Decision Science Letters 12 (2023) 41–54

Contents lists available at GrowingScience

Decision Science Letters

homepage: www.GrowingScience.com/dsl

Two-objective optimization of preventive maintenance orders scheduling as a multi-skilled resource-constrained flow shop problem

Masoud Fekri[a], Mehdi Heydari[a*] and Mohammad Mahdavi Mazdeh[a]

[a]*School of Engineering, Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran*

C H R O N I C L E

Article history:
Received June 12, 2022
Received in revised format: August 30, 2022
Accepted October 27, 2022
Available online October 28, 2022

Keywords:
Flow shop Scheduling
Maintenance Scheduling
Genetic Algorithm
Multi-Skilled Resource-Constrained Scheduling

A B S T R A C T

In this article, the application of the Multi-Skilled Resource-Constrained Flow Shop Scheduling Problem (MSRC-FSSP) in preventive maintenance as a case study has been investigated. In other words, to complete each maintenance order at each stage, in addition to the machine, a set of required human resources with different skills must be available. According to human resources skills, each of them can perform at least one order or at most N orders, and each maintenance order must be done by a set of human resources with different skills. To carry out a maintenance order, different human resources must be in communication and cooperation so that a preventive maintenance order can be completed. In this article, these resources are considered as technical supervisors, repairmen and maintenance managers who complete all maintenance orders in a flow shop environment as a job. For this problem, a new Mixed Integer Linear Programming (MILP) model has been formulated with the two-objective functions, minimizing total orders completion time and the human resources idle time. To solve the model on a small scale, CPLEX is used, and to solve it on a large scale, due to the fact that this problem is NP-Hard, a meta-heuristic algorithm named Genetic Algorithm (GA) is presented. Finally, the computational results have been done to validate the model, along with the analysis of the human resources idle time.

1. Introduction

In general, scheduling problems are known based on three parameters α|β|γ. The first parameter, α, shows the environment in which the problem is formulated, which in this article is the flow shop. The second parameter, β, details the processes, constraints, and assumptions of the model. In the real world, one of the important issues in the flow shop scheduling problems is the point that in some stages of the job processing, there must be several human resources available to be able to do that job, and this important issue has always been simplified in scheduling problems. Most of the researchers assumed that job processing in flow shop or job shop models depends on having a resource, which is often known as a machine. In fact, in this research, the innovation presented is to consider multi-skilled resource-constrained to operate any job at each stage as a β parameter. Finally, the third parameter, γ, shows the objective functions of the model, which are the objective functions used in this problem are minimizing the total completion time of maintenance orders and minimizing the human resources idle time. Due to the high maintenance costs with a large number of equipment with advanced technologies and their own maintenance instructions in a company, maintenance order scheduling with the availability of specific limited resources is a high priority for these companies. Certainly, in a very large company with many equipment, each with different technologies, there will be a large number of maintenance orders with different resources. Managing and

* Corresponding author.
E-mail address: mheydari@iust.ac.ir (M. Heydari)

doi: 10.5267/dsl.2022.10.007

scheduling them is very important for the high productivity of the system. In fact, the innovation of this research can be summarized as follows:

- The simultaneous presence of a set of human resources with different skills is necessary to carry out maintenance orders at every stage.
- This research is MSRC-FSSP which, in addition to the objective function of completion time, shows a complete analysis of the idle time of available resources according to different human resource costs.

The outline of this study is as below:
In section (2), an overview of the relevant literature review is given. In section (3), the application of the problem for preventive maintenance is stated. Formulation of MSRC-FSSP is described in section (4). In section (5) two numerical examples in a preventive maintenance environment with solution analysis are presented. At the end, a summary of our investigations and the outlook of upcoming research is presented in section (6).

2. Literature Review

Maintenance scheduling problems often require consideration of multiple objective functions such as maximizing productivity and reliability or minimizing cost and failure risk. For example, Azadeh et al. (2016) have presented a new mathematical model for maintenance strategies to increase reliability and reduce the failure risk of energy flows of the electrical power sector. It should also be added that complex technical systems maintenance easily turns into a multi-objective optimization (MOO) problem (Zio, 2009). The MOO approach is commonly used to solve design and organizational problems in reliability, availability, maintainability, and safety engineering (Coit & Zio, 2019). Keizer et al. (2017) have described that resource dependence occurs when components such as spare parts, tools, or maintenance repairmen are jointly required to operate a job. Repair tools, spare parts (Nguyen et al., 2017), and budget availability (Mild & Salo, 2009) were recognized as instances of resource dependencies (De Jonge & Scarf, 2020).

Award and Ertem (2017) have developed a model with the objective function of maximizing the number of completed jobs and minimizing the average resource shortage for scheduling preventive maintenance orders considering human resources with different skills and costs. Also, the time of processing maintenance orders is considered as a probability distribution. Indeed, the researchers simplified the hypothesis of the availability of several human resources with different skills to process orders. Witteman (2021) has researched an aircraft fleet maintenance task allocation model with a multi-year time horizon. The author made her model closer to the reality by adding the constraints of job delay, job interval, and repeatability of preventive maintenance orders. Indeed, it should be acknowledged that in reality, the scheduling and implementation of preventive maintenance is not a one-step model and must be done as a flow shop model with multi-step. Urbani et al. (2022) have developed a maintenance scheduling model as a network system where nodes represent machines or workers, and edges represent the exchange of materials, information or work between these nodes. They formulated a two-objective optimization problem to find efficient maintenance schedules. They proposed an algorithm that was a good approximation of the Pareto frontier in terms of cost and efficiency considering the limited availability of maintenance repairmen. Also, through sensitivity analysis, they showed how much the addition of maintenance repairmen improves system productivity despite the increase in maintenance costs and idle time of some resources. Ertem et al. (2022) have devised two mixed-integer mathematical models along with efficient solution algorithms for the flexible multi-skill resource-constrained shutdown maintenance scheduling problem. They have investigated how to better allocate high-skilled resources to tasks to reduce the average shutdown completion time. These researchers have not considered the importance of resource idle time and have simplified the allocation of multi-skilled resources to process a task.

Another scheduling problem is scheduling in flexible production systems. The main reference for resource-constrained Flexible Manufacturing Systems (FMS) is written by (Blazewicz et al, 2019), where job execution is considered regardless of precedence constraints. Cheng et al. (2012) have analyzed the process of disposal and renovation of buildings as a flow shop problem with limited resources. The authors formulated the problem as mixed integer programming and regarding their complexity, they developed polynomial algorithms. Costa et al. (2020) have modeled a flexible job shop problem with limited human resources such that there are several machines in each stage, the job on each of the machines can be done, and the number of workers is less than the number of machines in each stage. In the event that the skill of each human resource is considered the same and the presence of a set of human resources to do the work has not been investigated. Yunusoglu and Yildiz (2022) have studied the multi-resource-constrained unrelated parallel machine scheduling problem under various operational constraints with the objective of minimizing maximum completion time among the scheduled jobs. The purpose of these researchers was to consider factors such as sequence-dependent setup times, precedence relations, machine eligibility restrictions and release date as operational constraints in the problem, and in this study to reduce the idle time of resources according to the cost of these resources has not been considered. Also, the allocation of several resources to operate a job was simplified in this research.

Also, most of the researches done on scheduling problems, other than the resource- constraint flow shop scheduling, are the problems related to project management and planning, which in the literature are called Resource- Constrained Multi-Project Scheduling Problem (RCPSP) is known to have grown faster in the past decades. RCPSP is a problem with constant processing time, activity with limited resource and precedence relation between all of activities that generally is shown through Activities on Node (AON) (Hans et al., 2007). Habibi et al. (2018) the topics related to RCPSP have been reviewed, recent developments in this field have been evaluated, and the results have been presented for future studies. They have investigated related developments presented from four aspects of resources, characteristics of activities, type of objective functions, and availability level of information.

Kazemipoor et al. (2012) have presented a MILP for a multi-skilled project scheduling problem (MSPSP). In their study, the idle time of resources was not considered as an objective function and, like other RCPSP models, the activities completion time was investigated. Also, they did not consider the constraints of the flow shop scheduling problem. Snauwaert and Vanhoucke (2022) have analyzed the multi-skilled resource-constrained project scheduling problem (MSRCPSP). They have studied the impact of skill availability, workforce size and multi-skilling on the makespan of the project.

