Decision Science Letters 10 (2021) 463–470

Contents lists available at GrowingScience

Decision Science Letters

homepage: www.GrowingScience.com/dsl

On ranking by using weighted self-normalizing distance metrics in multi-attribute decision-making

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C H R O N I C L E

Article history:
Received February 2, 2021
Received in revised format: June 2, 2021
Accepted July 6 2021
Available online July 6, 2021

Keywords:
Distance metric
LSP
MADM
Normalization
SAW
Reference ranking
Self-normalizing

A B S T R A C T

Preliminary normalization is central to the decision process of several popular, recent or completely new multi-attribute decision-making (MADM) methods. However, a number of authors have pointed out serious pitfalls attributed to normalization methods. One major pitfall, which has been identified, is that normalization methods may lead to different final rankings of alternatives when a ranking procedure (RP) based on them is used for solving a MADM problem. The current paper aims to ascertain and illustrate the effectiveness of some RPs based on prominent primary WEighted Self-NORmalizing Distance (WESNORD) metrics and their averages. The effectiveness of the selected RPs is demonstrated by solving a logistics service provider (LSP) selection problem taken from the literature. The results reveal that the RPs considered deliver final rankings of alternatives, which are very similar to the SAW-produced reference ranking.

1. Introduction

Multi-attribute decision-making (MADM) is a prominent branch of operations research and management science. It refers to "making preference decisions (such as evaluation, prioritization, and selection) over the available alternatives that are characterized by multiple, usually conflicting attributes" (Hwang & Yoon, 1981). In MADM, preliminary normalization (i.e., mathematical transformation of all initial attribute values to eliminate the effects of different scales of measurement before using a given method) is central to the decision process of various well-established, recent or completely new methods. Some of these are: the Simple Additive Weighting (SAW) (McCrimmon, 1968), the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) (Hwang & Yoon, 1981), the TOmada de Decisao Interativa e Multicriterio (TODIM) (Gomes & Lima, 1992), the Complex Proportional Assessment (COPRAS) (Zavadskas et al., 1994), the VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) (Opricovič, 1998), the Multi-Objective Optimization on the basis of Ratio Analysis (MOORA) (Brauers et al, 2006), the Relative Ratio (RR) (Li, 2009), the Performance Selection Index (PSI) (Maniya & Bhatt, 2010), the Additive Ratio Assessment (ARAS) (Zavadskas & Turskis, 2010), the Weighted Aggregated Sum Product Assessment (WASPAS) (Zavadskas et al., 2012), the Weighted Euclidean Distance Based Approach (WEDBA) (Rao & Singh, 2012), the Multi-Attribute Range Evaluations (MARE) (Hodgett, 2013), the Multi-Attributive Border Approximation area Comparison) (MABAC) (Pamucar & Cirovic, 2015), the Combinative Distance-based Assessment (CODAS) (Keshavarz-Ghorabaee et al., 2016), the Total Area based on Orthogonal Vectors (TAOV) (Hajiagha et al., 2016), the Double Normalization-based Multiple Aggregation (DNMA) (Liao & Wu, 2017), the Combined Compromise Solution (CoCoSo) (Yazdani et al., 2019), the Proximity Indexed Value (PIV) (Mufazzal & Muzakkir, 2018), the Simultaneous Evaluation of Criteria and Alternatives (SECA) (Keshavarz-Ghorabaee et al., 2018), the Mixed Aggregation by Comprehensive Normalization Technique (MACONT) (Wen et al., 2020), the

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doi: 10.5267/j.dsl.2021.7.003

Measurement of Alternatives and Ranking According to Compromise Solution (MARCOS) (Stevic et al., 2020). Numerous types of normalization methods can be found in the MADM literature (see e.g., Aytekin, 2021; Ploskas & Papathanasiou, 2019; Shekhovtsov & Kołodziejczyk, 2020). The most common classical normalization methods are vector normalization, linear max normalization, linear max-min normalization and linear sum normalization.

In the literature, a number of authors have pointed out serious pitfalls associated with normalization methods (e.g., Aytekin, 2021; Bhowmik et al., 2018; Çelen, 2015; Ginevičius, 2008; Jafaryeganeh, 2020; Jahan & Edwards, 2015; Kaplinski & Tamošaitienė, 2015; Kosareva et al., 2018; Lakshmi et al., 2019; Milani et al., 2005; Mokotoff et al., 2010; Palczewski & Sałabun, 2019; Pavlicic, 2001; Podviezko, 2014; Podviezko & Podvezko, 2015; Vafaei et al., 2018; Shekhovtsov & Kołodziejczyk, 2020). One major pitfall attributed to normalization methods is that they may lead to different final rankings of alternatives when a ranking procedure (RP) based on them is used for solving a MADM problem.

