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Investigating the collective value at risk model (CVaR) and its application on real data for life insurance

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A B S T R A C T

Life insurance is designed to reduce the risk of financial loss due to unforeseen consequences related to the insured's death. In life insurance, the insurer provides death benefits as a claim when the insured suffers death. The claim is the compensation for a risk loss. Individual claim in one-period insurance is called aggregation claim, while aggregation claim is a collective risk. Collective risk is usually measured using a variance. However, the variance risk measure cannot often accommodate any event risk because there is a risk of claims beyond the amount of variance. Using the proposed method CVaR and confidence level are taken from $\alpha = 0.25\%$ until 4%. This study found that the proposed method CVaR scored more fairly than Collective Risk. In conclusion, this study indicated that the collective risk model is just included using mean and variance without any confidence level. Therefore, only one result for the Collective Risk model, which automatically shows the model using mean, variance and standard deviation, could not accommodate all risk events.

1. Introduction

Insurance can be defined as the undertaking of risks by an insurance company which agrees to indemnify the insured or the risk transferor for specified losses upon their occurrence, subject to the payment of insurance premiums by the insured (Albrecher, 2008). Bon *et al.* (2018) and Dionne (2013) mentioned that insurance can be seen from two points of view which are first as protection for the finances provided by the insurer and second, as a risk pooling tool of two or more persons or companies through promised donations to establish funds to pay. Insurance is one of the methods of managing risks, in particular, one of the methods of risk transfer. Risk is generally measured by variance and standard deviation (Dickson, 2016). It should be highlighted that variance and standard deviations measure the average risk size and do not accommodate all of the risks, thus, there is a need to find an alternative measure. The variance or standard deviation is a measure of the average deviation, which is often not able to accommodate all events deviation (risk) (Andreas de Vries, 2000). Therefore, the idea emerged to quantify the risk carried by the quintile or better known as Value-at-Risk (VaR) (Li, 2000; Aktas & Sjostrand, 2011; Diers *et al.*, 2012; Dowd & Blake, 2006; Meyers, 2008).

According to Riaman *et al*. (2013), there is much research on the collective risk model conducted for instance the application of collective risk theory to estimate variability in deciding loss of savings, and the collective risk model for savings claim distribution. They also studied Collective Risk Models Analysis in Life Insurance Credit By Using Claim Model Aggregate.

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In his research, he applied the Collective Risk Model to decide and anlayze the claim number of risk borne by the insurance company where the object is the credit life insurance (Bortoluzzo *et al*., 2011). Bon *et al.* (2018) had propose a development model of Collective Value-at-Risk (CVaR) (Dokov *et al*., 2007). The development model is based on the collective risk model contained in research by Dickson (2016) and Kahn (1992). This paper will discuss a proposed method called Collective Value-at-Risk (CVaR). We will apply the Collective Value-at-Risk (CVaR) in real data analysis. Data obtained is from Bank Negara Malaysia (BNM) period from 2009 until 2017.

2. Literature review

Agustini *et al.* (2020) applied Bayes method to estimate risk premiums for critical disease insurance. Further, Sukono *et al*. (2017) determined and estimated the risk premium to be paid to the insurance company using the method of Empirical Bayes Credibility Theory (EBCT). Also, Sukono *et al*. (2017) estimated the risk model of life insurance claims for cancer patients by using the Bayesian method. In this research, there were two models of risk discussed namely Collective and Individual Risks. The problem found is that the cost of cancer treatment is higher, and the number of claims from year to year is increasing which will affect the insurer in estimating future claims trends to determine the risk premium. Therefore, they did research on the risks claims, particularly in cancer patients (Buckham *et al*., 2010; Masci, 2011; Nino & Paolo,2010).

