|\} \tag{1}$$

and

$$L = \frac{1}{n}\sum_{i=1}^{n}(2i-1)[\ln\{F_X(x_i\} + \ln\{1 - F_X(x_{n+1-i})\}], \tag{2}$$

where $F_e(x)$ represents the exact CDF of the data, n represents the data size, and x_i is the i-th single flood catastrophe loss data.

4) Making decision. Reject the null hypothesis of the KS and AD tests if the statistical values are greater than the critical values and vice versa. If the null hypothesis of the KS test and AD test is rejected, the exact probability distribution of the data is not fit to be represented by the X probability distribution, and vice versa.

The fittest probability distribution has the smallest KS and AD test statistic values.

2.5 Flood Loss Aggregate Modelling Using Compound Poisson Process

In this research, the flood loss aggregate is designed into the compound Poisson process (CPP). CPP is chosen because it can simultaneously accommodate the loss and frequency of flood measurements (Sukono et al., 2022). Flood loss aggregate expressed by CPP is mathematically expressed as follows (Tang & Yuan, 2019):

$$S_t = \sum_{i=1}^{N_t} X_i, \tag{3}$$

where $\{N_t; t \geq 0\}$ is a Poisson process with intensity λt representing the frequency of flood catastrophes that occur until time t, $\{X_i; i = 1, 2, \ldots, N_t\}$ is independent and identically distributed random variables representing the i-th single flood catastrophe loss. There are basic assumptions imposed in CPP, where the frequency of catastrophe is assumed not to depend on catastrophic loss (Tse, 2009). The CDF of CPP S_t is mathematically expressed as follows (Ma & Ma, 2013; Klugman et al., 2008):

$$F_{S_t}(x) = \sum_{n=0}^{\infty} e^{\lambda t} \frac{(\lambda t)^n}{n!} F_X^{*n}(x), \quad (4)$$

where $F_X^{*n}(x) = \Pr\{X_1 + X_2 + \cdots + X_n \le x\}$. Meanwhile, the probability density function of S_t is mathematically expressed as follows (Dickson, 2005):

$$f_{S_t}(x) = \sum_{n=0}^{\infty} e^{\lambda t} \frac{(\lambda t)^n}{n!} f_X^{*n}(x), \quad (5)$$

where $f_X^{*n}(x) = \Pr\{X_1 + X_2 + \cdots + X_n = x\}$. The following equation can be used to determine λ (Ross, 1996):

$$\lambda = \frac{E(N_t)}{t}, \quad (6)$$

where $E(N_t)$ is the expected value of N_t.

2.6 The Approximation Method of the Loss Aggregate Distribution

Calculating the value of CDF S_t in Eq. (4) is generally difficult to estimate analytically. Therefore, alternative methods can be used to calculate it. One of the fastest calculation methods is the loss aggregate distribution approximation method (Dickson, 2005). This method has the advantage of a fast-computational process. This computational speed occurs because there is no recursive process in the calculation. In other words, this method only requires one calculation. We use four methods of approximating the loss aggregate's CDF: the approximation methods of normal, gamma, normal-power (N-P2), and gamma inverse Gaussian (GIG). The normal approximation method can be applied using the following equation (Dickson, 2005):

$$F_{S_t}(x) \approx \Phi(z) = \Phi\left(\frac{x - \mu_{S_t}}{\sigma_{S_t}}\right), \quad (7)$$

where $\Phi(\cdot)$ represents the standard normal CDF, μ_{S_t} represents the mean of S_t, σ_t represents the standard deviation of S_t, and $z = \frac{x-\mu_{S_t}}{\sigma_{S_t}}$. Usually, this method has the lowest accuracy of the other methods. Then, the gamma distribution approximation method can be applied using the following equation (Chaubey et al., 1998):

$$F_{S_t}(x) \approx F_G(x) = \frac{1}{\Gamma(\alpha)} \int_0^{\alpha+z\sqrt{\alpha}} e^{-y} y^{\alpha-1} dy, \quad (8)$$

where $\Gamma(\cdot)$ represents the gamma function, and $\alpha = \frac{4}{\gamma_{S_t}^2}$, where γ_{S_t} represents the skewness of S_t. Furthermore, the N-P2 distribution approximation method can be applied using the following equation (Chaubey et al., 1998):

$$F_{S_t}(x) \approx \Phi\left(\sqrt{\frac{9}{\gamma_{S_t}^2} + \frac{6z}{\gamma_{S_t}} + 1} - \frac{3}{\gamma_{S_t}}\right). \quad (9)$$

