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Analysis of COVID-19 rapid antigen and PCR detection policy

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A B S T R A C T

After the outbreak of COVID-19, Taiwan has implemented rigorous border control and taken specific measures such as virus detection, contact tracing, and quarantine since 2020. Its epidemic prevention performance has been quite outstanding. Even in May 2021, when the epidemic situation worsens, the people in Taiwan fully cooperate with the government's control measures so as to successfully alleviate and control the epidemic in less than three months. Among them, the detection policy has played a pivotal role. We analyze and discuss the false positive and false negative problems from rapid antigen and PCR detection in the screening policy as well as the timing of using these two instruments. This paper provides theoretical verification of the appropriateness of screening policy in Taiwan, offering a few feasible suggestions for related policies in other countries or regions at different stages of this and other potential epidemics.

1. Introduction

We are facing a world full of uncertainty. The sudden raging of the COVID-19 virus around the world is the best portrayal of this scene. Since the virus spread across the globe in early 2020, it has caused millions of lives lost, and the economic loss is even more challenging to estimate. People can't help asking whether the generation and spread of the virus is a natural disaster or not? Regardless of the answer, if human beings cannot introspect on themselves and leave their living environment to continue to deteriorate, another massive disaster worse than the COVID-19 is only a matter of time. The assumption of self-interest and rational behavior in economics does not hinder human beings' mutual assistance and cooperation, since coexistence and co-prosperity often contribute to greater self-interest achieved. Extreme and irrational self-interest, at best, can only lead to short-term benefits for selfish individuals. But in the long run, the behavior will be identified and boycotted by the public.

Among the variety of epidemic prevention measures to combat the spread of the COVID-19, screening policy is essentially a very important one. It becomes the focus of the paper. In this study, we set up a two-stage COVID-19 screening model similar to the two-stage credit risk assessment model used in Chen, Guo & Huang (2009). The latter employs a low-cost but low-accuracy statistical forecasting model as the first-stage credit risk assessment mechanism, and then decides whether to perform the second-stage costly manual evaluation operations but with high accuracy based on the first-stage evaluation results.

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Interestingly, both of the two papers have information asymmetry problems in conventional agency theory, but the situation is somehow different. In Chen, Guo & Huang (2009), there is an information asymmetry between the borrower and the lender, and the borrower holds private information about his credit status. However, this paper's information asymmetry "probably" exists between the screened subjects and the executing unit. But the critical point is that those who are screened may not know whether they are infected or not. Those who really hold private information may be only the virus itself. Therefore, the executing unit is faced with the virus hidden in the infected person, and the private information holder (virus) can be found out only through the screening measure.

There have been many discussions in the past audit literature on alleviating the information asymmetry problems. In general, conventional audit policy analysis employs a three-tier agency structure (principal, auditor, and agent). Among them, the principal regards the auditor as the second agent. In the previous studies, such as Baiman, Evans & Noel (1987), Baiman, Evans & Nagarajan (1991), Baron & Besanko (1984), Demski & Sappington (1987), Kofman & Lawarree (1993 & 1996), Chen, Guo & Huang (2009), Guo, Chen & Lee (2013), Guo & Chen (2015), and Guo, Wu & Lee (2021), and others, can be classified as the analysis and application of the related agency structure. In this study, Taiwan's CDC (equivalent to the principal in the agency structure) is responsible for formulating appropriate screening policies, and those who implement screening policies are equivalent to the role of auditors. Since the subjects screened may not know whether they are infected or not, they can only be regarded as quasi-agents. Hence, the analysis model used in this paper will be somehow different from the research models of the past audit literature.

In terms of screening policies for the COVID-19, "rapid antigen screening" and "PCR nucleic acid testing" are commonly used in practice. The former has the advantages of low cost and quick results, but because the accuracy rate is not so good, it tends to cause false positive and false negative problems, resulting in nonnegligible social costs. In contrast, although the accuracy of the latter is higher than that of the former, it also has the problem of time-consuming, high cost, and false negative result caused by test timing. In this study, we discuss the policies related to the two screening tools. Since the two policy tools have their own strengths and weaknesses, we set up an appropriate analysis model to derive the optimal policy under the variety of possible conditions. In addition to verifying the appropriateness of the screening policies implemented by Taiwan's CDC in the past, we attempt to offer possible screening policy recommendations for countries or regions in different stages of the epidemic.

The structure of this paper is divided into five parts: The introduction is presented in Section 1. In Section 2, we describe the basic assumptions and settings of the model used in this study. The related results derived from the model are elaborated in Section 3. Simple numerical examples of the models are illustrated in Section 4. Finally, some concluding remarks are addressed in Section 5.