For at least this reason, the current work aims to ascertain and illustrate the effectiveness of some RPs based on prominent primary WEighted Self-NORmalizing Distance (WESNORD) metrics and their averages (see Section 2 and Section 3 for details). The WESNORD metrics of interest are listed below.

I) The three prominent primary distance metrics involved are:
- Weighted Canberra distance;
- Weighted Gower distance;
- Weighted Wave-Hedges distance.

II) The four averages of distance metrics considered are:
- The average of weighted Canberra and Gower distances;
- The average of weighted Canberra and Wave-Hedges distances;
- The average of weighted Gower and Wave-Hedges distances;
- The average of weighted Canberra, Gower, and Wave-Hedges distances.

In the above, all seven of these distance metrics are normalized and computationally quite simple, and therefore suitable for practical applications.

The main contribution of this work is threefold: *First*, we introduce the **cornerstone concept** of WEighted Self-NORmalized Distance (WESNORD) metrics. *Second*, we define an **original ranking index** exploiting the duality of normalized similarity and distance metrics. *Third*, we ascertain that the WESNORD metrics based methodology is **worthwhile**. We organize the rest of the paper as follows. The next section presents the definitions, notation and ideas related to the WESNORD metrics based methodology. The third section explains how to rank alternatives by using WESNORD metrics. To ascertain and illustrate the effectiveness of the use of WESNORD metrics, a logistics service provider (LSP) selection problem adapted from the paper by Hidouri and Rebaï (2019) is solved in the fourth section. The resulting rankings are then compared to the SAW-produced *reference ranking*. Finally, the fifth section concludes the article and points out two directions for future research.

2. Basic mathematical definitions

Let us start by presenting some basic preliminary mathematical definitions such as normalized distance metrics (NDMs) and normalized similarity metrics (NSMs).

Definition 1 (Muscat, 2014). Let $\mathbb{R}$ denote the set of all real numbers and let X be an arbitrary nonempty set. A function $d : X \times X \to \mathbb{R}$ is called a *distance metric* on X if, for all $x, y, z \in X$, it holds:

- $d(x, y) = d(y, x)$ (symmetry),
- $d(x, y) \leq d(x, z) + d(z, y)$ (triangle inequality),
- $d(x, y) = 0$ if and only if $x = y$ (identity of indiscernibles).

Consequently, we have that $d(x, y) \geq 0$ (non-negativity) for any $x, y \in X$.

Definition 2 A distance metric $d(x, y)$ is said to be *normalized* if and only if $d(x, y) \leq 1$.

Note that for nonnegative real numbers, all three below-mentioned distances are WESNORD metrics.

Definition 3 (Lance & Williams, 1967). A function $d_C : \mathbb{R}^n_{0+} \times \mathbb{R}^n_{0+} \to \mathbb{R}_{0+}$ is an n-dimensional *Canberra distance* if

$$d_C(x, y) = \sum_{j=1}^{j=n} \frac{|x_j - y_j|}{x_j + y_j} \tag{1}$$

where $\mathbb{R}_{0+}$ denotes the set of all nonnegative real numbers, and where $x = (x_1, x_2, ..., x_n)$ and $y = (y_1, y_2, ..., y_n) \in \mathbb{R}^n_{0+}$.

Definition 4 (Gower, 1971). A function $d_G : \mathbb{R}^n_{0+} \times \mathbb{R}^n_{0+} \rightarrow \mathbb{R}_{0+}$ is an n-dimensional *Gower distance* if

$$d_G(x.y) = \sum_{j=1}^{j=n} \frac{|x_j - y_j|}{R_j} \tag{2}$$

where R_j is a normalizer, usually equal to the range of the *j*th attribute.

Definition 5 (Cha, 2007). A function $d_{WH} : \mathbb{R}^n_{0+} \times \mathbb{R}^n_{0+} \rightarrow \mathbb{R}_{0+}$ is an n-dimensional *Wave-Hedges distance* if

$$d_{WH}(x, y) = \sum_{j=1}^{j=n} \frac{|x_j - y_j|}{\max(x_j, y_j)} \tag{3}$$

In the above, it seems obvious (1) that all three of these distances self-normalize separately the absolute difference between the components of the vectors x and y prior to summing and (2) that the mathematical convention employed is that $\frac{0}{0} = 0$. We now present the axiomatic system introduced by Rozinek and Mareš (2021) for NSMs.