This method is a form of development of the normal approximation method. The last is the inverse Gaussian approximation method. This method integrates the gamma and inverse Gaussian distributions. The inverse Gaussian distribution used is expressed as follows (Reijnen et al., 2005):

$$F_{IG}(x) = \Phi\left(\frac{x - x_0 - m}{\sqrt{b(x - x_0)}}\right) + e^{\frac{2m}{b}} \Phi\left(-\frac{x - x_0 + m}{\sqrt{b(x - x_0)}}\right), \quad (10)$$

where $m = \frac{3c_{2S_t}^2}{c_{3S_t}}$, $b = \frac{c_{3S_t}}{3c_{2S_t}}$, and $x_0 = c_{1S_t} - m$. In detail, c_{1S_t}, c_{2S_t}, and c_{3S_t} are 1st, 2nd, and 3rd moments of S_t, respectively. The CDF of S_t approximated by the gamma inverse Gaussian distribution approximation method is applied using the following equation (Reijnen et al., 2005; Chaubey et al., 1998):

$$F_{S_t}(x) \approx (1 - \omega) F_{IG}(x) + \omega F_G(x), \quad (11)$$

where $\omega = \frac{\kappa_{S_t}-\kappa_{IG}}{\kappa_G-\kappa_{IG}}$. In detail, κ_{S_t}, κ_{IG}, and κ_G represent the kurtosis of S_t, the inverse Gaussian random variable, and the gamma random variable, respectively. This approximation method of the gamma inverse Gaussian distribution is the most modern (Ibrahim et al., 2022). Selecting the most suitable approximation method can be conducted visually and formally. The visual selection is based on the method whose CDF graph most closely coincides with the exact CDF. Then, the formal selection can be based on the method with the smallest root-mean-squared error (RMSE) value. This RMSE value is calculated using the following equation:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(F_e(x_i) - F_X(x_i)\right)^2}{n}}. \tag{12}$$

2.7 The FCB Price Model

The estimated FCB prices in this study are divided into two types. The first type is zero-coupon FCB prices. Investors' acceptance of a zero-coupon FCB on the maturity date (R_T) is mathematically expressed as follows:

$$R_T = \begin{cases} \theta P & ; S_T > a_S \\ P & ; S_T \leq a_S \end{cases}, \tag{13}$$

where $T = 1, 2, \dots$ is the term of the zero-coupon FCB in years, P represents the principal of the zero-coupon FCB, a_S represents the attachment point of the flood loss aggregate, and θ represents the proportion of principal received by investors when the flood loss aggregate exceeds the attachment point in the zero-coupon FCB period. Eq. (13) verbally explains that the investor will receive the entire principal amount if the flood loss aggregate does not exceed the attachment point within the term of the zero-coupon FCB. Then, if the flood loss aggregate exceeds the attachment point within the term of zero-coupon FCB, the investor will only receive a proportion of the principal amount. The zero-coupon FCB price is determined using the concept of the present value of the principal expected value on the maturity date. Mathematically, this is written as follows:

$$\begin{aligned} V_T &= E(R_T)v^T, \\ &= \left[\theta P\left\{1 - F_{S_T}(a_S)\right\} + PF_{S_T}(a_S)\right](1+r)^{-T}, \\ &= P\left[\theta\left\{1 - F_{S_T}(a_S)\right\} + F_{S_T}(a_S)\right](1+r)^{-T}, \end{aligned} \tag{14}$$