2. The model

In a single-period analysis model, we assume that the total number of members of a certain unit to be tested (e.g. community residents, firm employees, market workers, etc., denoted by M) is π. Also, it is assumed the proportion of people who have been infected with COVID-19 pneumonia virus at the time of testing is k, and the proportion of people who have not been infected is $1-\mathrm{k}$, where $0<\mathrm{k}<1$. Taiwan's Centers for Disease Control (CDC) performs the first phase of rapid antigen screening (T1) for all subjects to be tested. Under the action of natural force (N), among the infected people, there will be the ratio of p_1 found positive, and the ratio of $1-\mathrm{p}_1$ found negative (false negative). Among the uninfected people, there will be the ratio of n_1 found negative, and the ratio of $1-\mathrm{n}_1$ found positive (false positive), where $0<\mathrm{p}_1\leq 1$ and $0<\mathrm{n}_1\leq 1$. After the completion of the first phase of rapid antigen screening (T1), the second phase of PCR nucleic acid detection (T2) will be under way. For those who are positive in the first stage rapid screening, it is assumed the rate of performing the second stage PCR test is α, and the rate of not performing the PCR test is $1-\alpha$. But for those who are negative in the first stage antigen rapid screening, the second stage PCR test is performed with the rate of β, and the rate without PCR detection is $1-\beta$. Due to the characteristics of the PCR test, among the infected people, there will be the ratio of p_2 found positive, and the ratio of $1-\mathrm{p}_2$ found negative (false negative); but among the uninfected people, there will be 100% found negative, and the ratio of 0% found positive (false positive), where $0<\mathrm{p}_1<\mathrm{p}_2\leq 1$.[1] In other words, the accuracy rate of PCR nucleic acid detection in the second stage is higher than that in the first stage of rapid antigen screening. However, the detection cost of PCR (C_2) is higher than the cost of rapid antigen screening (C_1), and the result generation time is longer, too. Hence, it is supposed to be much more economical and effective if PCR detection can be used with rapid antigen screening. Furthermore, for those who are actually uninfected but judged as positive in the first stage of rapid screening, there remains a false positive problem if they have not undergone PCR detection. The derived social cost (or opportunity cost) is assumed to be "Sp" per (false positive) person. As for those who are actually infected and diagnosed as positive by PCR, they will be isolated for observation or sent to hospital for treatment based on their risk status. We assume that they will not incur other social costs. However, those who are infected and have not been diagnosed as positive by PCR remain to have false negative problems, and the false negative infected persons may spread the virus to other people. The

[1] Due to the incubation period of the COVID-19 pneumonia virus, the infected person may not be diagnosed at the first PCR test, so there is a false negative situation.

related social cost is assumed to be "Sn" per (false negative) person. To simplify the analysis, it is assumed that Sp and Sn are constants, and they are the same for each subject. In fact, Sp will be affected by the individual socioeconomic status. The higher the socioeconomic status, the higher the individual opportunity cost of being isolated due to false positive result will be. It leads to a larger Sp. Similarly, Sn is also obviously affected by the Ct value (or Rt value and fatality rate) of the false negative person. The smaller the Ct value (or the larger the Rt value and the fatality rate), the larger the Sn will be. The development sequence of related events analyzed in this study is briefly described as follows:

1. The total number of subjects (such as community residents, firm employees, market workers, etc., denoted by M) that CDC intends to collect is π. The proportion of subjects who have been infected with COVID-19 pneumonia virus at the time of testing is k, and the proportion of people who have not been infected is 1 - k, where $0 < k < 1$.
2. CDC performs the first phase of rapid antigen screening (T1) for all subjects (M) to be collected. Under the action of natural force (N), among the infected people, there will be the ratio of p_1 found positive ($\widehat{P}_{o1}$), and the ratio of $1 - p_1$ found negative (false negative) ($\widehat{N}_{e1}$); but among the uninfected people, there will be the ratio of n_1 found negative ($\widehat{N}_{e1}$), and the ratio of $1 - n_1$ found positive (false positive) ($\widehat{P}_{o1}$), where $0 < p_1 \leq 1$ and $0 < n_1 \leq 1$.
3. For those who are positive in the first-stage rapid antigen screening, the sampling rate for the second-stage PCR test is α, and the rate for not performing the PCR test is $1 - \alpha$. Also, for those who are negative in the first-stage rapid screening, the sampling rate for the second-stage PCR test is β, and the rate without PCR detection is $1 - \beta$.
4. In the second stage of the PCR test, among the infected people, there will be the ratio of p_2 found positive, and the ratio of $1 - p_2$ found negative (false negative); but among the uninfected people, there will be 100% found negative, and the ratio of 0% found positive (false positive), where $0 < p_1 < p_2 \leq 1$. However, if the first-stage rapid screening positive person is actually uninfected and no further PCR test is performed, there remains a false positive problem, and the social cost derived from it is assumed to be "Sp" per (false positive) person.
5. Those diagnosed as positive by PCR test will be isolated for observation or sent to hospital for treatment based on their risk status. We assume that they will not incur other social costs. However, those who are infected and have not been diagnosed as positive by PCR remain to have false negative problems, and the false negative infected persons may spread the virus to other people. The related social cost is assumed to be "Sn" per (false negative) person.
6. All pay-offs are realized.

Related decision tree diagram is shown in Fig. 1. Among them, P_o represents the subjects actually infected at the time of testing; and N_e represents the subjects actually uninfected at the time of testing. After the action of nature, $\widehat{P}_{o1}$ represents the subjects judged as positive after the first-stage rapid screening, and $\widehat{N}_{e1}$ represents the subjects judged as negative after the first-stage rapid screening. If the result of the second-stage PCR test is positive, or the result of the first-stage rapid screening is positive but no PCR test is performed, the final judgment will be $\widehat{P}_o$. On the other hand, if the result of the second-stage PCR test is negative, or the result of the first-stage rapid screening is negative but no PCR test is performed, it will eventually be judged as $\widehat{N}_e$. The notation ( ) in the bottom layer of the tree diagram represents the social costs that may be derived after testing.