Golab et al. (2022) have developed a multi-layer feed-forward neural network to solve the standard single- mode RCPSP. Their developed MLFNN learns based on eight project parameters, namely network complexity, resource factor, resource strength, average work per activity, percentage of remaining work, etc., which are calculated at each step of project scheduling, and identified priority rules, which are the outputs of the developed neural network.

In RCPSP, there are a large number of limited renewable resources which should be used in proportion to the volume of the activities while the idle time of the multi-skilled resources in a project is not important. To the best of our knowledge, researchers insist on resource balancing in comparison with resource idle time. If the purpose of this research is to consider the costs of resources idle time, the presence of a set of multi-skilled resources to process the job and the constraints of a flow shop scheduling problem.

3. Problem Definition

In this article, we will investigate the application of MSRC-FSSP in preventive maintenance as a case study. In order to carry out a maintenance order, different human resources must be in cooperation so that a preventive maintenance order can be completed. These resources can be technical supervisors, repairmen and maintenance managers. Processing maintenance orders is considered as a flow shop environment, the steps of which include the initial approval of technical supervisors for order instructions, the execution of order by repairmen and technical supervisors, and the supervision of the correct performance of maintenance order by maintenance managers and technical supervisors with different skills. It should also be noted that each of the maintenance orders is performed on one of the equipment, and the equipment must be one of the available non-human resources. Also, the following assumptions are considered in this research:

- In this research, for the equipment, only the orders related to the preventive maintenance, which are in the maintenance manual of each equipment, are considered. So the movement path of order in each stage of the flow shop environment and their operation time by each set of resources are determined.
- In the orders related to the preventive maintenance, no assumptions about the orders delays, the time intervals of the order have been considered, and only the total time of the execution of the orders, which is the most important parameter in the scheduling of the preventive maintenance orders, has been examined.
- All spare parts and consuming materials are available in stock for preventive maintenance.
- It is assumed that the order related to the corrective or predictive maintenance when operating the preventive maintenance can also be done during the same review, and this will increase the duration of the job and the human resource required according to the "unusual rates" estimated. It is considered from historical data.

According to Fig (1), the concept of multi-skilled human resources can be explained. For the case study reviewed in this article, three different human resources of technical supervisors, repairmen and maintenance managers are defined. The resource of type one is the technical supervisors to confirm the order instructions. According to the type of maintenance orders, these supervisors must cooperate with each other and without the presence of even one required supervisor, this order will not be done. For example, for the initial approval of an order according to its instructions, S_1, S_2, and S_3 technical supervisors must be present, and for other types of maintenance orders, S_6 and S_3 technical supervisors must be present to approve the relevant instructions. Now, due to their limitations and expertise, these technical supervisors may have the necessary cooperation in approving some special instructions. For example, the S_1 supervisor must be present in the approval of the instructions for two maintenance orders, and without the presence of this supervisor as a resource, those maintenance orders will not be carried out. Resources of type two are considered as maintenance repairmen in the implementation of maintenance orders. These ten repairmen may cooperate with the six technical supervisors in the second stage of the maintenance order implementation. For example, to process the maintenance order in the second stage, S_3 and S_6 technical supervisors must cooperate with repairmen R_8 and R_9. Even if a required resource is not available at the execution time of that order, the operation of the maintenance order is postponed so that four resources are present at the same time. Also, the third type of resources are the maintenance managers who can cooperate with the technical supervisors or repairmen in the third stage of the maintenance order processing.

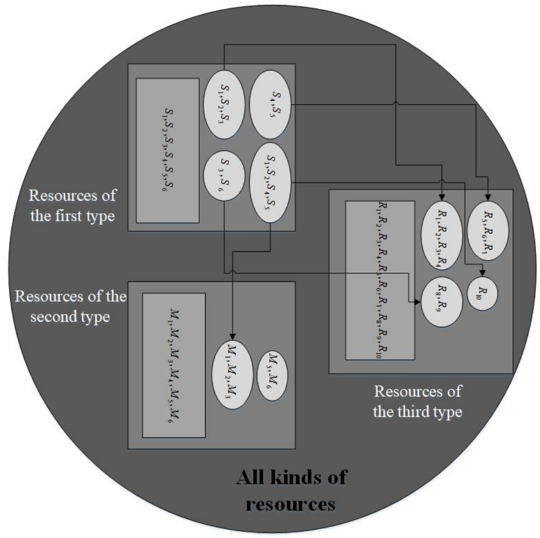

Fig 1. Concept of multi-skilled human resources

4. Mixed Integer Linear Programming Considering Human Resource Constraints

In this section, a MILP is modeled where a set of multi-skilled resources cooperate to process the order in each machine. It should be noted in the flow shop problem, the machine refers to the steps of job processing. Here, the meaning of machine has a different meaning than equipment. In the following sections, indices, parameters, decision variables and the developed model are described.

In this section, to formulate the mathematical model of this problem, a matrix is needed that shows the commonalities and interference of human resources of each order in each machine. This matrix is an upper triangular square matrix whose main diameter is zero. The commonalities and interactions of human resources required to perform orders on each machine are specified with values of 0 or 1. If there is at least one resource in common to compare the resources required to perform two orders on each machine, the value 1 and otherwise the value 0 is written in the matrix for comparing these two orders. Also, the sequence of rows and columns of this matrix is such that first, the sequence of orders and then the sequence of machines are determined. For example, the first order is considered in the first machine and then the second order in the first machine and so on. As a result, we will have a square matrix with dimension $N \times Q$ so that N represents the total number of orders and Q represents the total number of machines or stages in the flow shop problem. For example, Fig (2) represents a matrix for performing N orders in Q machines. This matrix shows the common interactions of multi-skilled human resources. For better understanding, for example, the first order in the first machine has resource interference with the second order in the second machine, and they cannot share time to operate the order, because the matrix element related to row 1, 1 and column 2, 2 has a value of 1.

	1,1	2,1	…	N,1	1,2	2,2	…	N,2	…..	1,Q	2,Q	…	N,Q
1,1	0	1		0	1	1		1		1	0		0
2,1	-	0		1	0	1		1		0	1		0
…	-	-	0										
N,1	-	-	-	0	0	1		1		0	1		1
1,2	-	-	-	-	0	0		1		1	0		0
2,2	-	-	-	-	-	0		1		0	1		1
…	-	-	-	-	-	-	0						
N,2	-	-	-	-	-	-	-	0		1	1		0
⋮	-	-	-	-	-	-	-	-	0				
1,Q	-	-	-	-	-	-	-	-	-	0	1		1
2,Q	-	-	-	-	-	-	-	-	-	-	0		0
…	-	-	-	-	-	-	-	-	-	-	-	0	
N,Q	-	-	-	-	-	-	-	-	-	-	-	-	0

Fig. 2. Common interactions of multi-skilled human resources matrix A

4.1. Indices

N	Number of orders.
Q	Number of Machines.
H	Number of required resources.
M	An extremely large number.
$k, l = \{1, 2, \dots, N\}$	Order indices.

$h = \{1, 2, \dots, H\}$	Resource indices.
$j, j' = \{1, 2, \dots, Q\}$	Machine indices.
$x = \{1,2, \dots, N \times Q\}$	The indices rows of the matrix *A*.
$y = \{1,2, \dots, N \times Q\}$	The indices columns of the matrix *A*.

4.2. Parameters

$P_{k,j}$	Processing time of order *k* at machine *j*.
$C_{k,j}$	Completion time of order *k* at machine *j*.
C_k	Completion time of order *k* in planning horizon.
α_k	Cost coefficient for completion time of order *k* in planning horizon.
ST_h	Start time of resource *h*.
FT_h	Finish time of resource *h*.
IT_h	Idle time of resource *h*.
λ_h	Cost coefficient for idle time of resource *h* in planning horizon.
a_{xy}	element related to row *x* and column *y* of matrix *A*.

4.3. Decision Variables

$X_{k,j}$	Start time of order *k* at machine *j*.
$P_{k,j,l,j'}$	if order *l* is processed at machine j' earlier than order *k* at machine *j*.