Definition 6 (Rozinek & Mareš, 2021). A function $sim(x, y)$: $X \times X \rightarrow [0, 1]$ is a NSM if, for all $x, y, z \in X$, it satisfies the following conditions:

- $sim(x, y) = sim(y, x)$ (symmetry),
- $sim(x, z) + 1 \geq sim(x, y) + sim(y, z)$ (triangle inequality),
- $sim(x, y) = 1$ if and only if $x = y$ (identity of indiscernibles),
- $sim(x, y) \geq 0$ (non-negativity).

According to Rozinek and Mareš (2021), the two statements below hold true:

- Associated with every NDM, $dist(x, y)$, is a NSM, $sim(x, y)$, in the sense of definition 6, given by the equation $sim(x, y) = 1 - dist(x, y)$.
- Convex combinations allow assembling different primary NSMs together to produce a composite NSM.

As a result, convex combinations allow assembling different primary NDMs together to produce a composite NDM.
The previously presented definitions and ideas lay the foundation for the WESNORD metrics based RPs as it is shown in the next section.

3. The WESNORD metrics based methodology

In what follows, we assume that m alternatives (A_i , i = 1, 2, ..., m) are judged upon n attributes (C_j , j = 1, 2, ..., n). Moreover, we denote by a_{ij} the nonnegative crisp attribute value of each alternative A_i with respect to the attribute C_j. We also denote by W by the vector of attribute weights, $(w_1, w_2, \ldots, w_n) \in [0, 1]^n$ satisfying $\sum_{j=1}^{n} w_j = 1$.
The following enumerated list represents the selected WESNORD metrics to be used:

I) The three primary WESNORD metrics:

1) The weighted Canberra distance $d_{WC}(x, y)$ defined as:

$$dist_1(x, y) = d_{WC}(x, y) = \sum_{j=1}^{j=n} w_j \times \frac{|x_j - y_j|}{x_j + y_j} \tag{4}$$

2) The weighted Gower distance $d_{WG}(x, y)$ defined as:

$$dist_2(x, y) = d_{WG}(x, y) = \sum_{j=1}^{j=n} w_j \times \frac{|x_j - y_j|}{R_j} \tag{5}$$

3) The weighted Wave-Hedges distance $d_{WWH}(x, y)$ defined as:

$$dist_3(x, y) = d_{WWH}(x, y) = \sum_{j=1}^{j=n} w_j \times \frac{|x_j - y_j|}{\max(x_j, y_j)} \tag{6}$$

II) The four averages of WESNORD metrics:

$$dist_4(x, y) = \frac{d_{WC}(x,y) + d_{WG}(x,y)}{2} \tag{7}$$

$$dist_5(x,y) = \frac{d_{WC}(x,y) + d_{WWH}(x,y)}{2} \tag{8}$$

$$dist_6(x,y) = \frac{d_{WG}(x,y) + d_{WWH}(x,y)}{2} \tag{9}$$

$$dist_7(x,y) = \frac{d_{WC}(x,y) + d_{WG}(x,y) + d_{WWH}(x,y)}{3} \tag{10}$$

It is clear that the corresponding NSMs are given by:

$$Sim_k(x,y) = 1 - dist_k(x,y) \;, \tag{11}$$

where $1 \le k \le 7$.

Finally, the determination of the degree of suitability $S_k(A_i)$ of the alternative A_i is calculated according to the formula:

$$S_k(A_i) = \sqrt{sim_k(A_i, DP) \times dist_k(A_i, UP)}, \tag{12}$$

where $1 \le i \le m$ and $1 \le k \le 7$.

As defined, the degree of suitability of the alternative A_i, $1 \le i \le m$, is expressed as the geometric mean of its similarity score $sim_k(A_i, DP)$ and its dissimilarity score $dist_k(A_i, UP)$, where *DP* denotes the desired point formed from the most preferable values of the attributes, and *UP* the undesired point formed from the least preferred values. The alternatives are then ranked according to the degree of suitability (the highest degree represents the most suitable alternative).
The figure below shows the flowchart of the WESNORD metrics based methodology.