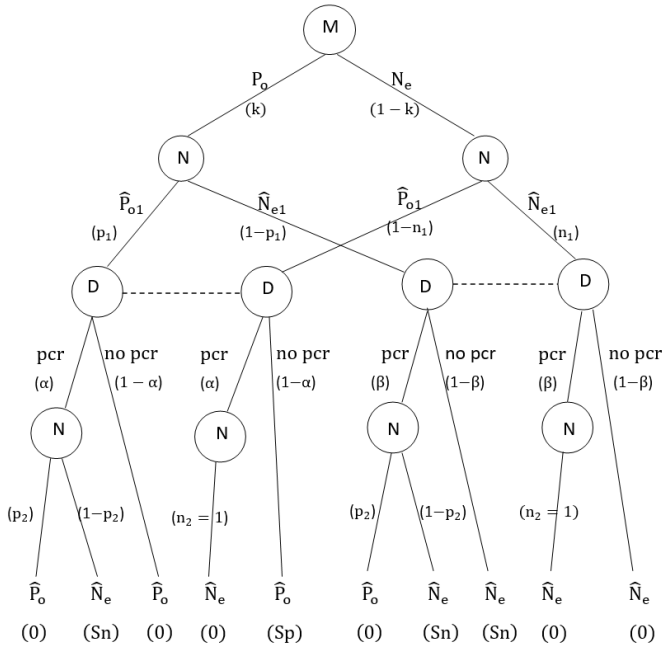

Fig. 1. Decision Tree for Two-Stage Screening

3. The results and Analysis

In this section, we first analyze and discuss the rapid antigen screening policy of COVID-19 pneumonia virus, and then combine rapid antigen screening with PCR testing to determine the most adequate screening policies in various situations.

Theorem 1.1

When the infection rate of the screened population reaches the threshold infection rate of rapid screening (i.e. $k > \underline{k}$), the rapid antigen screening for all members of COVID-19 pneumonia virus will be economically effective; otherwise, it will not be economically effective. Among them, the threshold infection rate of rapid screening is $\underline{k} \equiv [(1-n_1)Sp + C_1]/[(1-n_1)Sp + p_1Sn]$.

Proof. See Appendix A.

From Theorem 1.1, it is known that if rapid antigen screening is economically effective, the infection rate of the screened population cannot be too low; otherwise, the false positive and false negative problems can lead to that the costs are over than the benefits.

Theorem 1.2

Rapid antigen screening will lead to a false positive probability
$(1-k)(1-n_1)/[kp_1 + (1-k)(1-n_1)]$, and a false negative probability
$k(1-p_1)/[k(1-p_1) + (1-k)n_1]$.
Ceteris paribus, the improvement of either the positive accuracy of rapid screening (p_1) or the negative accuracy of rapid screening (n_1) will contribute to reducing the false positive or false negative incidence.
Also, a decline in the infection rate (k) of the screened population will lead to an increase in the false positive incidence (and a decrease in the false negative incidence) in the all-member rapid antigen screening, and vice versa.

Proof. See Appendix B.

In Theorem 1.2, it is worth noting that a decline in the infection rate of the screened population will lead to an increase in the false positive incidence for rapid antigen screening, but an increase in the infection rate of the screened population will lead to a rise in the false negative incidence for rapid antigen screening. It implies that if the infection rate of the screened population is too low, rapid screening will tend to cause a serious false positive problem, especially in metropolitan areas with higher socio-economic status. The social costs resulting from the problem will be much larger so that we have to be pretty cautious. In contrast, if the infection rate of the screened population is quite high, rapid screening is likely to lead to a serious false negative problem, causing the false negative people to spread the virus everywhere. Finally, it triggers the number of infections in the group to keep rising, and results in a significant social cost burden. Due to the false positive and false negative problems of rapid antigen screening, it becomes crucial to use more accurate PCR nucleic acid detection for confirmation after rapid screening. In this section, we analyze and discuss the possible optimal combination of the two detection tools. At first, we summarize the possible factors affecting the threshold infection rate of rapid screening for all members in theorem 1.3 and 1.4.

Theorem 1.3

Ceteris paribus, an increase in the cost of rapid antigen screening (C_1) will lead to a raise in the threshold infection rate ($\underline{k}$) of rapid screening; and an increase in the accuracy of rapid screening (p_1) or the social cost derived from false negative subjects (Sn) will be contributive to lowering the threshold infection rate ($\underline{k}$) of rapid screening.

Proof. See Appendix C.

The higher the threshold infection rate of the rapid screening, the more detrimental to the economic effectiveness of the rapid antigen screening for all members will be. Hence, a higher cost of rapid antigen screening, a lower accuracy rate of rapid screening, or an insignificant social cost derived from false negative result will be unfavorable for the economic effectiveness of the rapid antigen screening for all members.

Theorem 1.4

Ceteris Paribus,
(1) If the “rapid screening cost” is less than the product of “positive accuracy rate of rapid screening” and “false negative

social cost" (i.e. $C_1 < p_1 \times Sn$), an increase in the negative accuracy rate of rapid screening (n_1) or a decrease in the false positive social cost (Sp) will contribute to reducing the threshold infection rate for rapid screening ($\underline{k}$).
(2) But if the "rapid screening cost" is not less than the product of "positive accuracy rate of rapid screening" and "false negative social cost" (i.e. $C_1 \geq p_1 \times Sn$), an increase in the negative accuracy rate of rapid screening (n_1) or a decrease in the false positive social cost (Sp) will lead to an increase in the threshold infection rate for rapid screening ($\underline{k}$).

Proof. See Appendix D.

It is shown in Theorem 1.4 that when the "rapid screening cost" is less than the product of "positive accuracy rate of rapid screening" and "false negative social cost", a higher negative accuracy rate of rapid screening or an insignificant false positive social cost will contribute to achieving the economic effectiveness of the rapid antigen screening for all members, and vice versa.
According to Theorem 1.1, when the infection rate of the screened population reaches the threshold infection rate of rapid screening, COVID-19 pneumonia virus rapid screening for all members has been economically effective. At this time, the CDC can think about how to make use of the rapid and low-cost characteristics of rapid antigen screening, and combine it with the time-consuming and high-cost, but high-accuracy PCR detection. This is the analysis of the rapid antigen screening and PCR detection policies discussed in the second half of this section.

Lemma 1

If $C_2 < Cp$, $\alpha^* = 1$; but if $C_2 \geq Cp$, $\alpha^* = 0$,
where $Cp \equiv [(1-k)(1-n_1)Sp - kp_1(1-p_2)Sn]/[kp_1 + (1-k)(1-n_1)]^2$.