4.4. MILP mathematical model

$$minZ_1 = \sum_{k=1}^{k=N} \alpha_k \, C_k \tag{1}$$

$$minZ_2 = \sum_{h=1}^{h=H} \lambda_h \, IT_h \tag{2}$$

$$C_{k,j} = X_{k,j} + P_{k,j} \qquad \forall k, j \tag{3}$$

$$C_k = X_{k,Q} + P_{k,Q} \qquad \forall k \tag{4}$$

$$ST_h = Min\{X_{k,j}\} \qquad All(k,j)\epsilon h \tag{5}$$

$$FT_h = Max\{X_{k,j} + P_{k,j}\} \qquad All(k,j)\epsilon h \tag{6}$$

$$IT_h = (FT_h - ST_h) - \sum_{All(k,j)\epsilon h} P_{k,j} \qquad \forall h \tag{7}$$

$$C_{k,j} \le X_{k,j+1} \qquad \forall(k\epsilon N), (j = \{1,2, \dots, Q-1\}) \tag{8}$$

$$a_{xy}[C_{k,j}] \le a_{xy}[X_{l,j'} + MP_{k,j,l,j'}] \qquad \forall(k \ne l, (k, l\epsilon N)), (j, j'\epsilon Q), (a_{xy}\epsilon A) \tag{9}$$

$$a_{xy}[C_{l,j'}] \le a_{xy}[X_{k,j} + M(1 - P_{k,j,l,j'})] \qquad \forall(k \ne l, (k, l\epsilon N)), (j, j'\epsilon Q), (a_{xy}\epsilon A) \tag{10}$$

$$X_{k,j}\epsilon\ Z(Integer\ Variable) \qquad \forall k, j \tag{11}$$

$$P_{k,j,l,j'}\epsilon\{0,1\} \qquad \forall(k \ne l, (k, l\epsilon N)), (j, j'\epsilon Q) \tag{12}$$

Constraint (1) shows the first objective function of the model, which is the minimization of the total weighted completion time, and constraint (2) shows the second objective function of the model, which is the minimization of the total weighted resources idle time. Constraint (3) specifies the completion time of the order *k* in the machine *j*, which is equal to the sum of the order *k* start time in the machine *j* and the order *k* processing time in the machine *j*. Constraint (4) also determines the total completion time of the order *k* in the planning horizon, which is equal to the amount of the completion time of the order *k* in the machine *Q*. Constraint (5) shows how to calculate the starting time of resource *h*, which is equal to the lowest value of the resource h starting time that is in machine *j*. Constraint (6) shows how to calculate the completion time of the resource *h*, which is equal to the maximum amount of the resource *h* completion time in the machine *j*. constraint (7) also shows how to calculate the idle time of each resource, which is the order completion time minus the order start time minus the total processing time of the resource *h* in a specific planning horizon. Constraint (8) creates the requirement that the prerequisite relations of the order *k* are established in the machine *j* so that before the operation of the order *k* is not finished in the machine *j*, the operation of that order cannot be started in the next machines *{j+1, j+2,…, Q}*. Constraints (9) and (10) cause that different orders in machines j, j' (j, j' could be equal) that have at least one common resource cannot have time interference with each other and the start time of one of them should be after the other completion time. In other words, orders that interfere with each other's resources cannot interfere with each other's time. Constraint (11) defines the problem decision variable as an integer and constraint (12) also defines the problem decision variable as an integer {0, 1}.

5. Analysis of numerical results

In this section, for the analysis of the presented mathematical model, a small-scale numerical example and a large-scale numerical example for the scheduling of preventive maintenance orders are given. The optimal and exact solution of the first numerical example was done with CPLEX, and the second numerical example was solved with GA due to the NP-hard of the problem, which will be explained in the next section.

5.1. Genetic Algorithm

GA belongs to the category of evolutionary algorithms. Evolutionary Algorithms (EAs), a class of heuristic search techniques inspired by survival-of-the-fittest Darwinian evolution principles, work iteratively on a population of candidate solutions of the given problem. Unlike some other efficient meta-heuristics, EAs are flexible and therefore they have been successfully applied to many single and multi-objective optimization problems. In optimization problems, GAs are used to find the strong solution for some scheduling problems and maximum utilization problems (Cormen et al., 2009). For this reason, a GA is proposed to solve the problem discussed in this research.

5.1.1. Solution encoding (Chromosome)

Encoding the solutions in the proposed GA consists of a *(k)* × *(j)* matrix, where *k* are orders and *j* are machines in a flexible flow shop environment. Columns *j* show the start time of orders in different machines. Table (1) shows an example solution encoded with *N=8* and *Q=3*.

Table 1
Structure of Solution in the proposed GA

	j=1	j=2	j=3
k=1	14	18	22
k=2	10	13	13
k=3	38	43	47
k=4	1	8	18
k=5	22	29	34
k=6	0	1	4
k=7	1	7	11
k=8	7	14	18

5.1.2. Initial population

The initial population is randomly and legally determined by selecting *npop* solution matrices, where *npop* is the population size. *npop* which is obtained by Taguchi method.

5.1.3. Fitness function

For the model discussed in this research, the fitness function is sum of the negative of total the orders completion time and total the resources idle time so that each of them has a coefficient $(-[\sum_{k=1}^{k=N} \alpha_k\, C_k + \sum_{h=1}^{h=H} \lambda_h\, IT_h])$ and the penalty for violating the restrictions is determined by the relative degree of infeasibility. If the solution is out of bounds, this penalizes the final solution by adding a relatively large amount of penalty to the objective function.

5.1.4. Crossover & mutation operator

In the GA designed in this research, parents are selected based on the performance of the fitness function and the crossover operator is performed with a predefined rate of *rC* which is obtained by Taguchi method.

14	18	22		19	28	41
10	13	13		7	10	10
_ _	_ _	_ _		_ _	_ _	_ _
38	43	47		23	33	37
1	8	18	And	1	7	15
22	29	34		21	28	33
0	1	4		2	3	6
1	7	11		1	9	13
7	14	18		7	14	18

↓

19	28	41		14	18	22
7	10	10		10	13	13
38	43	47		23	33	37
1	8	18	And	1	7	15
22	29	34		21	28	33
0	1	4		2	3	6
1	7	11		1	9	13
7	14	18		7	14	18

Fig 2. Single-point crossover operator

For more variety, a single-point crossover operator, a two-point crossover operator, and a uniform crossover operator are used. In a single-point crossover operator, two parents are selected. Then a row between 1 and s-1 is randomly selected, and all the rows after this point in each parent are exchanged with the same rows in the other parent. An example for single-point crossover is shown in Fig 2. In double-point crossover, two points are randomly selected and the genes related to the jobs between them in the parents are exchanged with each other. In uniform crossover, all the genes related to each job are exchanged in the parents with the probability of 0.5. In the other words, the genes related to each job for each offspring are selected from one of the parents with an identical probability, and the genes related to the job for the other offspring are selected from the other parent. Also, the mutation operator is characterized by a predefined rate of *rM* which is obtained by Taguchi method. For the purposes of this study, a random mutation was used. This mutation is implemented by randomly selecting parents. Mutation operator is used at a predefined rate of rM which is obtained by Taguchi method. Use has been made, for the purposes of this study, of a random mutation. The mutation is implemented by random selection of parents. In the random mutation, the gene values of the parent chromosome are subject to change with the probability of pr_m. In this operator, the value of all the genes of each job are mutated with the probability of pr_m. An example for random mutation is shown in Fig. 3, in which the gene values of jobs 1, 4 and 6 are mutated.

→	19	28	41		2	11	26
	7	10	10		7	10	10
	38	43	47		38	43	47
→	1	8	18	→	12	19	29
	22	29	34		22	29	34
→	0	1	4		2	3	6
	1	7	11		1	9	11
	7	14	18		7	14	18

Fig. 3. Random mutation operator

5.1.5. Selection

Crossover and mutation operators transfer one generation to the next. For this purpose, the *npop* best solutions among the previous generation and the new children are kept according to their fitness function and the roulette wheel method so that the next generation is produced.

5.1.6. Stopping condition

The GA should stop after a certain predefined number of iterations. In this research, the GA ends with the number of iterations of 200.

5.2. Taguchi Method

Moreover, Taguchi method is used to tune GA parameters such as population size, crossover rate and mutation rate.

5.3. Optimal solution of small-scale numerical example with CPLEX

Table 2 shows a numerical example for the solution and analysis of the MILP model, which is a small scale of preventive maintenance orders. In this model, each of the orders in each of the machines can be done by a set of multi-skilled human resources on several specific equipment. In this numerical example, we have 3 machines, so $Q = 3$, there are 8 preventive maintenance orders to be processed, so $N = 8$, 7 human resources and 3 equipment resources to operate orders, so $H = 10$. To solve this example, the cost coefficients of the two-objective functions, the total completion time of 8 orders and the total idle time of 7 human resources are considered the same.