where V_T represents the price of zero-coupon FCB with a term of T years, v represents the discount factor, and r represents the interest rate per annum. Finally, the second type is coupon-paying FCB price. Investors' acceptance of coupon-paying FCB on the maturity date (R'_T) is mathematically expressed as follows:

$$R'_T = \begin{cases} \theta P & ; S_T > a_S \\ P + cP & ; S_T \leq a_S \end{cases}, \tag{15}$$

where c represents the coupon rate. Eq. (15) verbally states that if the flood loss aggregate does not exceed the attachment point within the term of the coupon-paying FCB, the investor will receive the entirety of the principal and the coupon. Then, if the flood loss aggregate exceeds the attachment point within the term of the coupon-paying FCB, the investor will only receive a proportion of the principal amount and not the coupon. The coupon-paying FCB price is determined using the concept of the present values of the expected value of the principal and the coupon on the maturity date. Mathematically, this is written as follows:

$$\begin{aligned} V'_T &= E(R'_T)v^T, \\ &= \left[\theta P\left\{1 - F_{S_T}(a_S)\right\} + (P + cP)F_{S_T}(a_S)\right](1+r)^{-T}, \\ &= P\left[\theta\left\{1 - F_{S_T}(a_S)\right\} + (1+c)F_{S_T}(a_S)\right](1+r)^{-T}, \end{aligned} \tag{16}$$

where V'_T represents the coupon-paying FCB price with a term of T years. Eq. (16) can be reformulated to be more straightforward as follows:

$$V'_T = V_T + cPF_{S_T}(a_S)(1+r)^T. \tag{17}$$

3. Results and Discussion

3.1 Fitting Single Flood Catastrophe Loss Data Distribution

The theoretical distributions used in this fitting are those presented in Table 1. Before the fitting process begins, the estimation of the parameters of the theoretical distributions is carried out using the MLE method. The results of the parameter estimates are presented in Table 2. The parameter estimates met the conditions for each likelihood function's maximum solution. Then, after the parameters of each theoretical distribution are obtained, the next step is to check their fit to represent the exact data distribution. This fit is formally carried out using the KS and AD tests. The significance level applied is 0.01. The test statistics of each theoretical distribution determined by Eq. (1) and Eq. (2) are also presented in Table 2. Table 2 shows that each theoretical distribution's KS test statistic values appear smaller than their critical value, 0.0814. It causes the null hypothesis on the KS test is not rejected. In other words, the theoretical distributions are fit to represent the distribution of single flood loss data. Then, Table 2 also shows that all the AD test statistical values of each theoretical distribution appear smaller than the critical value, 3.9074. It means that the null hypothesis in the AD test is not rejected. It also indicates that the theoretical distributions are fit to represent the distribution of flood loss data. However, the fittest one should be chosen. The fittest theoretical distribution has the smallest KS and AD test statistics. The Weibull distribution appears to have the smallest KS and AD test statistic values. Therefore, the Weibull distribution with parameters $\alpha = 0.9596$ and $\beta = 1.1308$ is used as an approximate distribution for the exact data distribution.

Table 2
Parameter Estimators and Statistical Values of KS and AD Statistical Tests of Each Theoretical Distribution

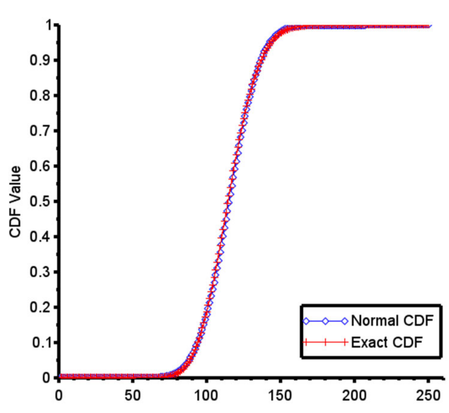
Distribution	Parameter Estimator(s)	KS Statistical Test	AD Statistical Test
Exponential	$\lambda = 0.8335$	0.0445	1.0931
Log-logistic	$\alpha = 1.3288, \beta = 0.6234$	0.0768	3.6290
Log-normal	$\sigma = 1.3138, \mu = -0.4648$	0.0664	3.6344
Pareto	$\alpha = 7.2871, \beta = 7.5361$	0.0235	0.1999
Weibull	$\alpha = 0.9596, \beta = 1.1308$	0.0199	0.1107