In addition, Sukono *et al*. (2018) estimated the risk model and premium for motor vehicle insurance by using the Bayesian method. They studied how to assess Collective Risk Models and non-life insurance by using the Bayesian method. The Bayesian method is used to estimate the parameters of the claim frequency model and the amount of the claim, in order to be used in the calculation of risk, and also to determine the premiums the insured must pay to the insurer (Djuric, 2013). Saputra *et al*. (2018) have done the research on risk adjustment model of credit life insurance by using a genetic algorithm. They have proposed the Algorithm to determine the distribution of life insurance claims and certain to minimize risk. Sukono *et al*. (2019) also have done another study about the Estimation of Value-at-Risk Adjusted under the Capital Asset Pricing Model Based on the ARMAX-GARCH Approach. They analyzed the measure of the risk identified as Value-at-Risk Adjusted or Modified Value-at-Risk (MVaR) (Guarda *et al*., 2012; Naufal *et al*., 2018), in which the shares analyzed are assumed to follow the CAPM form (Polanski *et al*., 2013).

Sidi *et al*. (2019) examined the Estimation of the Aggregate Claim Risk Model. The study is to examine the Aggregate Claim Risk Model on insurance and the case study is on the damaged buildings due to the flooding of the Citarum River in Bandung Indonesia. In the research, they examined by applying the Collective Risk Model as mentioned in equation (2.5). Another study by Sukono *et al*. (2019) is Supply Chain Strategy for Managing Risk for Health Insurance: An Application of Bayesian Model. In this study, they propose to estimate the health insurance claims for people who live in the south Bandung area, West Java, Indonesia. They have mentioned that the research is important to get a Mathematical model which can predict the amount of health insurance claims so that insurance companies do not suffer any losses. Also, it can be used to determine the amount of net premium of health insurance reasonably. The study was conducted using the Bayesian method because they believe this method is suitable to estimate the health insurance risk model.

Sukono *et al*. (2020) used Risk Surplus Analysis in credit life insurance using the Bayesian method. Their study aimed to determine the credit life insurance surplus obtained by companies using the Bayesian method. The Bayesian method is used to estimate the model parameters of many claims and the size of the claims used for the calculation of the risk model and is useful for determining the price of premiums that must be paid by the insured to the insurance company. Brahmantyo *et al*. (2021) used logistic regression model with parameters estimated Maximum Likelihood Estimation (MLE) based on Newton Raphson approximation to generate Willingness top Pay (WTP) insurance to the fishermen.

2.2 Collective Risk

Referring to Sukono *et al.* (2019), Bon *et al.* (2018) and Dickson (2016) S is defined as the sum of a collection of random variables total of claims incurred within one year of the risk. In this case, that N random variable is assumed to indicate the number of claims of risks this year and let the random variable X_i be used to declare the amount of the claims. The aggregate claim amount is the sum of the number of individual claims, can therefore written as:

$$S = \sum_{i=1}^{N} X_i \tag{1}$$

Next, two important assumptions are made. First, it is assumed $\{X_i\}_{i=1}^{\infty}$ a that the random variable is a sequence of independent and identically distributed, and second, it is assumed that the random variable N is independent of $\{X_i\}_{i=1}^{\infty}$. These assumptions reveal that the amount of any claim does not depend on the number of other claims, and that the distribution of the number of claims do not change throughout the year. The assumption also states that the number of claims has no effect on the amount of the claim (Dickson, 2016; Bowers *et al*., 1997). Bon *et al.* (2018) that moments and the moment generating function of S can be calculated using conditional expectation argument. The key result is that for any two random variables Y and Z, there is a relevant moment:

$$E[Y] = E[E(Y \mid Z)] \tag{2}$$

and

$$V[Y] = E[V(Y \mid Z)] + V[E(Y \mid Z)] \tag{3}$$

As the closest application of Eq. (2) is obtained

$$E[S] = E[E(S \mid N)]$$

Then suppose that $m_k = E[X_i^k]$ for $k = 1,2,3,...; i = 1,2,...,n$, is the kth moment. Therefore, obtained

$$E[S \mid N = n] = E\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E[X_i] = \sum_{i=1}^{n} m_1 = nm_1$$

and since it was formed for $n = 0,1,2,...$. Therefore $E[S \mid N] = Nm_1$ and therefore

$$E\left[S\right] = E[E(S \mid N)] = E\left[Nm_1\right] = E[n]m_1 \tag{4}$$

Bon *et al.* (2018) mention by using a similar method, with $\{X_i\}_{i=1}^{\infty}$ is independent random variables,

$$V[S \mid N = n] = V\left[\sum_{i=1}^{n} X_i\right] == \sum_{i=1}^{n} V[X_i] = \sum_{i=1}^{n}\left(E[X_i^2] - (E[X_i])^2\right) = n(m_2 - m_1^2).$$