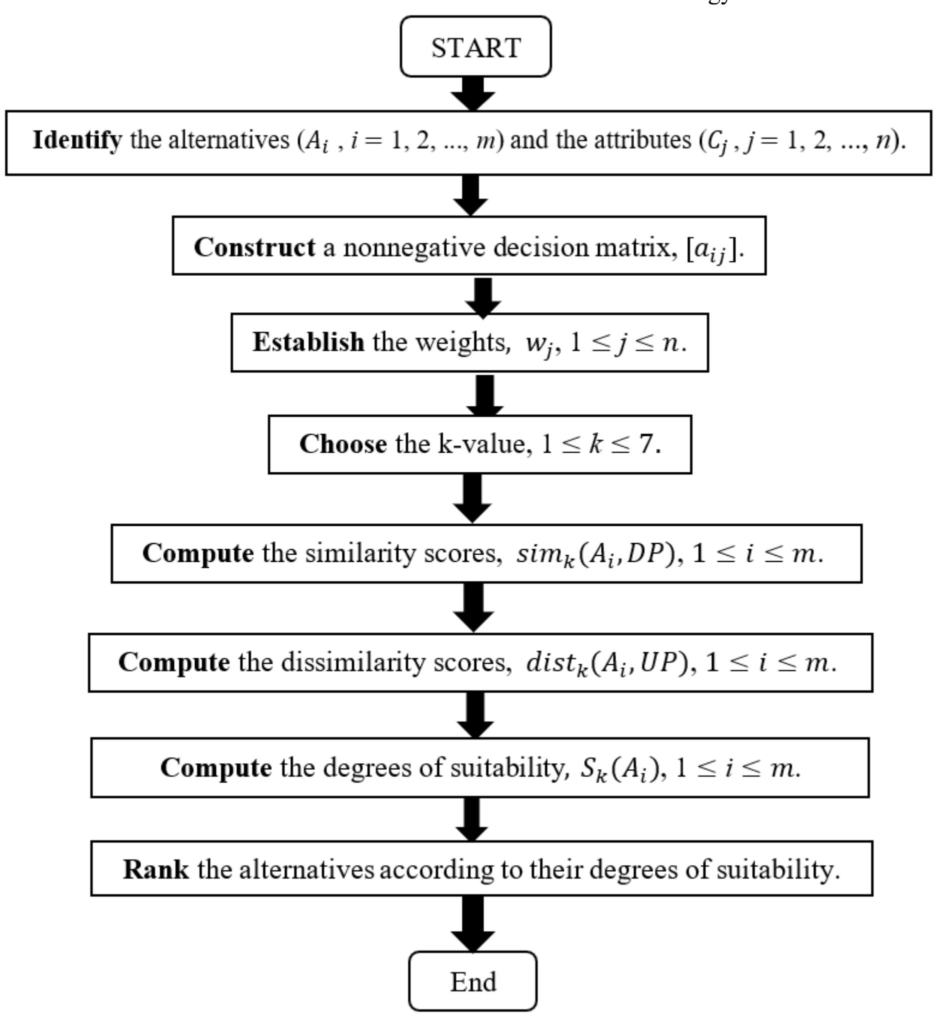

Fig. 1. Flowchart of the WESNORD metrics based methodology

For demonstration purposes, we provide the following logistics service provider (LSP) selection problem adapted from the paper by Hidouri and Rebaï (2019).

4. Illustrative example

4.1 Problem description

The problem at hand is to rank thirteen competing logistics service providers (LSPs) (P_i, $i = 1$ to 13). Each LSP is evaluated in terms of his ratings according to five attributes. The five attributes are Responsiveness (C_1), Price (C_2), Delivery time (C_3), Services (C_4), and Quality (C_5). The respective attribute weights are $w_1 = 0.50$, $w_2 = 0.20$, $w_3 = 0.15$, $w_4 = 0.10$, and $w_5 = 0.05$. The LSPs ratings are expressed in the same unitless scale from 0 (worst) to 10 (best). Table 1 below shows the ratings assigned to the thirteen LSPs.

Table 1
Ratings of the LSPs with respect to the attributes

Attribute	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}
C_1	9	0	1	7	0	1	5	8	8	5	7	5	0
C_2	8	6	7	10	6	6	7	8.5	8.5	7	6	6	5
C_3	9	0	2	5	0	1	5	5	8	1	0	0	0
C_4	5	0	0	8	0	8	3	7	6	1	0	0	0
C_5	5	0	0	9	1	1	9	8	8	1	7	7	5

As said earlier, after using the WESNORD metrics based RPs to rank the thirteen competing LSPs (from most to least suitable), we will compare the resulting rankings to the reference ranking produced by applying the SAW method with ''raw'' (not normalized) ratings of the LSPs, using the following three rankings similarity coefficients (see Shekhovtsov and Kołodziejczyk, 2020):

- Spearman coefficient (r_S);
- Weighted Spearman coefficient (r_W);
- Rank similarity coefficient (WS).