3.2 Determining Flood Catastrophe Intensity

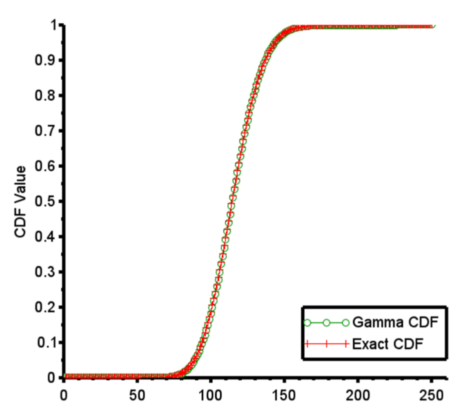
First, flood intensity must be determined to apply the CPP in measuring the risk of flood loss aggregate. The intensity is determined using Eq. (6). In short, the intensity of flood catastrophe obtained based on the data used is 33.3333. It means that, on average, Indonesia has $\lambda = 33.3333$ flood catastrophes yearly.

3.3 Determining the Approximation Method for Flood Loss Aggregate Distribution

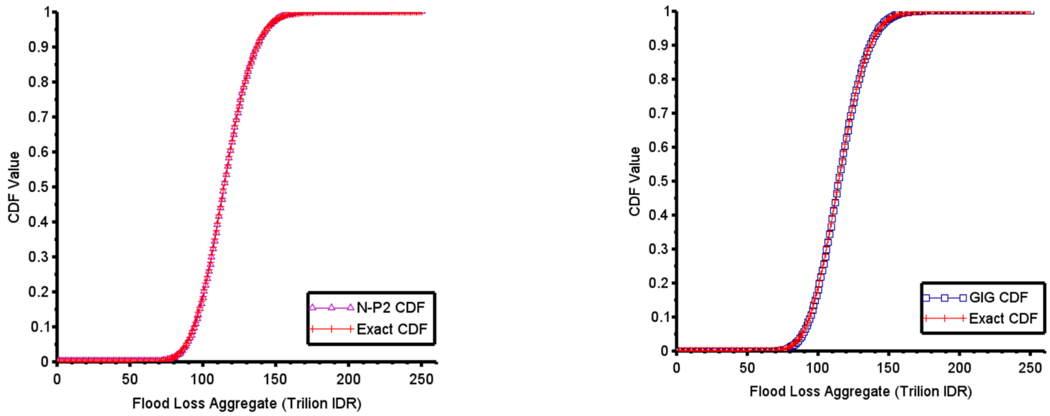
Suppose that the term of the FCB is $T = 3$ years. Thus, an approximation method is used to approximate the CDF of S_3. The first step is determining the exact CDF values of the flood loss aggregate, as shown in Eq. (4). We use the Strötter algorithm to resolve these values via the "actual" package in R software version 4.1.2. More about this method can be seen in Panjer and Willmot (1992). Next, we present the selection of the most suitable approximation method to visually represent the exact distribution of flood loss aggregate. The visualisation is created using Scilab software version 6.1.1. and presented in Fig. 2.



(a)



(b)



(c) (d)

Fig. 2. Visualisation of Approximation Method for Normal **(a)**, Gamma **(b)**, N-P2 **(c)**, and Gamma Inverse Gaussian **(d)** Distributions with Exact Distribution of Flood Loss Aggregate

Fig. 2 shows that almost every approximation method visually coincides with the exact CDF of the flood loss aggregate. It makes the chosen suitable one challenging to determine. Therefore, we used the root-mean-squared-error (RMSE) measure presented in Eq. (12). The method with the smallest RMSE value is the one we chose. The RMSE values of each approximation method are shown in Table 3. Table 3 shows that the N-P2 approximation method has the smallest RMSE value. Therefore, we use the N-P2 approximation method to approximate the flood loss aggregate distribution.