Table 2
Order sequencing considering required human resources and equipment for small-scale example.

$k = 1 \rightarrow \overbrace{P_{1,1} = 4}^{M_1,R_1,R_2} \rightarrow \overbrace{P_{1,2} = 3}^{M_1,R_4} \rightarrow \overbrace{P_{1,3} = 7}^{M_1,R_1,R_2,R_6}$	$k = 5 \rightarrow \overbrace{P_{5,1} = 7}^{M_3,R_3} \rightarrow \overbrace{P_{5,2} = 5}^{M_3,R_1,R_5} \rightarrow \overbrace{P_{5,3} = 4}^{M_3,R_2,R_6,R_7}$
$k = 2 \rightarrow \overbrace{P_{2,1} = 3}^{M_2,R_2,R_3} \rightarrow \overbrace{P_{2,2} = 0}^{Bypass} \rightarrow \overbrace{P_{2,3} = 1}^{M_2,R_2,R_6,R_7}$	$k = 6 \rightarrow \overbrace{P_{6,1} = 1}^{M_2,R_2,R_3} \rightarrow \overbrace{P_{6,2} = 3}^{M_2,R_4,R_5} \rightarrow \overbrace{P_{6,3} = 0}^{Bypass}$
$k = 3 \rightarrow \overbrace{P_{3,1} = 5}^{M_1,R_1,R_2} \rightarrow \overbrace{P_{3,2} = 4}^{M_1,R_1,R_5} \rightarrow \overbrace{P_{3,3} = 4}^{M_1,R_1,R_2,R_6}$	$k = 7 \rightarrow \overbrace{P_{7,1} = 6}^{M_1,R_1,R_2} \rightarrow \overbrace{P_{7,2} = 4}^{M_1,R_4} \rightarrow \overbrace{P_{7,3} = 2}^{M_1,R_3,R_7}$
$k = 4 \rightarrow \overbrace{P_{4,1} = 2}^{M_3,R_3} \rightarrow \overbrace{P_{4,2} = 6}^{M_3,R_1,R_5} \rightarrow \overbrace{P_{4,3} = 4}^{M_3,R_2,R_6,R_7}$	$k = 8 \rightarrow \overbrace{P_{8,1} = 2}^{M_2,R_2,R_3} \rightarrow \overbrace{P_{8,2} = 3}^{M_2,R_4,R_5} \rightarrow \overbrace{P_{8,3} = 2}^{M_2,R_1}$

According to the sequence of processing the eight orders with different resources, common interactions of multi-skilled human resource matrix for this numerical example is in the form of Fig. 4.

	1,1	2,1	3,1	4,1	5,1	6,1	7,1	8,1	1,2	2,2	3,2	4,2	5,2	6,2	7,2	8,2	1,3	2,3	3,3	4,3	5,3	6,3	7,3	8,3
1,1	0	1	1	0	0	1	1	1	1	0	1	1	1	0	1	0	1	1	1	1	1	0	1	1
2,1	-	0	1	1	1	1	1	1	0	0	0	0	0	1	0	1	1	1	1	1	1	0	1	1
3,1	-	-	0	0	0	1	1	1	1	0	1	1	1	0	1	0	1	1	1	1	1	0	1	1
4,1	-	-	-	0	1	1	0	1	0	0	0	1	1	0	0	0	0	0	0	1	1	0	1	0
5,1	-	-	-	-	0	1	0	1	0	0	0	1	1	0	0	0	0	0	0	1	1	0	1	0
6,1	-	-	-	-	-	0	1	1	0	0	0	0	0	1	0	1	1	1	1	1	1	0	1	1
7,1	-	-	-	-	-	-	0	1	1	0	1	1	1	0	1	0	1	1	1	1	1	0	1	1
8,1	-	-	-	-	-	-	-	0	0	0	0	0	0	1	0	1	1	1	1	1	1	0	1	1
1,2	-	-	-	-	-	-	-	-	0	0	0	0	0	1	1	1	1	0	1	0	0	0	1	0
2,2	-	-	-	-	-	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3,2	-	-	-	-	-	-	-	-	-	-	0	1	1	1	1	1	1	0	1	0	0	0	1	1
4,2	-	-	-	-	-	-	-	-	-	-	-	0	1	1	0	1	1	0	1	1	1	0	0	1
5,2	-	-	-	-	-	-	-	-	-	-	-	-	0	1	0	1	1	0	1	1	1	0	0	1
6,2	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1	1	0	1	0	0	0	0	0	1
7,2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1	1	0	1	0	0	0	1	0
8,2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	1	0	0	0	0	0	1
1,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1	1	1	1	0	1	1
2,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1	1	1	0	1	1
3,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1	1	0	1	1
4,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1	0	1	0
5,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	1	0
6,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	0
7,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0
8,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0

Fig 4. Common interactions of multi-skilled human resource matrix for small-scale example

With CPLEX, the optimal solution of the two-objective functions, which is the total orders completion time and the total resources idle time, is 281. For this numerical example, we have 363 constraints with 632 decision variables and 576 integer variables. Table 3 shows orders start time, orders completion time and the optimal solution of the two-objective functions for this numerical example.

Table 3
Details of the optimal solution of the small-scale numerical example obtained by CPLEX

Number of orders	Cost coefficient of objective functions	Orders start time				Orders completion time				Optimal Solution	Running Time
			j=1	j=2	j=3		j=1	j=2	j=3		
8	$\alpha_k \& \lambda_h = 1$ $\forall k,h$	k=1	19	28	41	k=1	23	31	48	$Z_1 = 193$ $Z_2 = 88$	1hrs 32min
		k=2	7	10	10	k=2	10	10	11		
		k=3	23	33	37	k=3	28	37	41		
		k=4	1	7	15	k=4	3	13	19		
		k=5	21	28	33	k=5	28	33	37		
		k=6	0	1	4	k=6	1	4	4		
		k=7	1	9	13	k=7	7	13	15		
		k=8	11	13	16	k=8	13	16	18		

Also, to evaluate the validity and the feasibility of the solution from this example, in Fig. 5, the optimal order sequence in different machines can be seen from this example.

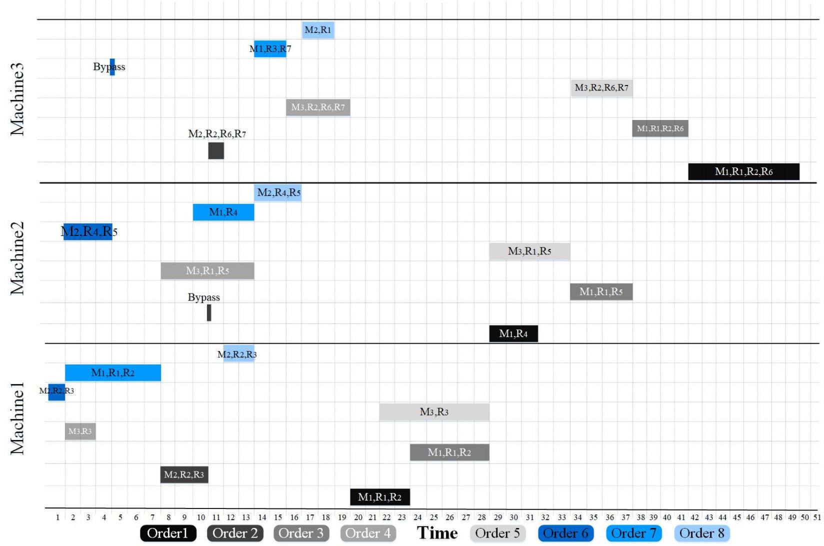

Fig. 5. Start time and order sequence of the small-scale numerical example

Fig. 6 shows the start time, finish time and idle time of each resource R_1 to R_7 according to the optimal orders sequence in planning horizon. In this example, taking into account that the cost coefficient of the resources idle time and the cost coefficient of the orders completion time are considered the same, the total resources idle time is 88.