Proof. See Appendix E.

Lemma 1 points out that when the PCR test cost (C_2) is lower than the threshold test cost (Cp) for positive rapid test result, those who are positive in the first stage of rapid test will experience the second stage of PCR test for confirmation. Otherwise, it is determined based on the positive result of the first stage rapid screening.

Lemma 2

If $C_2 < Cn$, $\beta^* = 1$; but if $C_2 \geq Cn$, $\beta^* = 0$,
where $Cn \equiv kp_2(1-p_1)Sn/[k(1-p_1) + (1-k)n_1]^2$.

Proof. See Appendix F.

Lemma 2 indicates that when the PCR test cost (C_2) is lower than the threshold test cost (Cn) for negative rapid test result, those who are negative in the first stage of rapid test will experience the second stage of PCR test for confirmation. Otherwise, it is determined according to the negative result of the first stage rapid screening.

Lemma 3

If $k \to 0$, $Cn < Cp$; but if $k \to 1$, $Cp < Cn$.

Proof. See Appendix G.

Lemma 3 shows that when the infection rate (k) of the screened population is quite low, the negative threshold test cost (Cn) will be lower than the positive threshold test cost (Cp). Hence, those judged as positive in the first-stage rapid test tend to experience the second-stage PCR testing. Conversely, when the infection rate (k) of the screened population is pretty high, the positive threshold test cost (Cp) will be lower than the negative threshold test cost (Cn), making it easier for those who are negative in the first stage-rapid test to experience the second-stage PCR testing.
On the basis of the above analysis, we can obtain the following most appropriate policy combinations for rapid antigen screening and PCR detection in different situations.

Theorem 2.1

As $k > \underline{k}$ and $Cn < Cp$,
(1) $\alpha^* = 1$ and $\beta^* = 1$ if $C_2 < Cn$;
(2) $\alpha^* = 1$ and $\beta^* = 0$ if $Cn \leq C_2 < Cp$;
(3) $\alpha^* = 0$ and $\beta^* = 0$ if $C_2 \geq Cp$.

Proof. Based on the results of Theorem 1.1 as well as Lemmas 1 and 2, Theorem 2.1 is proved.

According to Theorem 2.1, when the negative threshold test cost (Cn) is lower than the positive threshold test cost (Cp), if the PCR test cost (C_2) is lower than the negative threshold test cost (Cn), then those who are either positive or negative in the first-stage rapid screening will experience the second-stage PCR testing for confirmation. However, if the PCR test cost (C_2) is lower than the positive threshold test cost (Cp) but not lower than the negative threshold test cost (Cn), only those who are positive in the first-stage screening will undergo the second-stage PCR test for confirmation. Those who are negative in the first-stage screening will only need to do self-health management. Finally, if the PCR test cost (C_2) is not lower than the positive threshold test cost (Cp), those who are either positive or negative in the first-stage screening will not undergo the second-stage PCR test. They will be treated directly on the basis of the first-stage screening results.

Theorem 2.2

As $k > \underline{k}$ and $Cp < Cn$,
(1) $\alpha^* = 1$ and $\beta^* = 1$ if $C_2 < Cp$;
(2) $\alpha^* = 0$ and $\beta^* = 1$ if $Cp \le C_2 < Cn$;
(3) $\alpha^* = 0$ and $\beta^* = 0$ if $C_2 \ge Cn$.

Proof. Based on the results of Theorem 1.1 as well as Lemmas 1 and 2, Theorem 2.2 is proved.

Based on Theorem 2.2, when the positive threshold test cost (Cp) is lower than the negative threshold test cost (Cn), if the PCR test cost (C_2) is lower than the positive threshold test cost (Cp), then those who are either positive or negative in the first-stage rapid screening will experience the second-stage PCR testing for confirmation. Nevertheless, if the PCR test cost (C_2) is lower than the negative threshold test cost (Cn) but not lower than the positive threshold test cost (Cp), only those who are negative in the first-stage screening will undergo the second-stage PCR test for confirmation. Those who are positive in the first-stage screening will be directly isolated for observation or treatment. At last, if the PCR test cost (C_2) is not lower than the negative threshold test cost (Cn), those who are either positive or negative in the first-stage screening will not undergo the second-stage PCR test. They will be treated directly according to the first-stage screening results.

Theorem 2.3

As $k > \underline{k}$ and $Cn = Cp$,
(1) $\alpha^* = 1$ and $\beta^* = 1$ if $C_2 < Cn = Cp$;
(2) $\alpha^* = 0$ and $\beta^* = 0$ if $C_2 \ge Cn = Cp$.

Proof. Based on the results of Theorem 1.1 as well as Lemmas 1 and 2, Theorem 2.3 is proved.
According to Theorem 2.3, when the negative threshold detection cost (Cn) is equal to the positive threshold detection cost (Cp), if the PCR detection cost (C_2) is lower than the negative threshold detection cost (Cn) (or the positive threshold detection cost (Cp)), then those who are either positive or negative in the first-stage rapid screening will experience the second-stage PCR testing for confirmation. However, if the PCR test cost (C_2) is not lower than the negative threshold test cost (Cn) (or the positive threshold test cost (Cp)), those who are either positive or negative in the first-stage screening will not undergo the second-stage PCR test. They will be treated directly according to the first-stage screening results. In other words, those who are positive in the first-stage screening will be directly isolated for observation or treatment, while those who are negative in the first-stage screening will be required to do their self-health management.