Table 3
The RMSE Values of Each Approximation Method

Approximation Method	RMSE
Normal	0.0058
Gamma	0.0022
N-P2	0.0021
GIG	0.0028

3.4 Estimated FCB Price in Indonesia

Before estimation is conducted, the values of the variables in Eq. (14) and Eq. (17) are determined first. The values of these variables are presented in Table 4.

Table 4
The Values of Variables for FCB Price Estimation

Variable	Value
P	1 IDR
C	$5\% \times F$ IDR
r	5%
T	3 Years
a_S	115.1743 Trillion IDR*
θ	0.5

Note: * is three times the average annual flood loss in Indonesia

The price of zero-coupon FCB is estimated using Eq. (14). The variables can be seen in Table 4. By applying the N-P2 approximation method, the estimated zero-coupon FCB price obtained is 0.6542 IDR. It means that the zero-coupon FCB price in Indonesia is 65.42% of the principal amount. It is quite cheap because the price is far below 100% of the principal received at maturity. For example, if the principal of the zero-coupon FCB is changed to 100,000,000,000 IDR, then the price of the zero-coupon FCB will be 100,000,000,000 × 65.42% = 65,420,000,000 IDR. Next is the estimation of coupon-paying FCB price using Eq. (17). The variables can also be seen in Table 4. By applying the N-P2 approximation method, the estimated coupon-paying FCB price obtained is 0.6764 IDR. It means that the price of coupon-paying FCB in Indonesia is 67.64% of the base money. It is quite cheap because the price is far below 100% of the principal received at maturity. For example, if the principal from coupon-paying FCB is changed to 100,000,000,000 IDR, then the coupon-paying FCB price to be paid is 100,000,000,000 × 67.64% = 67,640,000,000 IDR.

4. Discussion

4.1 The Return and Loss Percentage of FCB in Indonesia

This study seeks to identify how much return and loss investors may experience in their investment in FCB. Using the variables in Table 4, the percentage of return and loss that investors may experience for each attachment point situation is presented in Table 5.

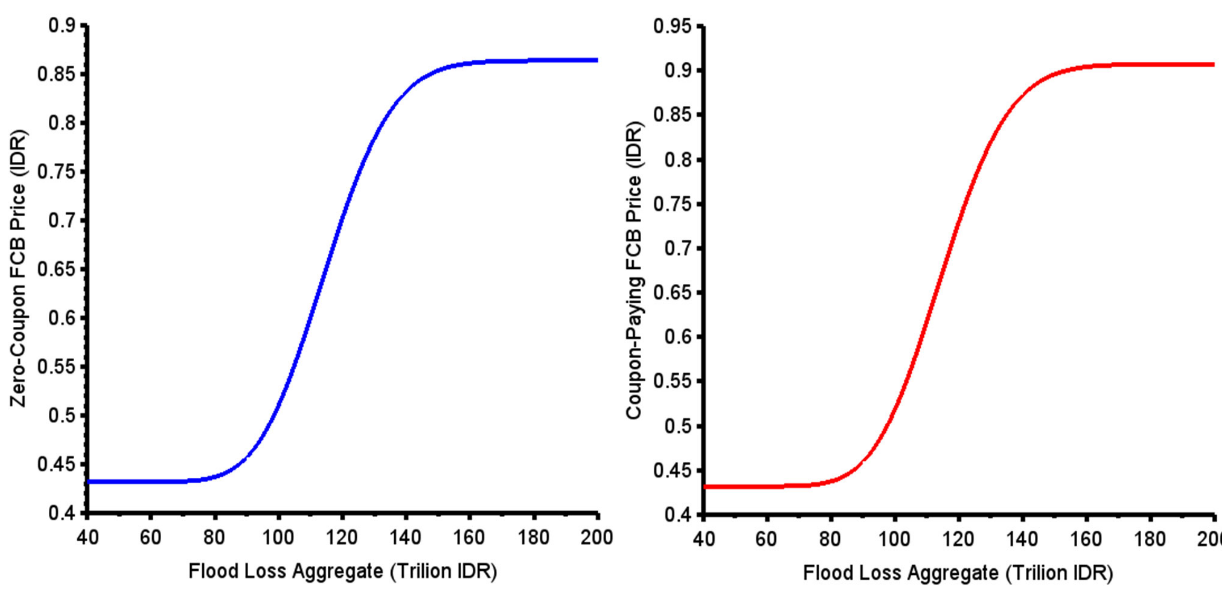
Table 5
The Percentage of Return and Loss that Investors May Experience for Each Attachment Point Situation