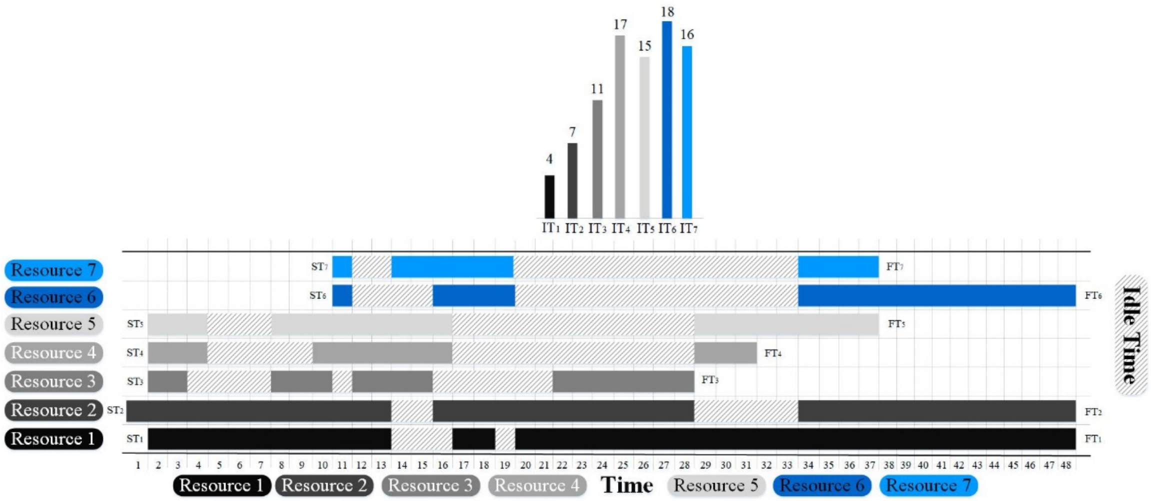

Fig. 6. Resources idle time according to the optimal orders sequence

As seen in Fig. 6, the idle time of the sixth resource is more than all other resources. Now it can be argued that if the idle cost coefficient of this resource is assumed to be very high compared to the idle cost coefficient of other resources and the cost coefficient of the orders completion time, the optimal sequence of orders will be according to Fig. 7. In other words, the first priority in the two-objective functions is that the idle of the sixth resource is not allowed, and the second priority is to minimize the total orders completion time. Table 4 shows the start time of orders processing, the finish time of the orders and the optimal solution of the new two-objective functions. According to the optimal sequence of orders, the idle time of the sixth resource is zero and the total orders completion time is 262.

Table 4
Details of the optimal solution of the small-scale numerical example considering the idle cost coefficient of the sixth resource is very high.

Number of orders	Cost coefficient of objective functions	Orders start time				Orders completion time				Optimal Solution
			j=1	j=2	j=3		j=1	j=2	j=3	
8	$\lambda_6 = \mathrm{M}$ $\alpha_k \& \lambda_h = 1$ $\forall k, h \neq 6$	k=1	22	31	40	k=1	26	34	47	$IT_6 = 0$ $Z_1 = 262$
		k=2	31	34	35	k=2	34	34	36	
		k=3	26	36	51	k=3	31	40	55	
		k=4	13	16	36	k=4	15	22	40	
		k=5	22	31	47	k=5	29	36	51	
		k=6	0	1	4	k=6	1	4	4	
		k=7	1	7	11	k=7	7	11	13	
		k=8	7	11	14	k=8	9	14	16	

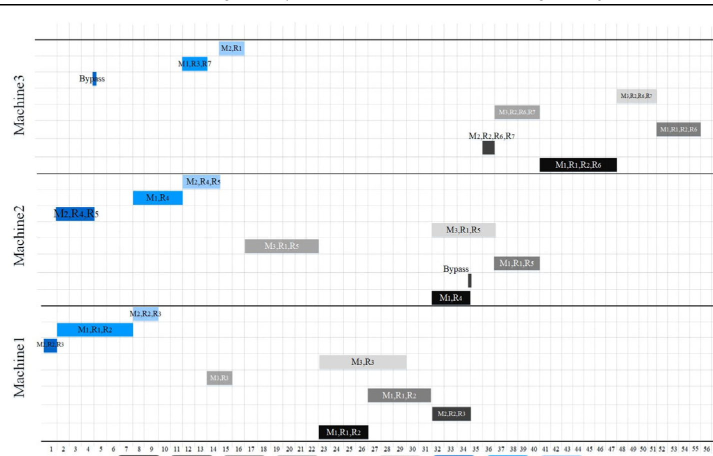

Fig. 7. Orders sequence of the small-scale numerical example considering the idle cost coefficient of the sixth resource is very high

5.4. Using the Taguchi method to tune the GA Parameters for small-scale example

In order to determine the value of the parameters of GA (including population size, crossover rate and mutation rate), three levels are considered for each of them. The suggested values are indicated in Table 5 as the levels of the parameters. The L9 array is selected with MINITAB based on the aforementioned levels. Fig 8 shows the signal-to-noise ratios (S/N). Since maximum S/N determines the optimal level for each factor, the optimal levels are shown in Table 5.

Table 5
Levels considered for GA parameters for small-scale example.

	Parameters		
	Factor 1. Population Size	Factor 2. Crossover Rate	Factor 3. Mutation Rate
Level 1	5000	0.2	0.05
Level 2	10000	0.3	0.1
Level 3	15000	0.4	0.15
The Best Level	**5000**	**0.3**	**0.15**

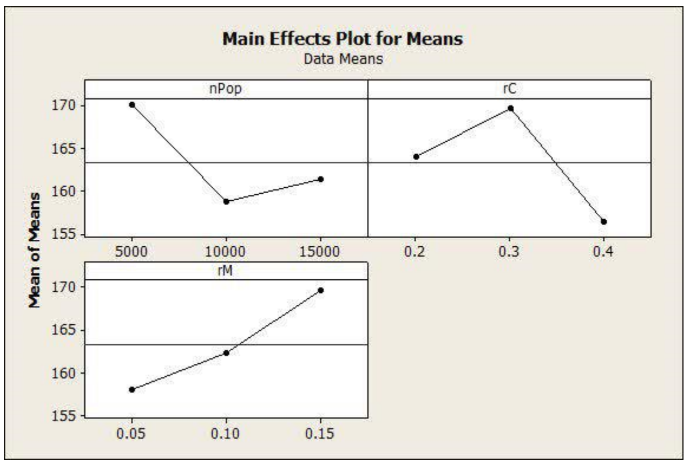

Fig 8. *S/N* ratios for tuning GA parameters.

5.5. Near-Optimal solution of small-scale numerical example with GA

Table 6 shows the near-optimal solution for the small-scale problem. Also, the gap between the objective function for the problem with 8 orders in the optimal solution obtained by CPLEX and GA shows the fact that GA has acceptable efficiency to solve the large-scale problem because the class of the problem is NP- hard. The proposed GA has the potential to solve each case study with any desired value for the number of orders and the number of human resources and equipment with different common interactions of the multi-skilled human resource matrix.

Table 6
Details of the near-optimal solution of the small-scale numerical example obtained by GA

Number of orders	Orders start time				Orders completion time				Optimal Solution	Gap Between CPLEX and GA
		j=1	j=2	j=3		j=1	j=2	j=3		
8	k=1	41	45	48	k=1	45	48	55	$Z_1 = 195$ $Z_2 = 102$	Gap $Z_1=2$ Gap $Z_2=14$
	k=2	7	10	10	k=2	10	10	11		
	k=3	19	24	28	k=3	24	28	32		
	k=4	3	7	15	k=4	5	13	19		
	k=5	23	32	37	k=5	30	37	41		
	k=6	0	1	4	k=6	1	4	4		
	k=7	1	8	13	k=7	7	12	15		
	k=8	11	13	16	k=8	13	16	18		

5.6. Large-scale numerical example

Table 7 shows a numerical example with the number of 16 orders in 3 machines whose operation time is specified in the table. As can be seen from the previous small-scale example, by calculating common interactions of the multi-skilled human resource matrix, it is possible to obtain the best sequence of orders in this problem. Appendix A shows the Common interactions of multi-skilled human resource matrix to process these 16 orders. Because CPLEX does not have the ability to solve the example on the scale of 16 orders, and according to Tao and Dong's research, it can be proved that the multi-skilled resource-constraint flexible flow shop scheduling problem on a large scale, is among NP-hard problems, a GA has been used to solve the problem on a large scale (Tao & Dong, 2017).

Table 7
Order sequencing considering required human resources and equipment for large-scale example.