Lemma 3 points out that when the infection rate (k) of the screened population is quite low, the negative threshold test cost (Cn) will tend to be lower than the positive threshold test cost (Cp), making the premise of Theorem 2.1 ($Cn < Cp$) easier to be satisfied. As a result, if the efficiency of PCR detection can be improved and the related cost (C_2) can be reduced, it will be naturally contributive to the result of Theorem 2.1(2). That is, only positive subjects in the first-stage rapid screening will undergo the second-stage PCR testing for confirmation. That is somehow similar to Taiwan's situation at the peak of the epidemic in June 2021. As for year 2020, due to the appropriate control of the Taiwanese epidemic during the whole year, the condition for the economic effectiveness of the rapid screening for all members in Theorem 1.1 cannot be met. Hence, the CDC in Taiwan did not implement rapid screening tests for all members in year 2020, and it is supposed to be a suitable policy. Anyway, the basic measures of epidemic prevention (such as wearing masks, washing hands frequently, and keeping safe social distance) should remain to be the best practice especially when vaccine coverage is not so sufficient.

4. Numerical Examples

In this section, two examples with hypothetical numerical analysis will be used to further illustrate the inference results obtained in the previous section.

Example 1

In this example, we assume that the positive accuracy rate (p_1) and the negative accuracy rate (n_1) of rapid antigen screening are both 0.9. The cost of rapid antigen screening (C_1) is 0.2 currency units per person, the social cost derived from false positive subject (Sp) is 10 currency units per person and the social cost derived from false negative subject (Sn) is 200 currency units per person.[2] According to Theorem 1.1, the threshold infection rate for fast screening can be inferred to be 0.0066. Under this situation, rapid screening for all members is not economically effective provided the current infection rate of the screened population is 0.001. However, based on Theorems 1.3 and 1.4, if the positive and negative accuracy rates of rapid antigen screening are both increased to 0.95, the cost of rapid antigen screening is reduced to 0.1 currency unit per person, the social cost derived from false positive subject is reduced to 5 currency units per person, and the social cost derived from false negative subject is enhanced to 500 currency units per person, all these changes will cause the threshold infection rate for fast screening to drop to 0.0007. As the current infection rate of the screened population remains at 0.001, rapid screening for all members will be economically effective. Hence, Example 1 verifies the appropriateness of Taiwan's CDC policy between 2020 and 2021. In other words, rapid antigen screening for all members is not performed when the epidemic is easing, but specific groups (such as employees in firms and markets that may be infected by the virus) need to do rapid screening and/or PCR detection when the epidemic heats up. The following continues to explain the timing of the combination between the rapid screening for all members and the PCR detection using a hypothetical situation.

Example 2

It is assumed that both the positive accuracy rate (p_1) and negative accuracy rate (n_1) of rapid antigen screening remain to be 0.9, but the positive accuracy rate (p_2) and the negative accuracy rate (n_2) of PCR detection are 0.99 and 1, respectively. The cost of rapid antigen screening (C_1) is 0.2 currency unit per person, the PCR test cost (C_2) is 1 currency unit per person, the social cost (Sp) derived from false positive subject is 10 currency units per person and the social cost (Sn) derived from false negative subject is 200 currency units per person. Under the different infection rates of screened population (k gradually increasing from 0.01 to 0.99), the corresponding Cp and Cn values are shown in Fig. 2.[3]

Judging from the relevant numerical values in Fig. 2, the corresponding k value (represented by $\bar{k}$ value) when Cp is equal to Cn is approximately located at some point between $k = 0.15$ ($Cn = 4.9 < Cp = 12.0$) and $k = 0.2$ ($Cn = 7.2 > Cp = 6.5$). In addition, the result of Theorem 2.1 applies when $k < \bar{k}$ and $Cn < Cp$; the result of Theorem 2.2 applies when $k > \bar{k}$ and $Cn > Cp$; finally, the result of Theorem 2.3 applies when $k = \bar{k}$ and $Cn = Cp$. In this example, the threshold infection rate ($\underline{k}$) of rapid screening for all members is 0.0066, and $Cn = 0.25$($<C_2=1$) as well as $Cp = 83.3$($>C_2=1$) when $k = 0.01$. Hence, if the current infection rate of the screened population is between 0.0066 and 0.01 (i.e., $k \in (0.0066, 0.01)$), then PCR test cost (=1) will be less than Cp but larger than Cn in addition to meeting the economic effectiveness of the rapid screening for all members. It implies that only those who are positive in the first-stage screening will undergo the second-stage PCR test for confirmation, and those who are negative in the first-stage screening will be required to do self-health management.

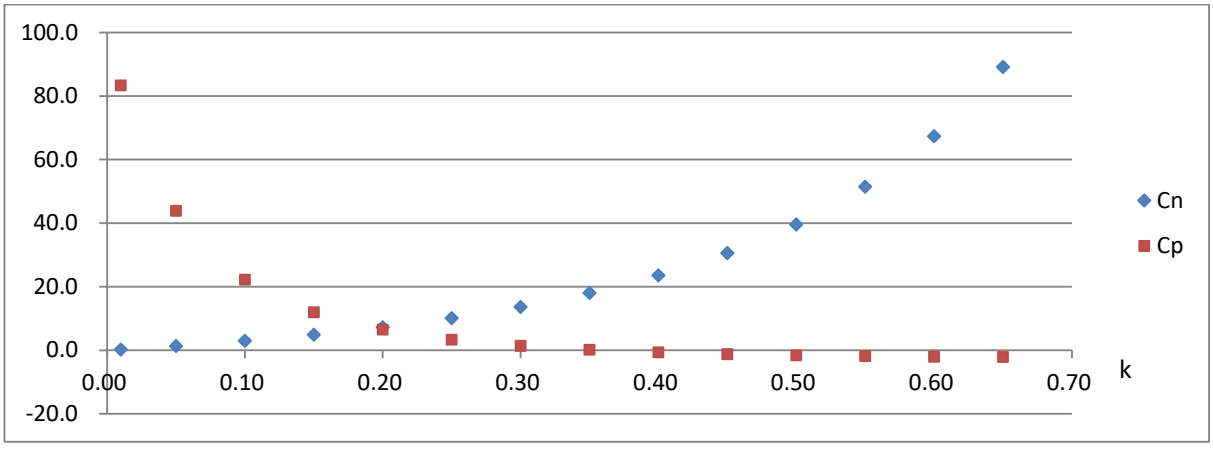

Fig. 2. Cp and Cn values corresponding to $k \in [0.01, 0{,}65]$
($p_1 = 0.9$, $n_1 = 0.9$, $p_2 = 0.99$, $n_2 = 1$, $C_1 = 0.2$, $C_2 = 1$, $Sp = 10$ and $Sn = 200$)

As a result, Example 2 also verifies the appropriateness of Taiwan CDC's policy during the peak of the epidemic in 2021.