Attachment Point Situation	Type of FCB	Return (%)	Loss (%)
Exceed	Zero-Coupon	-	23.57
	Coupon-Paying	-	26.08
Not Exceed	Zero-Coupon	52.86	-
	Coupon-Paying	47.84	-

Table 5 shows that the percentage return of both types of FCB is zero if the attachment point is exceeded. It happens because the principal received is half, and the coupon will not be paid. Instead of return, FCB investors will lose 23-27% of their price if the attachment point is exceeded. The percentage loss is enormous. Then, the return percentage from both types of FCB is 47-53% if the attachment point is not exceeded. It happens because the principal and coupons received are intact. The return percentage is significant and exceeds the loss percentage from the investor that may occur. Thus, FCB has a high risk of loss but a higher probability of return. If investors are good at analysing this, then FCB investments can benefit them.

4.2 The Effect of Attachment Point to FCB Price in Indonesia

This section analyses how attachment points affect FCB prices in Indonesia. The variables used are as presented in Table 4, with slight changes to the attachment point. Attachment points are changed to real number intervals $[40, 200]$ trillion IDR. Zero-coupon and coupon-paying FCB prices for each attachment point in the interval are presented visually in Fig. 3.



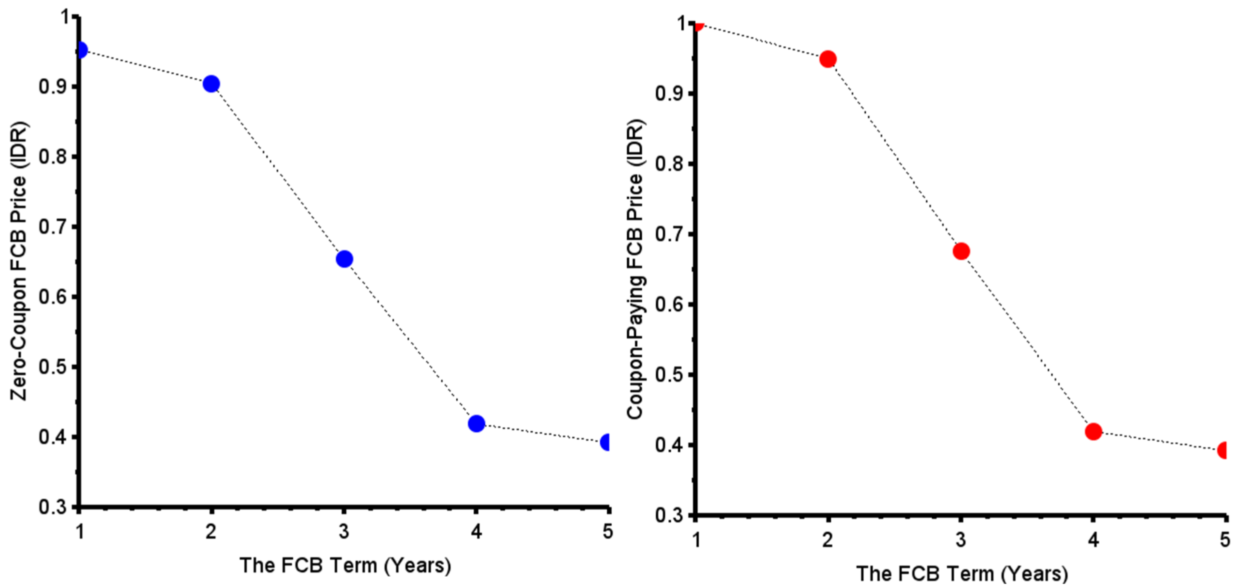
(a) **(b)**

Fig. 3. Zero-Coupon and Coupon-Paying FCB Prices for Each Attachment Point in the Interval $[40, 200]$