$k=1 \rightarrow \overbrace{P_{1,1}=4}^{M_1,R_1,R_2} \rightarrow \overbrace{P_{1,2}=3}^{M_1,R_4} \rightarrow \overbrace{P_{1,3}=7}^{M_1,R_1,R_2,R_6}$	$k=9 \rightarrow \overbrace{P_{9,1}=3}^{M_4,R_4,R_8} \rightarrow \overbrace{P_{9,2}=2}^{M_4,R_1,R_2,R_9} \rightarrow \overbrace{P_{9,3}=3}^{M_4,R_1}$
$k=2 \rightarrow \overbrace{P_{2,1}=3}^{M_2,R_2,R_3} \rightarrow \overbrace{P_{2,2}=0}^{Bypass} \rightarrow \overbrace{P_{2,3}=1}^{M_2,R_2,R_6,R_7}$	$k=10 \rightarrow \overbrace{P_{10,1}=4}^{M_3,R_5,R_7,R_9} \rightarrow \overbrace{P_{10,2}=2}^{M_3,R_8} \rightarrow \overbrace{P_{10,3}=0}^{Bypass}$
$k=3 \rightarrow \overbrace{P_{3,1}=5}^{M_1,R_1,R_2} \rightarrow \overbrace{P_{3,2}=4}^{M_1,R_1,R_5} \rightarrow \overbrace{P_{3,3}=4}^{M_1,R_1,R_2,R_6}$	$k=11 \rightarrow \overbrace{P_{11,1}=2}^{M_2,R_1,R_4} \rightarrow \overbrace{P_{11,2}=4}^{M_2,R_4} \rightarrow \overbrace{P_{11,3}=2}^{M_2,R_9}$
$k=4 \rightarrow \overbrace{P_{4,1}=2}^{M_3,R_3} \rightarrow \overbrace{P_{4,2}=6}^{M_3,R_1,R_5} \rightarrow \overbrace{P_{4,3}=4}^{M_3,R_2,R_6,R_7}$	$k=12 \rightarrow \overbrace{P_{12,1}=5}^{M_4,R_2,R_7} \rightarrow \overbrace{P_{12,2}=3}^{M_4,R_9} \rightarrow \overbrace{P_{12,3}=1}^{M_4,R_3}$
$k=5 \rightarrow \overbrace{P_{5,1}=7}^{M_3,R_3} \rightarrow \overbrace{P_{5,2}=5}^{M_3,R_1,R_5} \rightarrow \overbrace{P_{5,3}=4}^{M_3,R_2,R_6,R_7}$	$k=13 \rightarrow \overbrace{P_{13,1}=2}^{M_1,R_2,R_4,R_7} \rightarrow \overbrace{P_{13,2}=5}^{M_1,R_8} \rightarrow \overbrace{P_{13,3}=4}^{M_1,R_8,R_9}$
$k=6 \rightarrow \overbrace{P_{6,1}=1}^{M_2,R_2,R_3} \rightarrow \overbrace{P_{6,2}=3}^{M_2,R_4,R_5} \rightarrow \overbrace{P_{6,3}=0}^{Bypass}$	$k=14 \rightarrow \overbrace{P_{14,1}=3}^{M_3,R_5,R_9} \rightarrow \overbrace{P_{14,2}=6}^{M_3,R_2,R_3,R_5} \rightarrow \overbrace{P_{14,3}=2}^{M_3,R_2}$
$k=7 \rightarrow \overbrace{P_{7,1}=6}^{M_1,R_1,R_2} \rightarrow \overbrace{P_{7,2}=4}^{M_1,R_4} \rightarrow \overbrace{P_{7,3}=2}^{M_1,R_3,R_7}$	$k=15 \rightarrow \overbrace{P_{15,1}=1}^{M_4,R_8} \rightarrow \overbrace{P_{15,2}=4}^{M_4,R_4,R_9} \rightarrow \overbrace{P_{15,3}=2}^{M_4,R_2,R_8}$
$k=8 \rightarrow \overbrace{P_{8,1}=2}^{M_2,R_2,R_3} \rightarrow \overbrace{P_{8,2}=3}^{M_2,R_4,R_5} \rightarrow \overbrace{P_{8,3}=2}^{M_2,R_1}$	$k=16 \rightarrow \overbrace{P_{16,1}=2}^{M_1,R_1} \rightarrow \overbrace{P_{16,2}=3}^{M_1,R_8} \rightarrow \overbrace{P_{16,3}=0}^{Bypass}$

5.7. Strong solution of large-scale numerical example with GA

Table 8 shows the tuning of three parameters of GA with the Taguchi method for large-scale problems. With the GA designed for large-scale problems, the optimal solution of the two-objective functions for a numerical example with 16 orders, which is the total orders completion time and the total resources idle time, is 1260. Table 9 shows the orders start time, the orders completion time, the resources idle time and the optimal solution of the two-objective functions for numerical examples with 16 orders. To solve this example, the cost coefficients of the two-objective functions, the total completion time of 16 orders and the total idle time of 9 human resources are considered the same. Also, the details of the GA implementation which are shown in Appendix (B).

Table 8
Levels considered for GA parameters for large-scale example.

	Parameters		
	Factor 1. Population Size	Factor 2. Crossover Rate	Factor 3. Mutation Rate
Level 1	5000	0.2	0.05
Level 2	10000	0.3	0.1
Level 3	15000	0.4	0.15
The Best Level	**5000**	**0.2**	**0.1**

Table 9
Details of the best solution of the large-scale numerical example obtained by GA

Number of orders	**Cost coefficient of objective functions**	**Orders start time**				**Orders completion time**				**Best Solution**
			j=1	j=2	j=3		j=1	j=2	j=3	
16	$\alpha_k \& \lambda_h = 1$ $\forall k, h$	k=1	83	88	91	k=1	87	91	98	$Z_1 = 679$ $Z_2 = 343$ $IT_1 = 45$ $IT_2 = 35$ $IT_3 = 34$ $IT_4 = 62$ $IT_5 = 30$ $IT_6 = 55$ $IT_7 = 43$ $IT_8 = 11$ $IT_9 = 28$
		k=2	20	23	23	k=2	23	23	24	
		k=3	68	73	79	k=3	73	77	83	
		k=4	40	44	50	k=4	42	50	54	
		k=5	13	20	26	k=5	20	25	30	
		k=6	37	41	44	k=6	38	44	44	
		k=7	54	61	66	k=7	60	65	68	
		k=8	10	13	16	k=8	12	16	18	
		k=9	27	30	32	k=9	30	32	35	
		k=10	32	36	38	k=10	36	38	38	
		k=11	1	3	7	k=11	3	7	9	
		k=12	3	9	12	k=12	8	12	13	
		k=13	8	12	17	k=13	10	17	21	
		k=14	54	60	66	k=14	57	66	68	
		k=15	35	36	41	k=15	36	40	43	
		k=16	25	30	33	k=16	27	33	33	

To evaluate the GA for solving the large-scale MSRC-FSSP problem, the lower bound for each of the objective functions was obtained separately and compared with the results of the GA for the two- objective functions. In other words, first the second objective function was removed and only the first objective function of the total completion time was considered, and then the first objective function was eliminated and only the second objective function of the resource idle time was considered. By calculating the results separately, the value of the first objective function was obtained 562 and the value of the second objective function was obtained 320. If these values are considered as the lower bound in the two-objective problem, the difference between the functions in the two-objective and single-objective problem is 117 for the objective function of jobs completion time and 23 for the objective function of resources idle time, which shows the efficiency of the GA for the two-objective problem.

6. Conclusion

In this article, the application of MSRC-FSSP in preventive maintenance is investigated as a case study. In this research, the MSRC-FSSP is formulated as a MILP model with two-objective functions, the total completion time of jobs and the total idle time of human resources. Because this large-scale problem belongs to NP-hard problems, a meta-heuristic GA was designed that can solve large-scale problems.

Future research on this article may be done on the time of processing maintenance orders is considered as a probability distribution considering human resource-constraints, uncertainty in maintenance orders considering corrective and predictive maintenance at each stage. Another alternative for future research could be to consider adding time window constraints for maintenance orders.

References

Awad, M., & Ertem, M. (2017). Stochastic scheduling of workforce-constrained preventive maintenance activities in petroleum plants. In *2017 Annual Reliability and Maintainability Symposium (RAMS) (pp.1-5). IEEE.*

Azadeh, A., Fekri, M., Asadzadeh, S. M., Barazandeh, B., & Barrios, B. (2016). A unique mathematical model for maintenance strategies to improve energy flows of the electrical power sector. *Energy Exploration & Exploitation, 34*(1), 19-41.