[2] The currency unit is any possible currency unit, such as Taiwan dollar, U.S. dollar, euro, Japanese yen, etc. In this paper, the PCR detection cost (C_2) is standardized to 1 currency unit per person. Hence, the rapid antigen screening cost (C_1) is 0.2 currency unit per person, which can be regarded as 1/5 of the PCR detection cost. Others can be deduced by analogy.

[3] For the convenience of observation, Figure 2 only shows the part of k value (from 0.01 to 0.65), and the part of $0.65<k\leq0.99$ is omitted since the trend remains unchanged and is not the focus of observation. For the omitted part, please refer to Figure 3 in Appendix H.

That is, proceed all-members rapid screening for specific high-risk groups (such as areas, firms, markets, etc., which are most likely to be infected by the virus), and then perform PCR test for those who are positive in the first-stage screening. In contrast, in countries or regions with more severe epidemics, the infection rate of the screened population is likely to exceed $\bar{k}$, making the Cn value greater than the Cp value. According to the assumption of Example 2 (as shown in Figure 2), in the high-risk range of $0.2 \leq k \leq 0.3$, we have $Cn > Cp > C_2 = 1$, and in the extremely high-risk range of $k > 0.3$, we have $Cn > C_2 = 1 > Cp$. It implies that, for those members of the screened population with a high infection rate, if their rapid antigen screening is negative, they need be confirmed by PCR to reduce the social costs derived from false negative subjects. However, for those who are positive in the first-stage screening, CDC will undergo the second-stage PCR test for confirmation only in the high-risk interval ($0.2 \leq k \leq 0.3$), rather than in the extremely high-risk interval ($k > 0.3$). In the extremely high-risk interval, those who are positive in the first-stage screening will be directly isolated for observation or medical treatment.

5. Concluding remarks

From the beginning of 2020, the COVID-19 virus triggered a serious threat all over the world, causing significant loss of millions of lives. In 2020, Taiwan implemented strict border controls as well as other measures such as virus detection, contact tracing, and quarantine. Its epidemic control performance is quite impressive. Even in 2021, when the epidemic situation worsened, the people of Taiwan remained fully cooperative with the government's 3rd Level control measures to gradually ease and control the epidemic. Among the control measures, the screening policy essentially played a pivotal role. The paper analyzes and discusses the false-positive and false-negative issues related to rapid antigen screening and PCR testing, as well as the timing of using the two tools in the screening policy. As a result, it is shown that the infection rate of the screened population cannot be too low for all-member rapid antigen screening to be economically effective. Otherwise, the false positive and false negative problems can lead to higher costs than benefits. Meanwhile, it is worth noting that a decline in the infection rate of the screened population leads to an increase in the false positive incidence for rapid antigen screening; and an increase in the infection rate of the screened population leads to a rise in the false negative incidence for rapid antigen screening.

It implies that if the infection rate of the screened population is too low, rapid screening will easily lead to serious false positive problem, especially in metropolitan areas with high socio-economic status. The social costs resulting from the problem will be much higher so that we have to be pretty cautious. In contrast, if the infection rate of the screened population is quite high, rapid screening is likely to result in a serious false negative problem, causing the false negative people to spread the virus everywhere. Finally, it triggers the number of infections in the group to keep rising, resulting in to a significant social cost burden. At this time, CDC should think about how to use the rapid and low-cost characteristics of rapid antigen screening, and appropriately combine it with the time-consuming and high-cost, but high-accuracy PCR detection.

In the numerical analysis, Example 1 reflects the suitability of Taiwan's CDC screening policy between 2020 and 2021. In other words, rapid antigen screening for all employees is not performed when the epidemic is easing, but it is undertaken with (or without) PCR detection for specific groups (such as employees in firms and markets who may be infected by the virus) when the epidemic heats up. Additionally, Example 2 also verifies the appropriateness of Taiwan's CDC screening policy during the peak of the epidemic in 2021. That is, CDC conducts all-member rapid screening for specific high-risk groups (ex., people working in firms or markets, who are most likely to be infected by the virus); and only for those found positive in the first-stage screening, CDC will undergo the second-stage PCR test for confirmation. But for those found negative in the first-stage screening, CDC will require them to do self-health management.

Finally, this study suggests possible screening policies for countries or regions with relatively severe epidemics. For those members of the screened population with a high infection rate, if their rapid antigen screening is negative, they must be confirmed by PCR to reduce the social costs that may be derived from false negatives. However, if the rapid antigen screen is positive, PCR confirmation will be performed only in the high-risk range. As for the extremely high-risk range, the most appropriate treatment is to isolate the rapid screen positive for observation or treatment.[4]

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[4] For the range of "high risk" and "extremely high risk", please refer to the explanation of Example 2 in the section on numerical analysis.