Thus

$$V[S \mid N] = N(m_2 - m_1^2).$$

Then, by applying Eq. (3) is obtained

$$V[S] = E[V(S \mid N)] + V[E(S \mid N)] = E[N(m_2 - m_1^2)] + V[Nm_1] = (m_2 - m_1^2)E[N] + m_1^2 V[N]. \tag{5}$$

Eq. (5) is called Collective Risk, where it is measured using a variance claim (Cunningham, *et al*., 2006; Dickson, 2016).

3. Materials and methods

3.1 Material

The data used in the research in the development of the Collective Value-at-Risk (*CVaR*) model, is the claim data from Bank Negara Malaysia (BNM) from the year 2009 until 2017. These claims data are secondary data, which can be grouped into two types, namely claims frequency or numbers, and claim severity or amount.

3.2 Method

3.2.1 Claims Distribution Model

According to Iqbal *et al.* (2021), Poisson distribution has some interesting properties that makes it easy to be used in various actuarial applications (Cunningham *et al*., 2006). Therefore, Iqbal *et al.* (2021), used the Poisson distribution (Yates and Goodman, 2014) to analyze the number of claims denoted by N, using the following equation:

$$P(N = n) = \frac{e^{-\lambda}\lambda^n}{n!},\ n = 0,1,2,....,\lambda > 0,$$

where λ is the average number of events per interval, *e* is the Euler's number used as the base of the natural logarithms which approximately equal to 2.71828... and $k!k \cdot (k-1) \cdot (k-2)...3 \cdot 2 \cdot 1$. This equation is also known as the probability mass function (PMF) for a Poisson distribution. Iqbal *et al.* (2021) defined a moment-generating function of Poisson distribution (Yates & Goodman, 2014).

$$M_X(t) = E[e^{tX}] = \sum_{n=0}^{\infty} e^{tn} P(X = n),$$

where X is a discrete random variable with a Poisson distribution, the moment generating function M_N of Poisson distribution N is given by

$$M_N(t) = E[e^{tN}] = \sum_{n=0}^{\infty} e^{tn} P(N=n) = \sum_{n=0}^{\infty} e^{tn} \frac{e^{-\lambda}\lambda^n}{n!} = e^{-\lambda} \sum_{n=0}^{\infty} \frac{\left(\lambda e^t\right)^n}{n!} = e^{\lambda}(e^{t-1}) = \exp\{\lambda(e^t - 1)\} \tag{6}$$

and its probability-generating function is given by $P_N(r) = \sum_{n=0}^{\infty} r^n e^{-\lambda} \frac{\lambda^n}{n!} = e^{-\lambda} \sum_{n=0}^{\infty} \frac{(\lambda r)^n}{n!} = e^{-\lambda} e^{\lambda r} = \exp\{\lambda(r-1)\}$. The moments of N can be found from the moment generating function because by the definition that the kth moment of N is the kth derivative of $M_N(t)$. Therefore, for example, from the moment generating function of Poisson distribution which is given by

$$M_N(t) = \exp\{\lambda(e^t - 1)\},$$

From the relationship between $M_N(t)$ and $E[e^{tN}]$ given by

$$M_N(t) = E[e^{tN}] = E\left[1 + tN + \frac{(tN)^2}{2!} + \ldots \frac{(tN)^n}{n!} + \right], t \in R$$

$$= E[1] + E[tN] + E\left[\frac{(tN)^2}{2!}\right] + \ldots + E\left[\frac{(tN)^n}{n!}\right] + \ldots = 1 + tE[N] + t^2 \frac{E[N^2]}{2!} + \ldots + t^n \frac{E[N^n]}{n!} + \ldots = 1 + tm_1 + \frac{t^2 m_2}{2!} + \ldots + \frac{t^n m_n}{n!} + \ldots$$