Fig. 3 shows that the attachment point aligns with the FCB price. In other words, the higher the attachment point, the more expensive the FCB will be, and vice versa. Why does this happen? Logically, this makes sense because if the attachment point is significant, the probability of investors losing their principal and coupons is also tiny. It causes the demand for FCB with the attachment point in Indonesia to be high despite little supply. Therefore, the price becomes expensive. It also applies to the opposite. The probability of investors losing the principal and coupons is also significant if the attachment point is tiny. It causes FCB requests with these attachment points in Indonesia to be low. Therefore, the price becomes cheap. The results of this analysis can be used as a reference for investors to choose FCBs with reasonable attachment points or not too cheap or expensive.

4.3 The Effect of the FCB Term on Its Price in Indonesia

This section analyses how the FCB term affects its price. The variables used are as presented in Table 4, with slight changes to the FCB term. The FCB term is changed to integer $\{1, 2, 3, 4, 5\}$ years. For a simple reason, the CDF of S_1, S_2, S_3, S_4, and S_5 is also approximated by the CDF of N-P2. Zero-coupon and coupon-paying FCB prices for each FCB term are presented visually in Fig. 4.



(a) **(b)**

Fig. 4. Zero-Coupon **(a)** and Coupon-Paying **(b)** FCB Prices for Each FCB Term in $\{1, 2, 3, 4, 5\}$

Fig. 4 shows that the FCB term does not align with its price. In other words, the longer the FCB term, the cheaper the FCB price will be, and vice versa. Why does this happen? Logically, this makes sense because investors will likely lose their principal and coupons if the FCB term is long. It causes the demand for FCB within that term in Indonesia to be low. Therefore, the price becomes cheap. It also applies to the opposite. If the FCB term is short, the probability of investors losing the principal and coupons is also tiny. It causes the demand for FCB within that term in Indonesia to be high. Therefore, the price becomes expensive. The results of this analysis can also be used as a reference for investors to choose FCB with terms that are not too long.

5. Conclusion

In conclusion, this study has estimated Indonesia's zero-coupon and coupon-paying FCB prices. The severity of a flood catastrophe is measured via its loss aggregate modelled by a compound Poisson process. There is no closed-form solution to the FCB pricing model used because the CDF value of loss aggregate is generally challenging to determine analytically. Therefore, this study uses the approximation method of loss aggregate distribution, such as the approximation methods of normal, gamma, N-P2, and gamma inverse Gaussian. With these methods, the estimated price of the FCB model can be computed more quickly and efficiently. The results show that for a 3-year FCB period, the N-P2 approximation method is the most suitable for estimating FCB prices in Indonesia. The estimated zero-coupon and coupon-paying FCB prices are 65.42% and 67.64% of the principal amount, respectively. Then, we conducted a further analysis of the FCB price. The loss and return percentage of FCB in Indonesia has a high risk of loss but has a higher probability of return. Then, the price of FCB in Indonesia is proportional to the attachment point. It shows that if the attachment point of the FCB is significant, the price is also high, and vice versa. Finally, the price of FCB in Indonesia is not proportional to its term. It shows that if the term of the FCB is long, its price is low, and vice versa. If investors are good at analysing these, then FCB investments can benefit them. This research is expected to help the Indonesian government, especially special-purpose vehicles, set FCB prices in Indonesia. This research can also be a reference for investors choosing FCB based on expected return, attachment point, and the term they want. As a suggestion for further study, flood parameter factors such as the area affected by the flood can be involved. It has never been done before, so it becomes an opportunity for future research.

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