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Appendix A. (Proof of Theorem 1.1)

Proof. With the rapid screening of antigens only, the expected social cost is
$\mathrm{E(SC(T1))} = \pi[\mathrm{ka}(1-\mathrm{p}_1)\mathrm{Sn} + \mathrm{k}(1-\mathrm{a})\mathrm{Sn} + (1-\mathrm{k})\mathrm{a}(1-\mathrm{n}_1)\mathrm{Sp} + \mathrm{aC}_1]$.
Also, π denotes the number of members in the group to be detected, and a is the detection rate of rapid screening, where $0 \le \mathrm{a} \le 1$.
To minimize $\mathrm{E(SC(T1))}$, $d[\mathrm{E(SC(T1))}]/d\mathrm{a} < 0 \Rightarrow \mathrm{a}^* = 1$, and

$d[\mathrm{E(SC(T1))}]/d\mathrm{a} < 0$
$\Leftrightarrow \mathrm{k}(1-\mathrm{p}_1)\mathrm{Sn} - \mathrm{kSn} + (1-\mathrm{k})(1-\mathrm{n}_1)\mathrm{Sp} + \mathrm{C}_1 < 0$
$\Leftrightarrow \mathrm{k} > [(1-\mathrm{n}_1)\mathrm{Sp} + \mathrm{C}_1]/[(1-\mathrm{n}_1)\mathrm{Sp} + \mathrm{p}_1\mathrm{Sn}] \equiv \underline{\mathrm{k}}$

Appendix B. Proof of Theorem 1.2

Proof. In the case of using rapid antigen screening only, π is the total number of members of the population screened, a is the detection rate of rapid antigen screening, and $0 \le \mathrm{a} \le 1$. The total number of positive subjects after rapid screening will be $\pi[\mathrm{kap}_1 + (1-\mathrm{k})\mathrm{a}(1-\mathrm{n}_1)]$, and the total number of false positive subjects after rapid screening will be $\pi(1-\mathrm{k})\mathrm{a}(1-\mathrm{n}_1)$. Therefore, the false positive probability is $(1-\mathrm{k})(1-\mathrm{n}_1)/[\mathrm{kp}_1 + (1-\mathrm{k})(1-\mathrm{n}_1)] \equiv \mathrm{A}$. Similarly, The total number of negative subjects after rapid screening will be $\pi[\mathrm{ka}(1-\mathrm{p}_1) + (1-\mathrm{k})\mathrm{an}_1]$, and the total number of false negative subjects after rapid screening will be $\pi\mathrm{ka}(1-\mathrm{p}_1)$. Hence, the false negative probability is $\mathrm{k}(1-\mathrm{p}_1)/[\mathrm{k}(1-\mathrm{p}_1) + (1-\mathrm{k})\mathrm{n}_1] \equiv \mathrm{B}$. Meanwhile, $\partial\mathrm{A}/\partial\mathrm{p}_1 < 0$, $\partial\mathrm{A}/\partial\mathrm{n}_1 < 0$, $\partial\mathrm{B}/\partial\mathrm{p}_1 < 0$, $\partial\mathrm{B}/\partial\mathrm{n}_1 < 0$, $\partial\mathrm{A}/\partial\mathrm{k} < 0$, and $\partial\mathrm{B}/\partial\mathrm{k} > 0$.

Appendix C. Proof of Theorem 1.3

Proof. Since $\underline{\mathrm{k}} \equiv [(1-\mathrm{n}_1)\mathrm{Sp} + \mathrm{C}_1]/[(1-\mathrm{n}_1)\mathrm{Sp} + \mathrm{p}_1\mathrm{Sn}]$, we have $\partial\underline{\mathrm{k}}/\partial\mathrm{C}_1 > 0$,
$\partial\underline{\mathrm{k}}/\partial\mathrm{p}_1 < 0$, and $\partial\underline{\mathrm{k}}/\partial\mathrm{Sn} < 0$.

Appendix D. Proof of Theorem 1.4

Proof. Since $\underline{\mathrm{k}} \equiv [(1-\mathrm{n}_1)\mathrm{Sp} + \mathrm{C}_1]/[(1-\mathrm{n}_1)\mathrm{Sp} + \mathrm{p}_1\mathrm{Sn}]$, we have
$\partial\underline{\mathrm{k}}/\partial\mathrm{n}_1 = [\mathrm{C}_1 - \mathrm{p}_1\mathrm{Sp}]/[(1-\mathrm{n}_1)\mathrm{Sp} + \mathrm{p}_1\mathrm{Sn}]^2 < 0$ (if $\mathrm{C}_1 < \mathrm{p}_1\mathrm{Sp}$),
$\partial\underline{\mathrm{k}}/\partial\mathrm{Sp} = [\mathrm{p}_1\mathrm{Sp} - \mathrm{C}_1]/[(1-\mathrm{n}_1)\mathrm{Sp} + \mathrm{p}_1\mathrm{Sn}]^2 > 0$ (if $\mathrm{C}_1 < \mathrm{p}_1\mathrm{Sp}$),
$\partial\underline{\mathrm{k}}/\partial\mathrm{n}_1 = [\mathrm{C}_1 - \mathrm{p}_1\mathrm{Sp}]/[(1-\mathrm{n}_1)\mathrm{Sp} + \mathrm{p}_1\mathrm{Sn}]^2 \ge 0$ (if $\mathrm{C}_1 \ge \mathrm{p}_1\mathrm{Sp}$), and
$\partial\underline{\mathrm{k}}/\partial\mathrm{Sp} = [\mathrm{p}_1\mathrm{Sp} - \mathrm{C}_1]/[(1-\mathrm{n}_1)\mathrm{Sp} + \mathrm{p}_1\mathrm{Sn}]^2 \le 0$ (if $\mathrm{C}_1 \ge \mathrm{p}_1\mathrm{Sp}$).