Since

$M_N(0) = \exp\{\lambda(e^0 - 1\} = 1$, $M'_N(0) = \lambda e^0 M_N(0) = \lambda$ and $M''_N(0) = \left(\lambda e^0 + (\lambda e^0)\right)^2 M_N(0) = \lambda + \lambda^2$. Thus,

$E[1] = 1$, $E[N] = \lambda$ and $E[N^2] = \lambda + \lambda^2$ so that $V[N] = E[N^2] - (E[N])^2 = \lambda + \lambda^2 - \lambda^2 = \lambda$. Furthermore, the notation $P(\lambda)$ is used to denote a Poisson distribution with parameter λ (Cunningham *et al*., 2006; Dickson, 2016). Also, Iqbal *et al.* (2021) in insurance, claim the amount is assumed to a continuous random variable. Suppose that X is the claim amount assumed lognormal distribution with parameters μ_X and σ_X, where $-\infty < \mu_X < \infty$ and $\sigma_X > 0$, its density function is given by

$$f(x) = \frac{1}{x\sigma_X\sqrt{2\pi}} \exp\left\{-\frac{(\log x - \mu_X)}{2\sigma_X^2}\right\}, x > 0$$

The distribution function can be obtained by integrating the density function as follows (Dutang, *et al*., 2009).

$$F(x) = \int_0^x f(y)dy = \int_0^x \frac{1}{y\sigma_X\sqrt{2\pi}} \exp\left\{-\frac{(\log y - \mu_X)^2}{2\sigma_X^2}\right\} dy.$$

By $z = \log y$, we have $z = -\infty$ when $y = 0$, $z = \log x$ when $y = x$ and $dz/dy = (1/y)$. Therefore, the function $F(x)$ can be written as

$$F(x) = \int_{-\infty}^{\log x} \frac{1}{\sigma_X\sqrt{2\pi}} \exp\left\{-\frac{(z - \mu_X)^2}{2\sigma_X^2}\right\} dz.$$

As the integrand is the $N(\mu_X, \sigma_X^2)$ density function,

$$F(x) = \Phi\left(\frac{\log x - \mu_X}{\sigma_X}\right)$$

Thus, probabilities under a lognormal distribution can be calculated from the standard normal distribution function (Dutang, *et al*., 2009; Eling, 2011). This relationship between normal and lognormal (*LN*) distributions is extremely useful, particularly in deriving moments. If $X \sim LN(\mu_X \sigma_X)$ and $Y = \log X_i$, then by some manipulation and perfect square, we can derive

$$E[X_i^k] = E[e^{\log X_i^k}] = E[e^{k \log X_i}] = E[e^{kY}] = M_Y(k).$$

But,

$$E[e^{kY}]=\int_{-\infty}^{\infty}\exp\{ky\}\frac{1}{\sigma_X\sqrt{2\pi}}\exp\{-(y-\mu_X)^2/(2\sigma_X^2)\}dy=\int_{-\infty}^{\infty}\frac{1}{\sigma_X\sqrt{2\pi}}\exp\{ky-\left((y-\mu_X)^2/(2\sigma_X^2)\right)dy$$

$$=\int_{-\infty}^{\infty}\frac{1}{\sigma_X\sqrt{2\pi}}\exp\left\{\frac{2ky\sigma_X^2-y^2+2y\mu_X-\mu_X^2}{2\sigma_X^2}\right\}dy=\int_{-\infty}^{\infty}\frac{1}{\sigma_X\sqrt{2\pi}}\exp\left\{-\frac{y^2-y(2\mu_X+2k\sigma_X^2+\mu_X^2)}{2\sigma_X^2}\right\}dy$$

$$=\int_{-\infty}^{\infty}\frac{1}{\sigma_X\sqrt{2\pi}}\exp\left\{-\frac{\left(y-y(\mu_X+k\sigma_X^2\right)^2}{2\sigma_X^2}\right\}.\exp\left\{\frac{\left(k(\sigma_X^2+2\mu_X)\right)}{2}\right\}dy=\exp\left\{\frac{k(2\mu_X+\sigma_X^2k)}{2}\right\}\int_{-\infty}^{\infty}\frac{1}{\sigma_X\sqrt{2\pi}}\exp\left\{-\frac{\left(y-(\mu_X+k\sigma_X^2)\right)^2}{2\sigma_X^2}\right|$$