4.2 Ranking results analysis

The ranking results obtained are summarized in Table 2.

Table 2
Rankings produced by the different RPs and SAW

LSP	$dist_1$	$dist_2$	$dist_3$	$dist_4$	$dist_5$	$dist_6$	$dist_7$	SAW
P_1	1	1	1	1	1	1	1	1
P_2	13	12	13	13	13	13	13	13
P_3	10	10	10	10	10	10	10	10
P_4	3	3	3	3	3	3	3	4
P_5	11	11	11	11	11	11	11	11
P_6	8	9	9	9	9	9	9	9
P_7	5	5	5	5	5	5	5	5
P_8	4	4	4	4	4	4	4	3
P_9	2	2	2	2	2	2	2	2
P_{10}	6	7	6	6	6	7	6	7
P_{11}	7	6	7	7	7	6	7	6
P_{12}	9	8	8	8	8	8	8	8
P_{13}	12	13	12	12	12	12	12	12

Based on Table 2, it can be indicated that all seven of these WESNORD metrics lead to the same most suitable LSP, P_1 and that at the same time, the LSP P_9 receives the second place, but additionally, except for the weighted Gower distance, the indication of the least suitable LSP is the same.

As said, to measure the resulting rankings similarity to the SAW-produced ranking, the following rank measures of correlation will be employed:

i) The Spearman coefficient (r_S) defined as:

$$r_S = 1 - \frac{6 \sum d_i^2}{m\,(m^2 - 1)} \tag{13}$$

where d_i is the difference in rankings for each object i, $i \in \{1, 2, ..., m\}$.

ii) The weighted Spearman coefficient (r_W) expressed as

$$r_W = 1 - \frac{6 \sum_{i=1}^{m} (Rx_i - Ry_i)^2 ((m - Rx_i + 1) + (m - Ry_i + 1))}{m\,(m^3 + m^2 - m - 1)} \tag{14}$$

Rx_i and Ry_i are defined as the position in the ranking of the ith element in ranking x and ranking y respectively.

iii) The rank similarity coefficient (WS) given by

$$WS = 1 - \frac{\sum_{i=1}^{m} 2^{-x_i} |Rx_i - Ry_i|}{max\{|1 - Rx_i| \,;\, |m - Rx_i|\}} \tag{15}$$

Table 3 below summarizes the correlation coefficients values obtained.

Table 3
Correlation coefficients between the rankings provided by the RPs and the reference ranking

WESNORD metric	r_S	r_W	WS
$dist_1$	0.901	0.976	0.976
$dist_2$	0.956	0.980	0.980
$dist_3$	0.956	0.977	0.978
$dist_4$	0.956	0.977	0.978
$dist_5$	0.956	0.977	0.978
$dist_6$	0.993	0.982	0.981
$dist_7$	0.956	0.977	0.978

From Table 3, we can obviously see that all seven WESNORD metrics perform very well. Besides, the weighted Canberra distance yields the least similarity values than the other ones, but the average of weighted Gower and Wave-Hedges distances leads to the best results. Further, the weighted Gower distance leads to the second best results. Finally, for all the remaining distance metrics, the results are similar to each other. In sum, the values of the similarity coefficients used ascertain the soundness and effectiveness of all seven WESNORD metrics as a useful tool for ranking alternatives.

5. Conclusions

To conclude, preliminary normalization in multi-attribute decision-making (MADM) is a necessary and unavoidable stage of the decision process of many methods. Nevertheless, a number of authors have pointed out significant pitfalls associated with normalization methods. One major pitfall, which has been pointed out is that normalization methods may lead to different final rankings of alternatives when a ranking procedure based on them is used for solving a MADM problem. We have investigated, in this work, whether three given primary WESNORD metrics and their averages can be employed effectively in MADM. To ascertain and illustrate the effectiveness of WESNORD metrics based RPs, we have solved a logistics service provider (LSP) selection problem. The obtained results show that the seven RPs used produce final rankings of alternatives, which are very similar to the reference ranking provided by the popular SAW method. The two avenues envisaged for future research involve:

- Carrying out a thorough performance comparison among WESNORD metrics based ranking procedures;
- Applying the WESNORD metrics based ranking procedures to solve real-world MADM problems in various practical fields.

Conflict of interests

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Funding

This research did not receive any specific from of funding.

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