Blazewicz, J., Ecker, K. H., Pesch, E., Schmidt, G., Sterna, M., & Weglarz, J. (2019). *Handbook on Scheduling: From Theory to Practice. Springer*.

Cheng, T. E., Lin, B. M., & Huang, H. L. (2012). Resource-constrained flow shop scheduling with separate resource recycling operations. *Computers & Operations Research, 39*(6), 1206-1212.

Coit, D. W., & Zio, E. (2019). The evolution of system reliability optimization. *Reliability Engineering & System Safety, 192,* 106259.

Cormen, T. H., Leiserson, C. E., Rivest, R. L., Stein, C. (2009). *Introduction to algorithms. MIT press.*

Costa, A., Fernandez-Viagas, V., & Framinan, J. M. (2020). Solving the hybrid flow shop scheduling problem with limited human resource constraint. *Computers & Industrial Engineering, 146,* 106545.

De Jonge, B., & Scarf, P. A. (2020). A review on maintenance optimization. *European journal of operational research, 285*(3), 805-824.

Golab, A., Gooya, E., Falou, A., & Cabon, M. (2022). A multilayer feed-forward neural network (MLFNN) for the resource-constrained project scheduling problem (RCPSP). *Decision Science Letters, 11*(4), 407-418.

Ertem, M., As' ad, R., Awad, M., & Al-Bar, A. (2022). Workers-constrained shutdown maintenance scheduling with skills flexibility: Models and solution algorithms. *Computers & Industrial Engineering, 172,* 108575.

Habibi, F., Barzinpour, F., & Sadjadi, S. (2018). Resource-constrained project scheduling problem: review of past and recent developments. *Journal of project management, 3*(2), 55-88.

Hans, E. W., Herroelen, W., Leus, R., & Wullink, G. (2007). A hierarchical approach to multi-project planning under uncertainty. *Omega, 35,* 563–577. doi:10.1016/j.omega.2005.10.004.

Kazemipoor, H., Tavakkoli-Moghaddam, R., & Shahnazari-Shahrezaei, P. (2002). Solving a mixed-integer linear programming model for a multi-skilled project scheduling problem by simulated annealing. *Management Science Letters, 2*(2), 681-688.

Keizer, M. C. O., Flapper, S. D. P., & Teunter, R. H. (2017). Condition-based maintenance policies for systems with multiple dependent components: A review. *European Journal of Operational Research, 261(*2), 405-420.

Mild, P., & Salo, A. (2009). Combining a multi attribute value function with an optimization model: An application to dynamic resource allocation for infrastructure maintenance. *Decision Analysis, 6*(3), 139-152.

Nguyen, K. A., Do, P., & Grall, A. (2017). Joint predictive maintenance and inventory strategy for multi-component systems using Birnbaum's structural importance. *Reliability Engineering & System Safety, 168,* 249-261.

Snauwaert, J., & Vanhoucke, M. (2022). A classification and new benchmark instances for the multi-skilled resource-constrained project scheduling problem. *European Journal of Operational Research.* https://doi.org/10.1016/j.ejor.2022.05.049

Tao, S., & Dong, Z. S. (2017). Scheduling resource-constrained project problem with alternative activity chains. *Computers & Industrial Engineering, 114,* 970 288–296. doi:10.1016/j.cie.2017.10.027.

Urbani, M., Brunelli, M., & Punkka, A. (2022). An approach for bi-objective maintenance scheduling on a networked system with limited resources. *European Journal of Operational Research, 305*(1), 101-113.

Witteman, M., Deng, Q., & Santos, B. F. (2021). A bin packing approach to solve the aircraft maintenance task allocation problem. *European Journal of Operational Research*, *294*(1), 365-376.

Yunusoglu, P., & Topaloglu Yildiz, S. (2022). Constraint programming approach for multi-resource-constrained unrelated parallel machine scheduling problem with sequence-dependent setup times. *International Journal of Production Research, 60*(7), 2212-2229.

Zio, E. (2009). Reliability engineering: Old problems and new challenges. *Reliability engineering & system safety, 94*(2), 125-141.

Appendix

Appendix A. Common interactions of multi-skilled human resource matrix for large-scale example

	1,1	2,1	3,1	4,1	5,1	6,1	7,1	8,1	9,1	10,1	11,1	12,1	13,1	14,1	15,1	16,1	1,2	2,2	3,2	4,2	5,2	6,2	7,2	8,2
1,1	0	1	1	0	0	1	1	1	0	0	1	1	1	0	0	1	1	0	1	1	1	0	1	0
2,1	-	0	1	1	1	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	1	0	1
3,1	-	-	0	0	0	1	1	1	0	0	1	1	1	0	0	1	1	0	1	1	1	0	1	0
4,1	-	-	-	0	1	1	0	1	0	1	0	0	0	1	0	0	0	0	0	1	1	0	0	0
5,1	-	-	-	-	0	1	0	1	0	1	0	0	0	1	0	0	0	0	0	1	1	0	0	0
6,1	-	-	-	-	-	0	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	1	0	1
7,1	-	-	-	-	-	-	0	1	0	0	1	1	1	0	0	1	1	0	1	1	1	0	1	0
8,1	-	-	-	-	-	-	-	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1	0	1
9,1	-	-	-	-	-	-	-	-	0	0	1	1	1	0	1	0	1	0	0	0	0	1	1	1
10,1	-	-	-	-	-	-	-	-	-	0	0	1	1	1	0	0	0	0	1	1	1	1	0	1
11,1	-	-	-	-	-	-	-	-	-	-	0	0	1	0	0	1	1	0	1	1	1	1	1	1
12,1	-	-	-	-	-	-	-	-	-	-	-	0	1	0	1	0	0	0	0	0	0	0	0	0
13,1	-	-	-	-	-	-	-	-	-	-	-	-	0	0	0	1	1	0	1	0	0	1	1	1
14,1	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	0	0	0	1	1	1	1	0	1
15,1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	0	0	0	0	0	0	0	0
16,1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1	0	1	1	1	0	1	0
1,2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	1	0	0	1	1	1
2,2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	0	0	0	0	0
3,2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1	1	1	1	0
4,2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1	1	0	1
5,2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1	0	1
6,2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1	1
7,2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1
8,2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0

	9,2	10,2	11,2	12,2	13,2	14,2	15,2	16,2	1,3	2,3	3,3	4,3	5,3	6,3	7,3	8,3	9,3	10,3	11,3	12,3	13,3	14,3	`15,3	16,3
1,1	1	0	0	0	1	1	0	1	1	1	1	1	1	0	1	1	1	0	0	0	1	1	1	0
2,1	1	0	1	0	0	1	0	0	1	1	1	1	1	0	1	0	0	0	1	1	0	1	1	0
3,1	1	0	0	0	1	1	0	1	1	1	1	1	1	0	1	1	1	0	0	0	1	1	1	0
4,1	0	1	0	0	0	1	0	0	0	0	0	1	1	0	1	0	0	0	0	1	0	1	0	0
5,1	0	1	0	0	0	1	0	0	0	0	0	1	1	0	1	0	0	0	0	1	0	1	0	0
6,1	1	0	1	0	0	1	0	0	1	1	1	1	1	0	1	0	0	0	1	1	0	1	1	0
7,1	1	0	0	0	1	1	0	1	1	1	1	1	1	0	1	1	1	0	0	0	1	1	1	0
8,1	1	0	1	0	0	1	0	0	1	1	1	1	1	0	1	0	1	0	1	1	0	1	1	0
9,1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	1	0	0	1	1	0	1	0
10,1	1	1	0	1	0	1	1	0	0	1	0	1	1	0	1	0	0	0	1	0	1	0	0	0
11,1	1	0	1	0	0	0	1	0	1	1	1	0	0	0	0	1	1	0	1	0	0	0	0	0
12,1	1	0	0	1	0	1	1	0	1	1	1	1	1	0	1	0	1	0	0	1	0	1	1	0
13,1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	1	1	1	0
14,1	1	1	0	1	0	1	1	0	0	0	0	1	1	0	0	0	0	0	1	0	1	1	0	0
15,1	1	1	0	1	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	1	1	1	1	0
16,1	1	0	0	0	1	0	0	1	1	0	1	0	0	0	1	1	1	0	0	0	1	0	0	0
1,2	0	0	1	0	1	0	1	1	1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0
2,2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3,2	1	0	0	0	1	1	0	1	1	0	1	0	0	0	1	1	1	0	0	0	1	0	0	0
4,2	1	1	0	0	0	1	0	0	1	0	1	1	1	0	0	1	1	0	0	0	0	1	0	0
5,2	1	1	0	0	0	1	0	0	1	0	1	1	1	0	0	1	1	0	0	0	0	1	0	0
6,2	0	0	1	0	0	1	1	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0
7,2	0	0	1	0	1	0	1	1	1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0
8,2	0	0	1	0	0	1	1	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0