$$=\exp\left\{k\mu_X+\frac{1}{2}k^2\sigma_X^2\right\} \tag{7}$$

where,

$$\int_{-\infty}^{\infty}\frac{1}{\sigma_X\sqrt{2\pi}}\exp\left\{-\frac{\left(y-(\mu_X+k\sigma_X^2\right)^2}{2\sigma_X^2}\right\}dy=1$$

being the integral of normal density with mean $(\mu_X+k\sigma_X^2)$ and variance σ_X^2. The Eq. (7) is followed by the moment of normal distribution (Dickson, 2016; Dutang, *et al*., 2009). If the moment generating function is generally used, then the moment generating function obtained is as follows:

$M_S(t)=E[e^{tS}]=E[E(e^{tS}\mid N)]$ and

$$E[e^{tS}\mid N=n]=E\left[\exp\left\{t\sum_{i=1}^{n}Xi\right\}\right]=E[\exp\{tX_1\}\cdot\exp\{tX_2\}...\exp\{tX_n\}]=\prod_{i=1}^{n}E[\exp\{tX_i\}],$$

where the obtained result is yielded by using the properties of independence $\{X_i\}_{i=1}^{\infty}$. Furthermore, because $\{X_i\}_{i=1}^{\infty}$ is identically distributed,

$$E[e^{tS}\mid N=n]=\left(M_X(t)\right)^n$$

where $M_X(t)=E[\exp\{tX_i\}]$, $(i=1,2,...,n)$. Based on these,

$$M_S(t)=E[e^{tS}]=E[E(e^{tS}\mid N)]=E\left[\left(M_X(t)\right)^N\right]=E\left[\exp\left\{\log\left(M_X(t)\right)^N\right\}\right]=E\left[\exp\left\{N\log\left(M_X(t)\right)\right\}\right]=M_N\left(\log(M_X(t))\right).$$

Thus, $M_S(t)$ expressed in the tribes of $M_N(t)$ and $M_X(t)$, (Constantinescu, 2013; Dickson, 2016). Poisson distribution has a probability function as:

$$P(N=n)=\frac{e^{-6.212}6.212^n}{n!}n=0,1,2,...$$

We obtain $E[N]=V[N]=\lambda=6.212$. Meanwhile, the claims amount data that match lognormal distribution *LN* (6.267, 21.42). Claims amount lognormal distribution results has the probability density function as:

$f(x)=\frac{1}{(21.42)x\sqrt{2\pi}}\exp\left\{-\frac{(\log x-6.267)^2}{2(6.267)}\right\},x>0.$ In addition,

$m_1=14423.89, m_2=1.74\times10^{11}, m_3=1.76\times10^{21}$ and $m_4=1.49\times10^{34}$.

3.2.2 Collective Value-at-Risk

Bon *et al.* (2018) defined Collective Value-at-Risk to be considered as the following

$$CVaR=-\tilde{N}\{E(S)+Z_\alpha(V[S]^{1/2}\}$$

with $\tilde{N}$ many claim they want to know of the level of risk, and Z_α percentile of the standard normal distribution when given a level of significance α. Due to the risk of claims related to the issue, the value of Z_α selected which is located on the left tail (Andreas, 2000; Casiglio et al., 2002; Manganelli & Engle, 2001). Bon *et al.* (2018) then substituted Eq. (4) and Eq. (5) into Eq. (6), and the *Collective Value-at-Risk* model obtained as the following equation:

$$CVaR=-\tilde{N}\left\{E[N]m_1+Z_\alpha\left(m_2-m_1^2E[N]+m_1^2V[S]^{1/2}\right)\right\} \tag{8}$$

4. Results and discussion

4.1 Collective Risk Calculation

Using the values of $E[N]$ and $V[N]$, and values m_1 and m_2. Value of collective risk can be calculated using Eq. (5) as follows:

$$V[S] = (6.212)\{1.741\times10^{11} - (14423.89)^2\} + (6.212)(14423.89)^2 = RM2.05\times10^{12}.$$

Also, use values $\lambda = 6.212, m_1 = 14423.89$ and $m_2 = 1.74\times10^{11}$, can obtain the values $\mu_S = 89601.21$ and $\sigma_S^2 = 1.082.10^{12}$.