Appendix E. Proof of Lemma 1

Proof. We assume that after the completion of the first stage of rapid antigen screening, the probability of positive subjects entering the second stage of PCR testing for confirmation is α. Hence, the expected social cost of performing PCR testing (T2) for positive subjects after rapid screening ($\widehat{\mathrm{P}}_{\mathrm{o1}}$) is
$\mathrm{E}\big(\mathrm{SC(T2}\big|\ \widehat{\mathrm{P}}_{\mathrm{o1}})\big)$
$= \pi\{[\alpha\mathrm{kp}_1(1-\mathrm{p}_2)\mathrm{Sn} + (1-\alpha)(1-\mathrm{k})(1-\mathrm{n}_1)\mathrm{Sp}]/[\mathrm{kp}_1 + (1-\mathrm{k})(1-\mathrm{n}_1)]\}$
$+\pi\alpha[\mathrm{kp}_1 + (1-\mathrm{k})(1-\mathrm{n}_1)]\mathrm{C}_2$.
To reduce the relevant expected social costs through PCR testing, the following condition must be satisfied, i.e.
$d\mathrm{E}\big(\mathrm{SC(T2}\big|\ \widehat{\mathrm{P}}_{\mathrm{o1}})\big)/d\alpha < 0$
$\Leftrightarrow [\mathrm{kp}_1(1-\mathrm{p}_2)\mathrm{Sn} - (1-\mathrm{k})(1-\mathrm{n}_1)\mathrm{Sp}]/[\mathrm{kp}_1 + (1-\mathrm{k})(1-\mathrm{n}_1)]$

$+[kp_1 + (1-k)(1-n_1)]C_2 < 0$
$\Leftrightarrow C_2 < [(1-k)(1-n_1)Sp - kp_1(1-p_2)Sn]/[kp_1 + (1-k)(1-n_1)]^2 \equiv Cp.$

Appendix F. Proof of Lemma 2

Proof. It is assumed that after the completion of the first stage of rapid antigen screening, the probability of negative subjects entering the second stage of PCR testing for confirmation is β. Hence, the expected social cost of performing PCR testing (T2) for negative subjects after rapid screening ($\widehat{N}_{o1}$) is

$E(SC(T2|\widehat{N}_{o1}))$
$= \pi[k\beta(1-p_1)(1-p_2) + k(1-\beta)(1-p_1)]Sn/[k(1-p_1) + (1-k)n_1]$
$+\pi\beta[k(1-p_1) + (1-k)n_1]C_2.$

To reduce the relevant expected social costs through PCR testing, the following condition must be satisfied, i.e.

$dE(SC(T2|\widehat{N}_{o1}))/d\beta < 0$
$\Leftrightarrow [k(1-p_1)(1-p_2)Sn - k(1-p_1)Sn]/[k(1-p_1) + (1-k)n_1]$
$+[k(1-p_1) + (1-k)n_1]C_2 < 0$
$\Leftrightarrow C_2 < kp_2(1-p_1)Sn/[k(1-p_1) + (1-k)n_1]^2 \equiv Cn.$

Appendix G. Proof of Lemma 3

Proof. If $k \to 0$, then $Cn \to 0$ and $Cp \to (1-n_1)Sp/(1-n_1)^2 > 0$. We have $Cn < Cp$. But if $k \to 1$, then $Cn \to p_2(1-p_1)Sn/(1-p_1)^2 > 0$ and
$Cp \to -p_1(1-p_2)Sn/{p_1}^2 < 0$. We have $Cp < Cn$.

Appendix H.

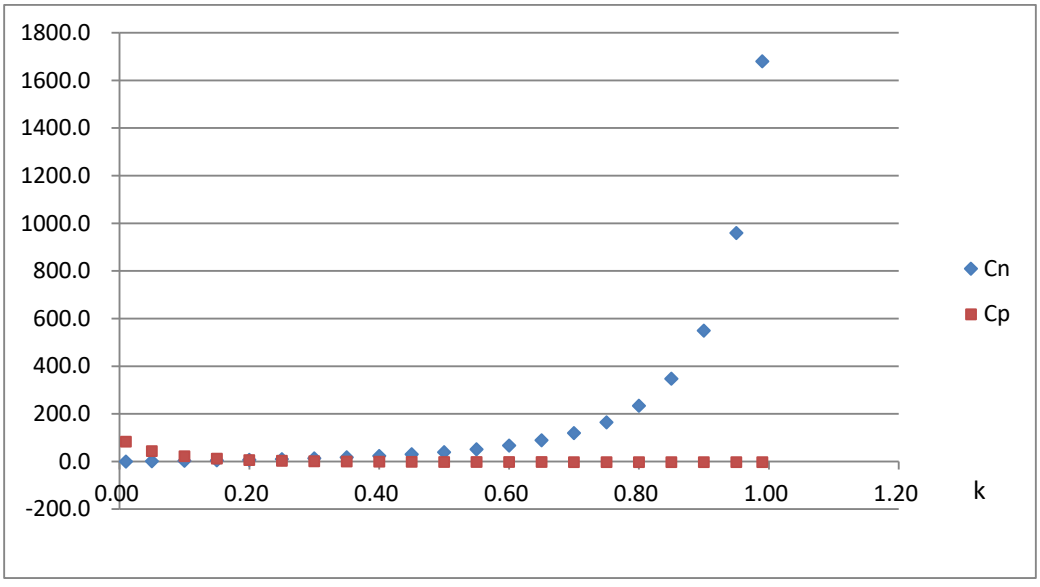

Fig. 3. Cp and Cn values corresponding to $k \in [0.01, 0{,}99]$
($p_1 = 0.9$, $n_1 = 0.9$, $p_2 = 0.99$, $n_2 = 1$, $C_1 = 0.2$, $C_2 = 1$, $Sp = 10$ and $Sn = 200$)