	9,2	10,2	11,2	12,2	13,2	14,2	15,2	16,2	1,3	2,3	3,3	4,3	5,3	6,3	7,3	8,3	9,3	10,3	11,3	12,3	13,3	14,3	`15,3	16,3
9,2	0	0	0	1	0	1	1	0	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1	0
10,2	-	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	1	1	0
11,2	-	-	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0
12,2	-	-	-	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	1	1	0	1	0
13,2	-	-	-	-	0	0	0	1	1	0	1	0	0	0	1	0	0	0	0	0	1	0	1	0
14,2	-	-	-	-	-	0	0	0	1	1	1	1	1	0	1	0	0	0	0	1	0	1	1	0
15,2	-	-	-	-	-	-	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	0	1	0
16,2	-	-	-	-	-	-	-	0	1	0	1	0	0	0	1	0	0	0	0	0	1	0	1	0
1,3	-	-	-	-	-	-	-	-	0	1	1	1	1	0	1	1	1	0	0	0	1	1	1	0
2,3	-	-	-	-	-	-	-	-	-	0	1	1	1	0	1	0	0	0	1	0	0	1	1	0
3,3	-	-	-	-	-	-	-	-	-	-	0	1	1	0	1	1	1	0	0	0	1	1	1	0
4,3	-	-	-	-	-	-	-	-	-	-	-	0	1	0	1	0	0	0	0	0	0	1	1	0
5,3	-	-	-	-	-	-	-	-	-	-	-	-	0	0	1	0	0	0	0	0	0	1	1	0
6,3	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	0	0	0	0	0	0	0	0	0
7,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	0	0	0	1	1	0	0	0
8,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1	0	1	0	0	0	0	0
9,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	0	1	0	0	1	0
10,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	0	0	0	0	0
11,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	1	0	0	0
12,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	0	1	0
13,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	1	0
14,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	1	0
15,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0
16,3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0

Appendix B. Implementation of the proposed GA, it has been executed ten times in 200 iterations

Fitness	Total Time	Total Idle	Feasibility	No	Fitness	Total Time	Total Idle	Feasibility	No	Fitness	Total Time	Total Idle	Feasibility
1736	768	388	No	68	1093	689	355	No	135	1039	687	353	Yes
1669	738	411	No	69	1093	689	355	No	136	1039	687	353	Yes
1610	695	435	No	70	1093	689	355	No	137	1039	687	353	Yes
1610	695	435	No	71	1088	689	362	No	138	1039	687	353	Yes
1610	695	435	No	72	1071	688	363	No	139	1039	687	353	Yes
1570	694	316	No	73	1071	688	363	No	140	1039	686	353	Yes
1570	694	316	No	74	1071	688	363	No	141	1039	686	350	Yes
1561	767	454	No	75	1071	688	363	No	142	1039	686	350	Yes
1444	677	387	No	76	1071	688	363	No	143	1039	686	350	Yes
1407	676	391	No	77	1070	690	360	No	144	1039	686	350	Yes
1407	676	391	No	78	1069	689	360	No	145	1039	686	350	Yes
1407	676	391	No	79	1069	689	360	No	146	1036	686	350	Yes
1407	676	391	No	80	1069	689	360	No	147	1035	685	350	Yes
1407	676	391	No	81	1069	689	360	No	148	1035	685	346	Yes
1383	705	398	No	82	1068	687	361	No	149	1035	685	346	Yes
1348	713	375	No	83	1052	691	361	Yes	150	1035	685	346	Yes
1341	685	376	No	84	1052	691	361	Yes	151	1035	685	346	Yes
1341	685	376	Yes	85	1052	691	361	Yes	152	1035	685	346	Yes
1313	672	381	No	86	1052	691	361	Yes	153	1032	686	346	Yes
1313	672	381	No	87	1052	691	361	Yes	154	1032	686	346	Yes
1294	694	380	No	88	1052	691	361	Yes	155	1032	686	346	Yes
1277	687	370	No	89	1050	688	362	Yes	156	1032	686	346	Yes
1277	687	370	No	90	1049	690	359	Yes	157	1032	686	346	Yes
1200	691	369	No	91	1047	687	360	Yes	158	1032	686	346	Yes
1200	691	369	No	92	1047	687	357	Yes	159	1030	684	346	Yes
1200	691	369	No	93	1046	689	357	Yes	160	1030	684	346	Yes
1192	690	362	No	94	1045	688	357	Yes	161	1030	684	346	Yes
1192	690	362	No	95	1045	688	357	Yes	162	1029	683	346	Yes
1192	690	362	No	96	1045	688	357	Yes	163	1029	683	346	Yes
1175	688	367	No	97	1045	688	357	Yes	164	1029	683	346	Yes
1175	688	367	No	98	1045	688	357	Yes	165	1029	683	346	Yes
1173	690	363	No	99	1045	688	357	Yes	166	1029	683	346	Yes
1171	688	363	No	100	1045	688	357	Yes	167	1027	683	345	Yes
1171	688	363	No	101	1045	688	357	Yes	168	1027	683	345	Yes
1171	688	363	No	102	1045	688	356	Yes	169	1027	683	345	Yes
1167	687	360	No	103	1045	688	356	Yes	170	1027	682	345	Yes
1152	689	363	No	104	1045	688	356	Yes	171	1027	682	345	Yes
1148	688	360	No	105	1044	688	356	Yes	172	1027	682	345	Yes
1136	693	363	No	106	1044	688	356	Yes	173	1026	682	344	Yes
1136	693	363	No	107	1044	688	356	Yes	174	1026	682	344	Yes
1134	688	366	No	108	1044	688	356	Yes	175	1026	682	344	Yes
1128	687	361	No	109	1044	688	356	Yes	176	1026	682	344	Yes
1115	691	364	No	110	1044	688	356	Yes	177	1026	682	344	Yes
1115	691	364	No	111	1044	688	356	Yes	178	1026	682	344	Yes
1115	691	364	No	112	1044	688	356	Yes	179	1026	682	344	Yes
1115	691	364	No	113	1044	688	355	Yes	180	1026	682	344	Yes
1115	691	364	No	114	1044	688	355	Yes	181	1025	681	344	Yes
1115	691	364	No	115	1044	688	355	Yes	182	1025	681	344	Yes
1113	690	363	No	116	1043	688	355	Yes	183	1025	681	344	Yes
1112	689	363	No	117	1043	688	355	Yes	184	1025	681	344	Yes
1112	689	363	No	118	1043	688	355	Yes	185	1025	681	344	Yes
1111	688	363	No	119	1042	687	355	Yes	186	1025	681	344	Yes
1111	688	363	No	120	1042	687	354	Yes	187	1025	681	344	Yes
1111	688	363	No	121	1042	687	354	Yes	188	1024	681	343	Yes
1111	688	363	No	122	1042	687	354	Yes	189	1024	681	343	Yes
1111	688	363	No	123	1041	687	354	Yes	190	1024	681	343	Yes
1111	688	363	No	124	1041	687	354	Yes	191	1024	681	343	Yes
1111	688	363	No	125	1041	687	354	Yes	192	1024	681	343	Yes
1111	688	363	No	126	1041	687	354	Yes	193	1024	681	343	Yes
1102	687	355	No	127	1041	687	352	Yes	194	1024	680	344	Yes
1102	687	355	No	128	1041	687	352	Yes	195	1024	680	344	Yes
1095	691	355	No	129	1041	687	352	Yes	196	1024	680	344	Yes
1095	691	355	No	130	1041	687	352	Yes	197	1024	680	344	Yes
1093	689	355	No	131	1041	687	352	Yes	198	1024	680	344	Yes
1093	689	355	No	132	1039	687	352	Yes	199	1024	680	344	Yes
1093	689	355	No	133	1039	687	352	Yes	200	1023	680	343	Yes
1093	689	355	No	134	1039	687	352	Yes					