4.2 Collective Value at Risk Calculation

Further, if the specified significance level of $\alpha = 0.005$, then percentile of the standard normal distribution is obtained $z_{0.005} = -2.576$.

$$CVaR = -250\left\{-2.576\left(1.082\times10^{12}\right)^{\frac{1}{2}} + 89601.213\right\} = RM6.30\times10^8$$

Table 1 summarizes the results of CvaR for other confidence levels.

Table 1
Results of CVaR

Confidence level	Result
0.5%	RM6.30×10^8
1	RM5.67×10^8
1.5%	RM5.27×10^8
2	RM4.98×10^8
2.5%	RM4.74×10^8
3%	RM4.54×10^8
3.5%	RM4.36×10^8
4%	RM4.21×10^8

Table 1 shows the maximum potential claim risk by using the CVaR approach model with confidence level $\alpha = 0.5\%$, for 250 claim events, the claim is RM6.30×10^8 while at the maximum potential claim $\alpha = 1\%$, the amount claim is RM5.67×10^8 while at the maximum potential claim risk at confidence level $\alpha = 1.5\%$, the claim is RM5.27×10^8 while at the maximum potential claim risk at $\alpha = 2\%$ the claim amount is RM4.98×10^8,- while at the maximum potential claim risk at confidence level $\alpha = 2.5\%$ the claim amount is RM4.74×10^8,- while at the maximum potential claim risk at confidence level $\alpha = 3\%$ the claim amount is RM4.54×10^8,- while at the maximum potential claim risk at confidence level $\alpha = 3.5\%$ the claim amount is RM4.36×10^8,- while at the maximum potential claim risk at confidence level $\alpha = 4\%$ the claim amount is RM4.21×10^8.

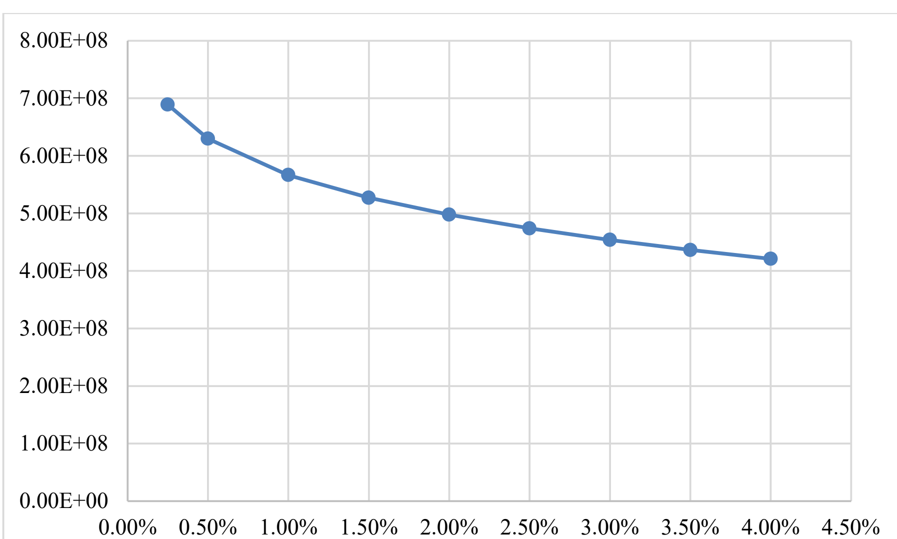

Fig. 1. Graph of CVaR Results.

Fig. 1 displays the potential risk claims from a straight line, showing that it decreases at every confidence level. The higher confidence level will be given effect the lower the potential risk claims. It means that the results are good.

5. Conclusion

In conclusion, this study indicated that the collective risk model is just included using mean and variance without any confidence level. Therefore, there are only one results for the Collective Risk model, which is automatically shown, and the model using mean, variance and standard deviation could not accommodate all risk events. While the proposed method CVaR, confidence levels are taken from α = 0.25% until 4%. As a result, Table 1 shows that the proposed method CVaR scores fairly more than Collective Risk.

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